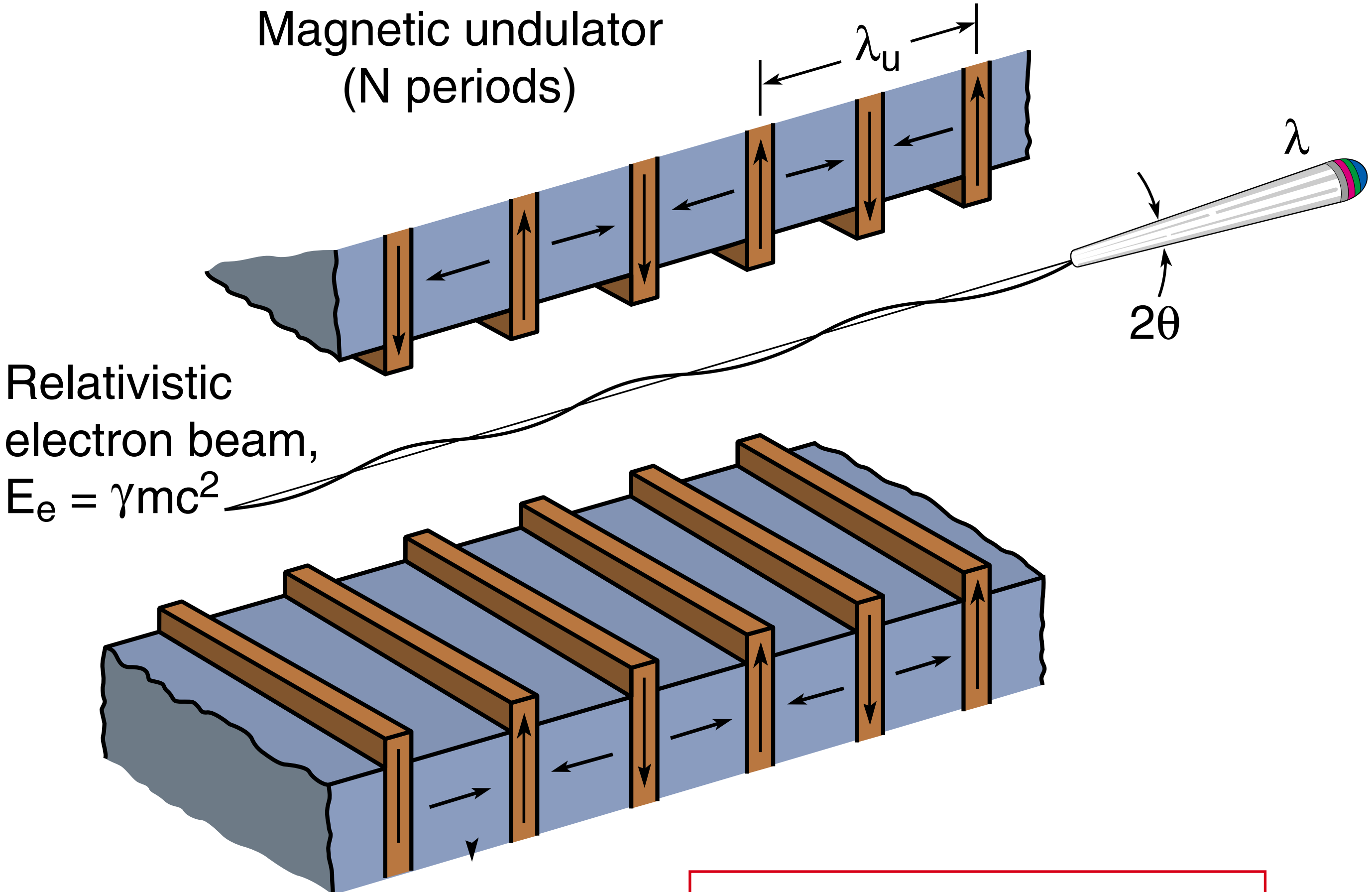
Undulator Radiation, Coherence, and Applications

