BL12 Impedance Considerations

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1 Resistive Wall Impedance

- The macro-particle transverse equation of motion

$$\frac{d^2 y(s)}{ds^2} + K(s)\, y(s) = -\frac{N r_0}{\gamma C} \sum_{k=1}^{\infty} y(s - kC)\, W_1(-kC)$$

when transformed to normal coordinates becomes

$$\ddot{w} + \nu_\beta^2 w = -\frac{N r_0}{2\pi\gamma} \nu_\beta \sum_{k=1}^{\infty} w(\phi - 2\pi k)\ \langle \beta W_1(-kC) \rangle$$

where the wake function is weighted by the local β function.

- Changing the scale of the frequency from ν_β to $\omega_\beta = \nu_\beta \omega_0$ one obtains the complex frequency shift

$$\begin{aligned} \Omega - \omega_\beta \ &= \ -i \frac{1}{2} \frac{N r_0}{\gamma T_0^2} \sum_{p=-\infty}^{\infty} \left\langle \beta Z_1^{\perp}(p\omega_0 + \Omega) \right\rangle \\ &= \ -i \frac{I_T}{2(E/e)} \frac{1}{T_0} \sum_{p=-\infty}^{\infty} \left\langle \beta Z_1^{\perp}(p\omega_0 + \Omega) \right\rangle \end{aligned}$$

and the growth rate

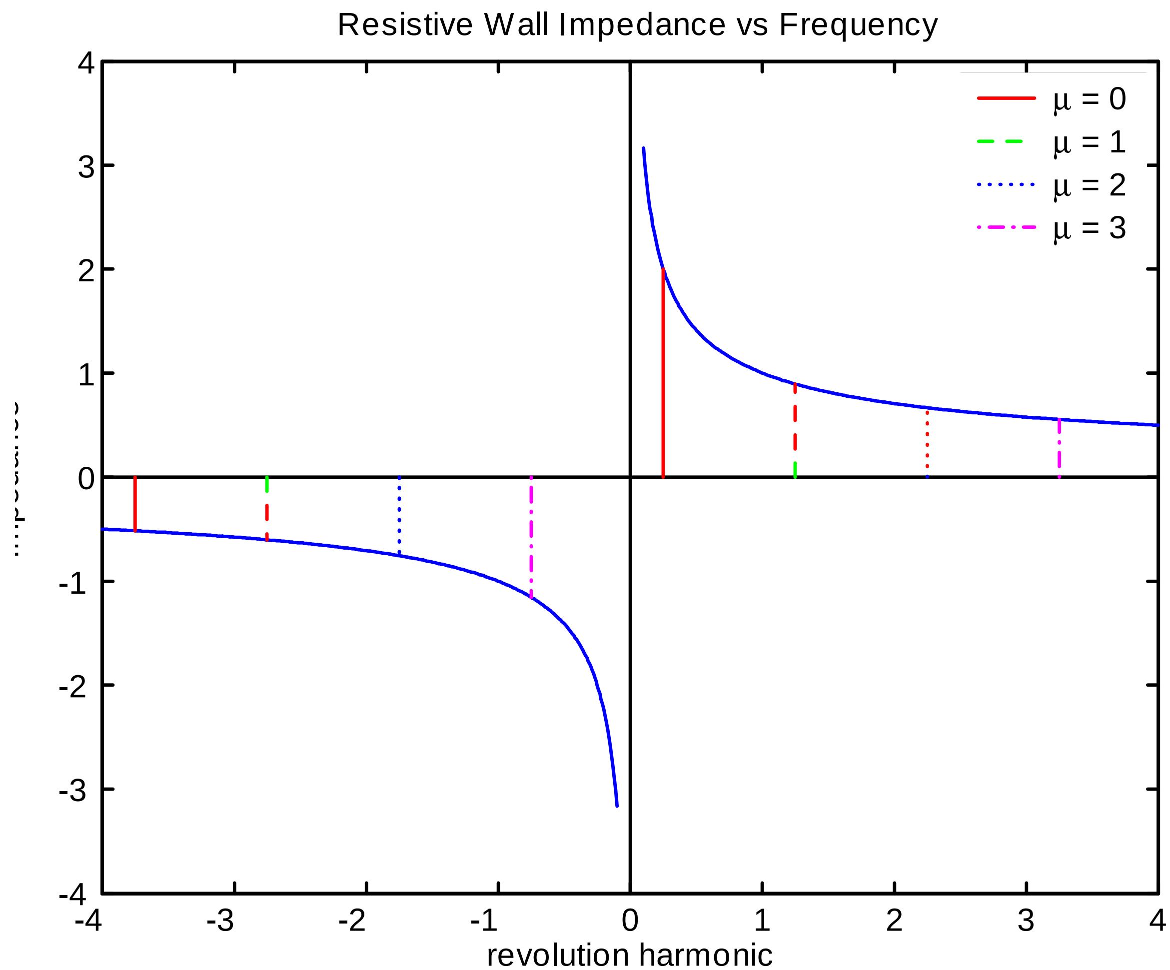
$$\tau^{-1} = -\frac{I_T}{2\,(E/e)\,T_0} \frac{1}{T_0}\sum_{p=-\infty}^{\infty} \mathrm{Re}\left\{\left\langle \beta Z_1^{\perp}\left(p\omega_0 + \Omega\right)\right\rangle\right\}$$