David Attwood

University of California, Berkeley
and
Center for X-Ray Optics
Lawrence Berkeley National Laboratory

www.coe.berkeley.edu/AST/sxreuv

Narrow Cone Undulator Radiation, Generated by Relativistic Electrons Traversing a Periodic Magnet Structure



$$\lambda \simeq \frac{\lambda_u}{2\gamma^2}$$

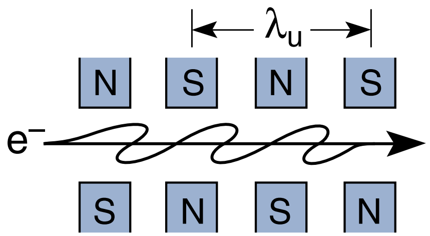
$$\theta_{cen} \simeq \frac{1}{\gamma\sqrt{N}}$$

$$\left[\frac{\Delta\lambda}{\lambda}\right]_{cen} = \frac{1}{N}$$

$$\boxed{\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)}$$

Undulator Radiation

Laboratory Frame of Reference

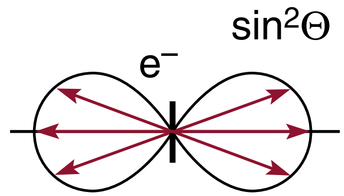


$E = \gamma mc^2$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

N = # periods

Frame of Moving e⁻



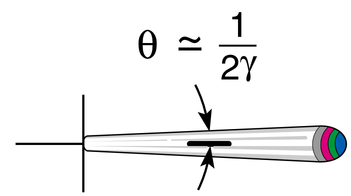
e^- radiates at the Lorentz contracted wavelength:

$$\lambda' = \frac{\lambda_u}{\gamma}$$

Bandwidth:

$$\frac{\lambda'}{\Delta\lambda'} \simeq N$$

Frame of Observer



Doppler shortened wavelength on axis:

$$\lambda = \lambda'\gamma(1 - \beta\cos\theta)$$

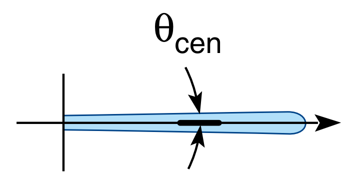
$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + \gamma^2\theta^2)$$

Accounting for transverse motion due to the periodic magnetic field:

$$\boxed{\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)}$$

where $K = eB_0\lambda_u/2\pi mc$

Following Monochromator



For $\frac{\Delta\lambda}{\lambda} \simeq \frac{1}{N}$

$$\theta_{cen} \simeq \frac{1}{\gamma\sqrt{N}}$$

typically

$$\theta_{cen} \simeq 40\ \mu\text{rad}$$

Physically, where does the $\lambda = \lambda_u/2\gamma^2$ come from?

The electron "sees" a Lorentz contracted period

$$\lambda' = \frac{\lambda_u}{\gamma} \tag{5.9}$$

and emits radiation in its frame of reference at frequency

$$f' = \frac{c}{\lambda'} = \frac{c\gamma}{\lambda_u}$$

Observed in the laboratory frame of reference, this radiation is Doppler shifted to a frequency

$$f = \frac{f'}{\gamma(1 - \beta\cos\theta)} = \frac{c}{\lambda_u(1 - \beta\cos\theta)} \tag{5.10}$$

On-axis ($\theta = 0$) the observed frequency is

$$f = \frac{c}{\lambda_u(1 - \beta)}$$

Physically, where does the $\lambda = \lambda_u/2\gamma^2$ come from?

$$f = \frac{c}{\lambda_u(1-\beta)}$$

By definition $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$; $\gamma^2 = \dfrac{1}{(1-\beta)(1+\beta)} \simeq \dfrac{1}{2(1-\beta)}$

thus

$$f = \frac{2\gamma^2 c}{\lambda_u}$$

and the observed wavelength is

$$\lambda = \frac{c}{f} = \frac{\lambda_u}{2\gamma^2} \qquad (5.11)$$

Give examples.

What about the off-axis $\Theta \neq 0$ radiation?

For $\theta \neq 0$, take $\cos\theta = 1 - \dfrac{\theta^2}{2} + \ldots$, then

$$f = \frac{c}{\lambda_u(1 - \beta\cos\theta)} \qquad (5.10)$$

$$f = \frac{c/\lambda_u}{1 - \beta\,(1 - \theta^2/2 + \ldots)} = \frac{c/\lambda_u}{1 - \beta + \beta\theta^2/2 - \ldots} = \frac{c/(1-\beta)\lambda_u}{1 + \beta\theta^2/2(1-\beta)\ldots}$$

$$f = \frac{2\gamma^2 c/\lambda_u}{1 + \gamma^2\theta^2} = \frac{2\gamma^2 c}{\lambda_u(1 + \gamma^2\theta^2)}$$

The observed wavelength is then

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + \gamma^2\theta^2) \qquad (5.12)$$

exhibiting a reduced Doppler shift off-axis, i.e., longer wavelengths.
This is a simplified version of the “Undulator Equation”.

The Undulator's "Central Radiation Cone"

□ With electrons executing N oscillations as they traverse the periodic magnet structure, and thus radiating a wavetrain of N cycles, it is of interest to know what angular cone contains radiation of relative spectral bandwidth

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{N} \tag{5.14}$$

Write the undulator equation twice, once for on-axis radiation ($\theta = 0$) and once for wavelength-shifted radiation off-axis at angle θ:

$$\lambda_0 + \Delta\lambda = \frac{\lambda_u}{2\gamma^2}(1 + \gamma^2\theta^2)$$

$$\lambda_0 = \frac{\lambda_u}{2\gamma^2}$$

divide and simplify to

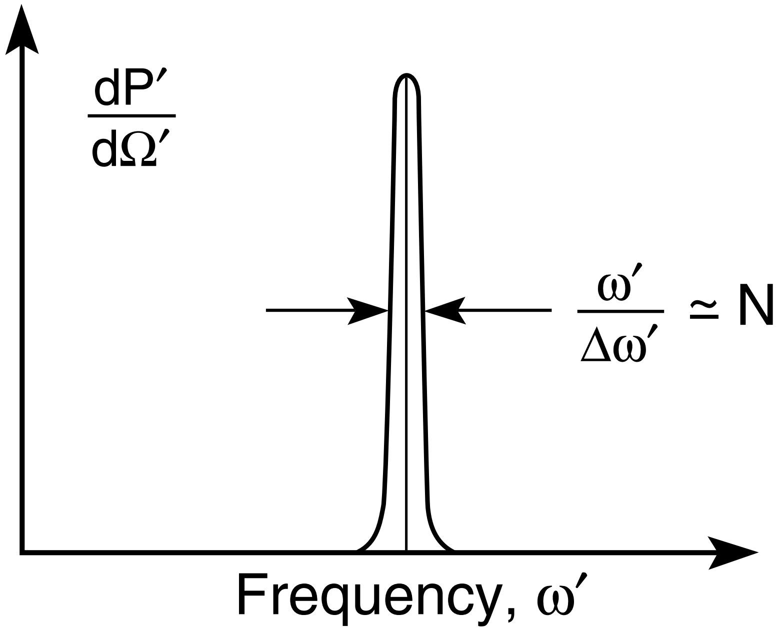
$$\frac{\Delta\lambda}{\lambda} \simeq \gamma^2\theta^2 \tag{5.13}$$

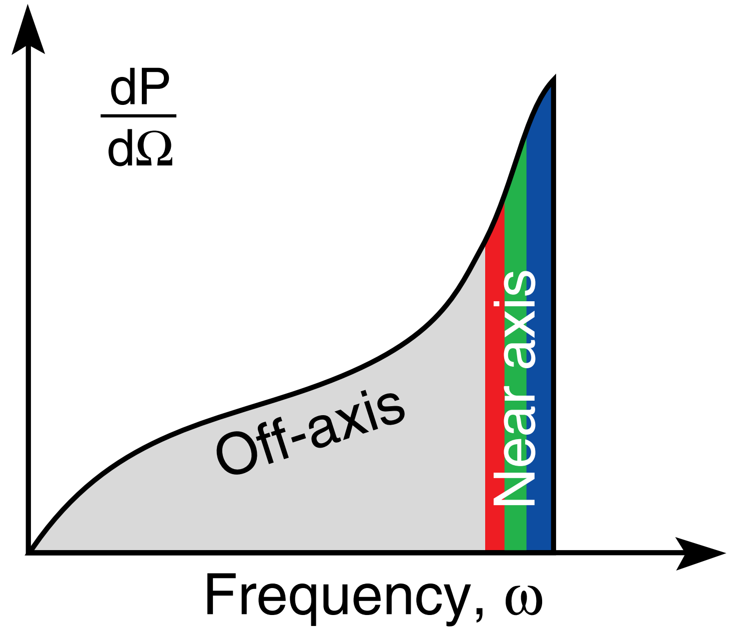
Combining the two equations (5.13 and 5.14)

$$\gamma^2\theta^2_{cen} = \frac{1}{N} \quad \text{, which gives} \quad \boxed{\theta_{cen} \simeq \frac{1}{\gamma\sqrt{N}}} \tag{5.15}$$

This is the half-angle of the "central radiation cone", defined as containing radiation of $\Delta\lambda/\lambda = 1/N$.