- The resistive wall impedance for a cylindrical chamber of radius, b, is

$$\frac{Z_1^{\perp}(\omega)}{L} \approx \frac{1 - \mathrm{sgn}(\omega)\, i}{2\pi b^3} Z_0 \delta$$

  where $Z_0 = 377\,\Omega$ and $\delta = \sqrt{\frac{2}{\mu\omega\sigma}}$ is the electrical skin depth of the chamber
  - Multiply $Z_1^{\perp}(\omega)$ by the geometric factor of $\pi^2/12$ to obtain the vertical parallel plate impedance

- Impedance scaling
  - $Z_1^{\perp}(\omega)$ varies with chamber radius b as b^{-3}
  - $Z_1^{\perp}(\omega)$ varies with conductivity σ as $\sigma^{-\frac{1}{2}}$
  - $Z_1^{\perp}(\omega)$ varies with frequency ω as $\omega^{-\frac{1}{2}}$



Some contributions of the resistive wall impedance for a four bunch fill pattern with a vertical betatron tune of 0.25.

- BL12 ID impedance
  - Stainless steel chamber with $5\,\mathrm{mm}$ gap has the equivalent impedance of standard vacuum chambers for 3 SPEAR rings
  - Must reduce the conductivity to that of Cu to reduce the impedance by a factor of 6.5