The Undulator Radiation Spectrum in Two Frames of Reference



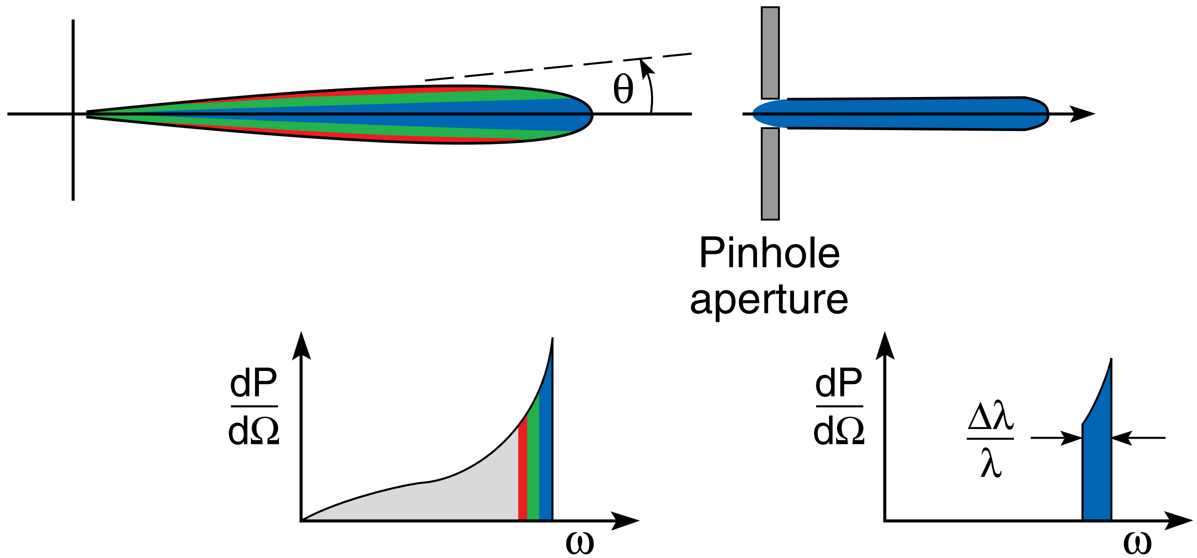


Execution of N electron oscillations produces a transform-limited spectral bandwidth, $\Delta\omega'/\omega' = 1/N$.

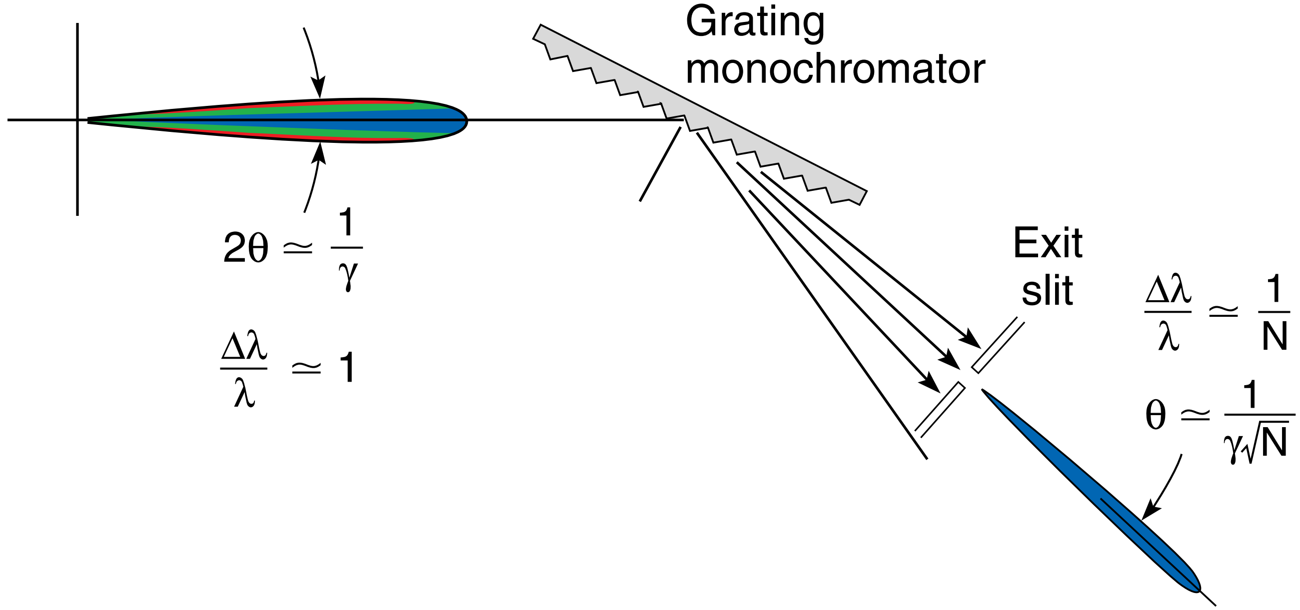
The Doppler frequency shift has a strong angle dependence, leading to lower photon energies off-axis.

The Narrow (1/N) Spectral Bandwidth of Undulator Radiation Can be Recovered in Two Ways

With a pinhole aperture



With a monochromator



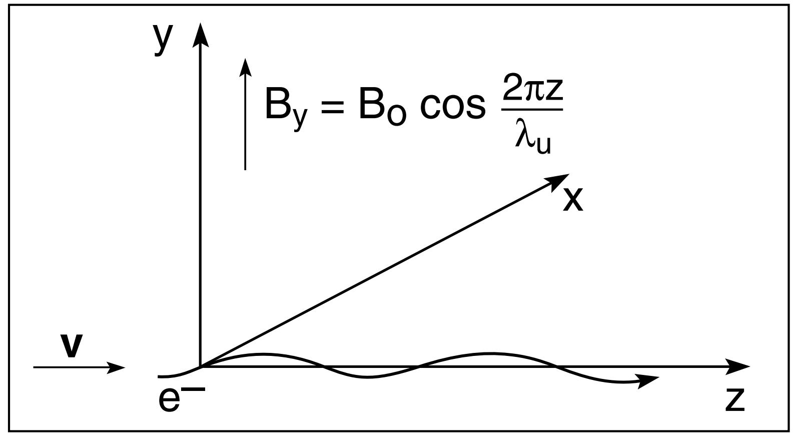
The Equation of Motion in an Undulator

□Magnetic fields in the periodic undulator cause the electrons to oscillate and thus radiate. These magnetic fields also slow the electrons axial (z) velocity somewhat, reducing both the Lorentz contraction and the Doppler shift, so that the observed radiation wavelength is not quite so short. The force equation for an electron is

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (5.16)$$

where $\mathbf{p} = \gamma m\mathbf{v}$ is the momentum. The radiated fields are relatively weak so that

$$\frac{d\mathbf{p}}{dt} \simeq -e(\mathbf{v} \times \mathbf{B})$$



Taking to first order $v \simeq v_z$, motion in the x-direction is

$$m\gamma \frac{dv_x}{dt} = +ev_z B_y$$

$$m\gamma \frac{dv_x}{dt} = e\frac{dz}{dt} \cdot B_0 \cos\left(\frac{2\pi z}{\lambda_u}\right) \qquad (0 \le z \le N\lambda_u)$$

$$m\gamma\, dv_x = e\, dz\, B_0 \cos\left(\frac{2\pi z}{\lambda_u}\right)$$

The Equation of Motion in an Undulator (continued)

$$m\gamma\, d\mathrm{v}_x = e\, dz\, B_0 \cos\left(\frac{2\pi z}{\lambda_u}\right)$$

integrating both sides

$$m\gamma \mathrm{v}_x = e B_0 \frac{\lambda_u}{2\pi} \int \cos\left(\frac{2\pi z}{\lambda_u}\right) \cdot d\left(\frac{2\pi z}{\lambda_u}\right)$$

$$m\gamma \mathrm{v}_x = \frac{e B_0 \lambda_u}{2\pi} \sin\left(\frac{2\pi z}{\lambda_u}\right) \qquad (5.17)$$

$$\boxed{\mathrm{v}_x = \frac{Kc}{\gamma} \sin\left(\frac{2\pi z}{\lambda_u}\right)} \qquad (5.19)$$

$$\boxed{K \equiv \frac{e B_0 \lambda_u}{2\pi mc} = 0.9337\, B_0(\mathrm{T}) \lambda_u(\mathrm{cm})} \qquad (5.18)$$

is the non-dimensional "magnetic deflection parameter."
The "deflection angle", θ, is

$$\theta = \frac{\mathrm{v}_x}{\mathrm{v}_z} \simeq \frac{\mathrm{v}_x}{c} = \frac{K}{\gamma} \sin k_u z$$

The Axial Velocity Depends on K

In a magnetic field γ is a constant; to first order the electron neither gains nor looses energy.