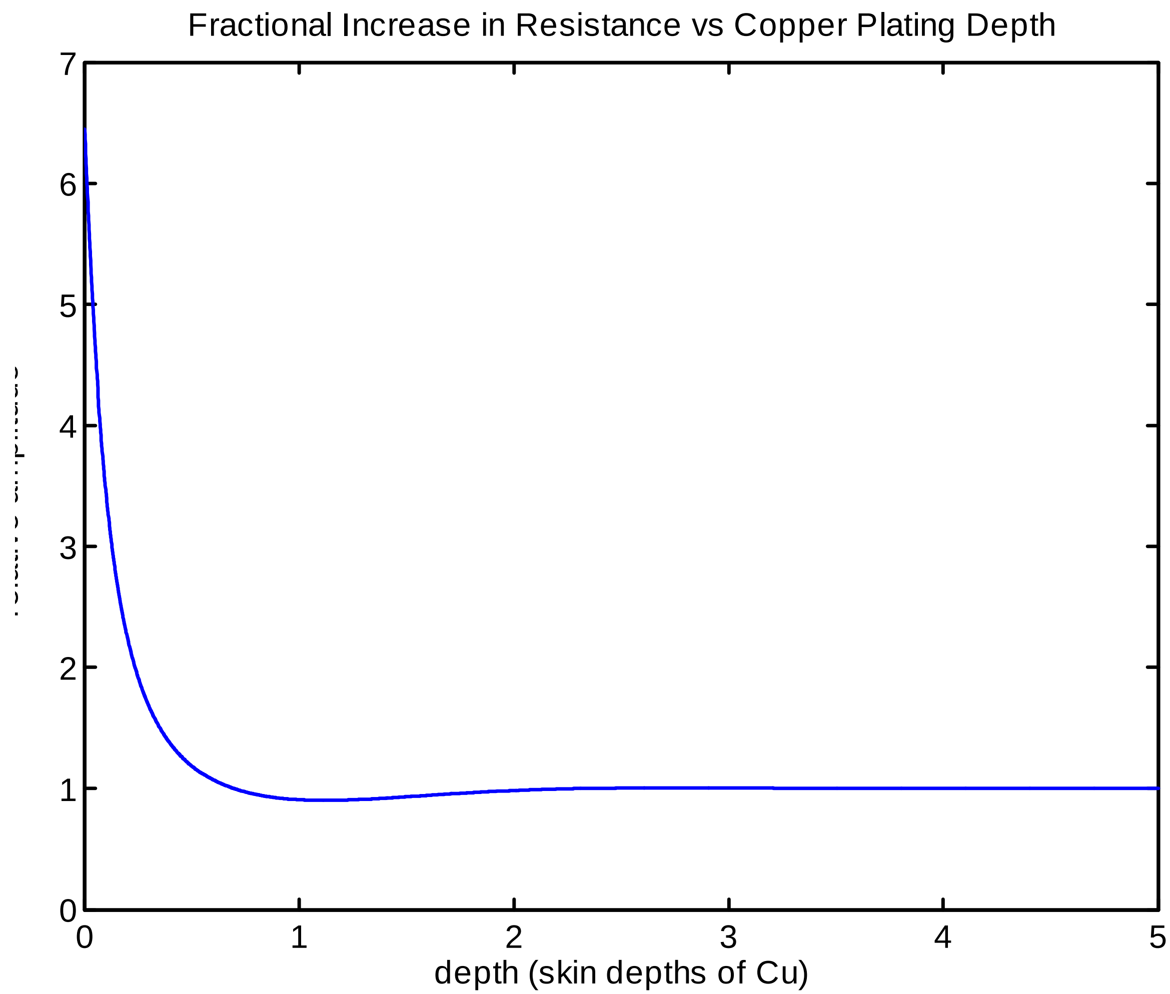
2 Plating

2.1 Plating Thickness

- If a thickness d of metal of conductivity σ_1 is plated on a metal of conductivity σ_2, the beam-generated longitudinal electric field, $\tilde{E}_s$, on the metal is

$$\tilde{E}_s = -\frac{(1-i)}{2}\sqrt{\frac{\omega}{2\pi\sigma_1}}\frac{2q}{b}\frac{\left(1+\sqrt{\frac{\sigma_2}{\sigma_1}}\right)+\left(1-\sqrt{\frac{\sigma_2}{\sigma_1}}\right)e^{2(i-1)d/\delta_1}}{\left(1+\sqrt{\frac{\sigma_2}{\sigma_1}}\right)-\left(1-\sqrt{\frac{\sigma_2}{\sigma_1}}\right)e^{2(i-1)d/\delta_1}}.$$

- The impedance of the compound wall has the same dependence on σ_1, σ_2, and d.
- Only about $66\,\mu\mathrm{m}$ of copper plating, one δ at $1\,\mathrm{MHz}$, is needed to obtain the desired ID resistive wall impedance



Decrease of resistive wall impedance, at a fixed frequency, of a stainless steel chamber as the plating thickness of copper increases.

2.2 Plating Width

- The Green function for the electric field generated by a charge between two parallel plates is

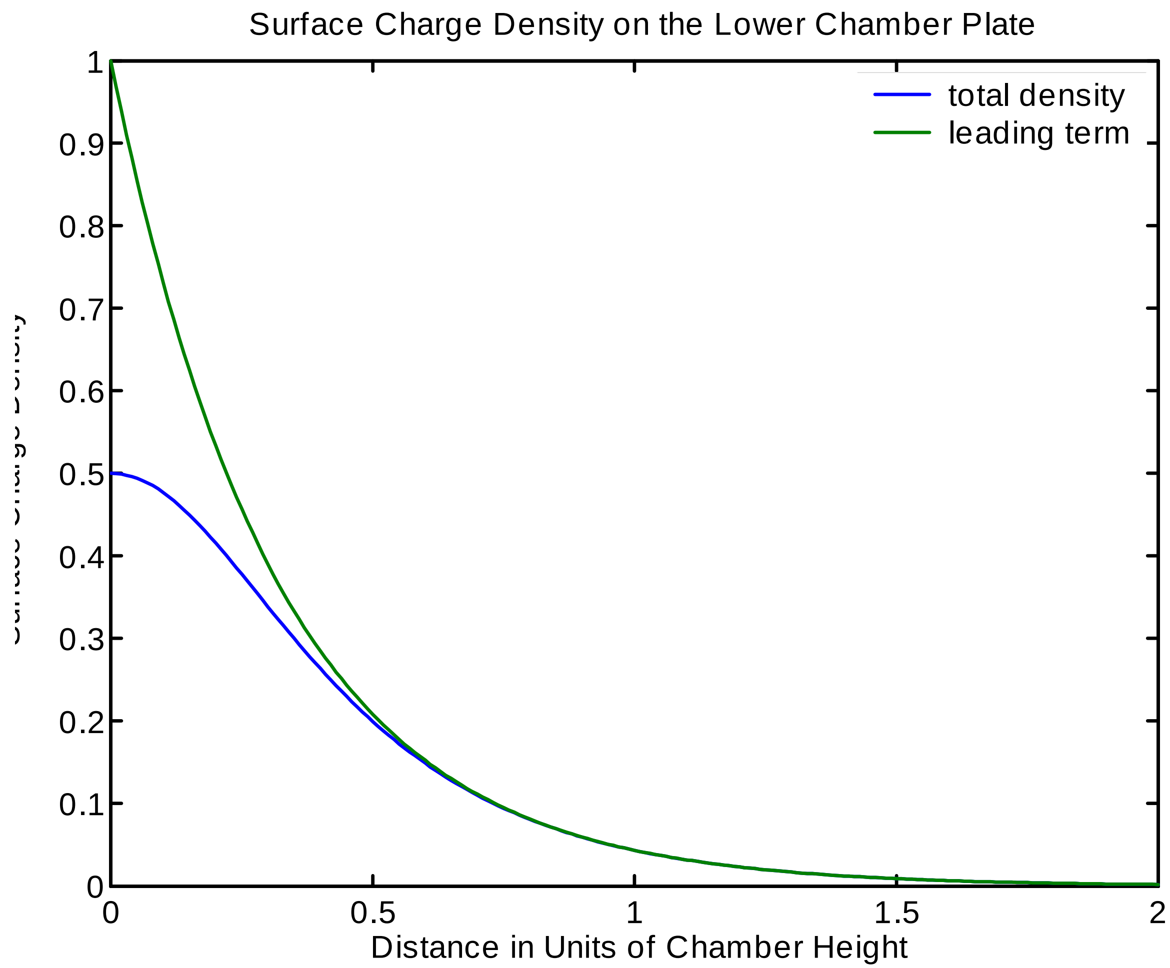
$$G\left(x,y;x',y'\right)=\sum_{k=1}^{\infty}\frac{4}{k}e^{-\frac{k\pi}{d}|x-x'|}\sin\frac{k\pi y}{d}\sin\frac{k\pi y'}{d}.$$

- The surface charge distribution on the plates is

$$\begin{aligned}\sigma\left(x,0;x',y'\right) &= -\frac{1}{4\pi}\frac{\partial}{\partial y}G\left(x,y;x',y'\right)\Big|_{y=0}\\ &= -\frac{1}{d}\sum_{k=1}^{\infty}e^{-\frac{k\pi}{d}|x-x'|}\sin\frac{k\pi y'}{d}\\ &= -\frac{1}{d}\frac{e^{-\frac{\pi}{d}|x-x'|}\sin\frac{\pi y'}{d}}{1-2e^{-\frac{\pi}{d}|x-x'|}\cos\frac{\pi y'}{d}+e^{-\frac{2\pi}{d}|x-x'|}}.\end{aligned}$$

  which drops off exponentially fast away from the source

- Plating width will be greater than four times the maximum gap opening



Surface charge density distribution for positive surface charge. The source particle is located at $(x, y) = (0, d/2)$ and the horizontal position is normalized in units of d. The leading term of this expansion is also plotted.

3 Spear Resistive Wall Contributions

BL	gap cm	len m	mat	β_y m	$Z_1^{\perp}(\omega_0)/L$ $\mathrm{k\Omega/m^2}$	$Z_1^{\perp}(\omega_0)$ $\mathrm{k\Omega/m}$	$\beta_y Z_1^{\perp}(\omega_0)$ $\mathrm{k\Omega}$
4	1.48	2.34	Cu	4.8	13.4	33.0	159
5	1.96	2.04	Al	4.8	3.9	7.9	38
6	1.19	2.46	Cu	4.8	13.4	33.0	159
7	1.19	2.46	Cu	4.8	13.4	33.0	159
9	1.78	2.27	ss	4.8	26.3	59.6	286
10	1.96	2.25	ss	4.8	19.7	44.3	213
11	1.48	2.40	ss	4.8	45.7	109.7	527
12	0.50	1.83	Cu	1.77	181.1	330.9	587
straights	3.40	28.0	Cu	10.0	0.6	11.7	117
vac cham	3.40	188.0	Cu	10.0	0.6	108.3	1083

- Total $\beta_y Z_1^{\perp}(\omega_0)$ around the ring is $3346\,\mathrm{k\Omega}$
- BL12 accounts for 18% of the total resistive wall contribution

4 Stability

- Relevant machine parameters
  - $\omega_0 = 2\pi \times 1.28\,\mathrm{MHz}$
  - $\nu_y = 6.22$
- Instability thresholds (at zero chromaticity)
  - $269\,\mathrm{mA}$ for existing configuration
  - $223\,\mathrm{mA}$ including BL12
- Calculated chromaticity (normalized) needed to stabilize beam (based on $1\,\Omega$, $Q = 1$ BBR model centered around $15\,\mathrm{GHz}$)
  - 0.09 for existing configuration
  - 0.13 for ring with BL12
- No instabillites seen at $200\,\mathrm{mA}$ with zero chromaticity during intial run gives some confidence to these calculations

5 Broad Band Impedance

- Taper from $34\,\mathrm{cm}$ to $5\,\mathrm{cm}$ extends over more than $35\,\mathrm{cm}$
- Slope is less than $1:20$
- ABCI calculations
  - Longitudinal impedance
    * $270\,\mathrm{pH}$
    * $k_{Loss} = 3.2\,\mathrm{mV}/\,\mathrm{pC}$
  - Transverse $(m = 1)$ impedance
    * $Z_1^{\perp}(\omega) \sim\ \mathrm{k}\Omega/\,\mathrm{m}$

6 BL12 Power Loss Calculations

- Power loss formula (MKS)

$$\frac{P_{tot}}{L} \approx \frac{1}{(2\pi)^2}\Gamma\left(\frac{3}{4}\right) T_0 \frac{I_{tot}^2}{N} \frac{1}{b\sigma_t^{3/2}} \sqrt{\frac{\mu_0}{2\sigma}}$$

- Operational modes
  - Normal running is $500\,\mathrm{mA}$ in 279 bunches
  - Possible "single bunch" mode of $150\,\mathrm{mA}$ in 6 bunches
- Normal total resistive wall power loss along BL12 chamber wall is $19\,\mathrm{W}$
- "Single bunch" power loss is $81\,\mathrm{W}$

7 Other Considerations (Bane/Krinsky 1993)

7.1 Tune Shift Across ID

- Maximum tune shift from resistive wall wake

$$\Delta\nu = \frac{Q}{4\pi\left(E/e\right)}\beta W_1^{\perp} l$$

where

$$W_1^{\perp} \approx \frac{2}{\pi b^3}\sqrt{\frac{c}{4\pi\varepsilon_0\sigma}}\sqrt{\frac{2}{\pi\sigma_z}}$$

- For head tail stability, $\Delta\nu < \nu_s = 8.2 \times 10^{-3}$
- Aggressive current
  - $25\,\mathrm{mA} = 1.2 \times 10^{11} e^{-} \times f_0$
- Calculated tune shift

$$\begin{aligned}\Delta\nu &\approx 1.7 \times 10^{-4} \\ &\ll \nu_s\end{aligned}$$

7.2 Emittance Blow-up

- Calculate vertical kick due to ID wake

$$\Delta y' = \frac{Q}{(E/e)} W_1^{\perp} l \, y$$

- Compare with angular divergence of
  - beam of $\sim 90\mu$rad
  - radiation of $\sim 12\mu$rad
- Use aggressive parameters
  - $Q = 1.2 \times 10^{11} e^-$
  - $y = 1\,\mathrm{mm}$
- Calculated angular kick

$$\Delta y' \approx 1.2\mu\text{rad}$$