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{\mathrm{v}_x^2+\mathrm{v}_z^2}{c^2}}}$$

thus

$$\frac{\mathrm{v}_z^2}{c^2} = 1 - \frac{1}{\gamma^2} - \frac{\mathrm{v}_x^2}{c^2} \tag{5.22}$$

$$\frac{\mathrm{v}_z^2}{c^2} = 1 - \frac{1}{\gamma^2} - \frac{K^2}{\gamma^2}\sin^2\left(\frac{2\pi z}{\lambda_u}\right)$$

Taking the square root, to first order in the small parameter K/γ

$$\frac{\mathrm{v}_z}{c} = 1 - \frac{1}{2\gamma^2} - \frac{K^2}{2\gamma^2}\sin^2\left(\frac{2\pi z}{\lambda_u}\right) \tag{5.23a}$$

Using the double angle formula $\sin^2 k_u z = (1 - \cos 2k_u z)/2$, where $k_u = 2\pi/\lambda_u$,

$$\frac{\mathrm{v}_z}{c} = \underbrace{1 - \frac{1 + K^2/2}{2\gamma^2}}_{\text{Reduced axial velocity}} + \underbrace{\frac{K^2}{4\gamma^2}\cos\left(2\cdot\frac{2\pi z}{\lambda_u}\right)}_{\text{A double frequency component of the motion}}$$

The first two terms show the reduced axial velocity due to the finite magnetic field (K). The last term indicates the presence of harmonic motion, and thus harmonic frequencies of radiation.

K-Dependent Axial Velocity Affects the Undulator Equation

Averaging the z-component of velocity over a full cycle (or N full cycles) gives

$$\frac{\bar{v}_z}{c} = 1 - \frac{1 + K^2/2}{2\gamma^2} \tag{5.25}$$

We can use this to define an effective Lorentz factor γ^* in the axial direction

$$\gamma^* \equiv \frac{\gamma}{\sqrt{1 + K^2/2}} \tag{5.26}$$

As a consequence, the observed wavelength in the laboratory frame of reference is modified from Eq. (5.12), taking the form

$$\lambda = \frac{\lambda_u}{2\gamma^{*2}}(1 + \gamma^{*2}\theta^2)$$

that is, the Lorentz contraction and relativistic Doppler shift now involve γ^* rather than γ

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)\left(1 + \frac{\gamma^2}{1 + K^2/2}\theta^2\right)$$

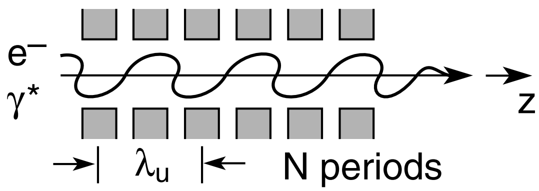
$$\boxed{\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)} \tag{5.28}$$

where $K \equiv e B_o \lambda_u / 2\pi mc$. This is the *undulator equation*, which describes the generation of short (x-ray) wavelength radiation by relativistic electrons traversing a periodic magnet structure, accounting for magnetic tuning (K) and off-axis ($\gamma\theta$) radiation. In practical units

$$\lambda(\text{nm}) = \frac{1.306\lambda_u(\text{cm})\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)}{E_e^2(\text{GeV})} \tag{5.29a}$$

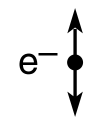
Calculating Power in the Central Radiation Cone: Using the well known "dipole radiation" formula by transforming to the frame of reference moving with the electrons

x, z, t laboratory frame of reference



Lorentz transformation

x', z', t' frame of reference moving with the average velocity of the electron



$$\lambda'_u = \frac{\lambda_u}{\gamma^*}$$

x', z', t' motion
$a'(t')$ acceleration

Determine *x, z, t* motion:

$$\frac{d\mathbf{p}}{dt} = -e\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$m\gamma\,\frac{dv_x}{dt} = e\,\frac{dz}{dt}\,B_0 \cos\frac{2\pi z}{\lambda_u}$$

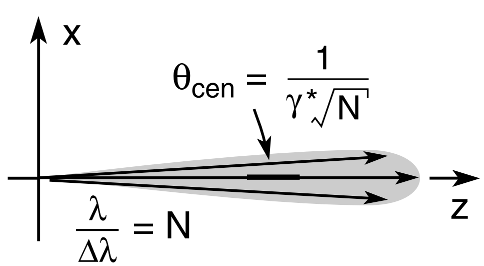
$$\mathrm{v}_x(t);\; a_x(t) = \cdots$$

$$\mathrm{v}_z(t);\; a_z(t) = \cdots$$

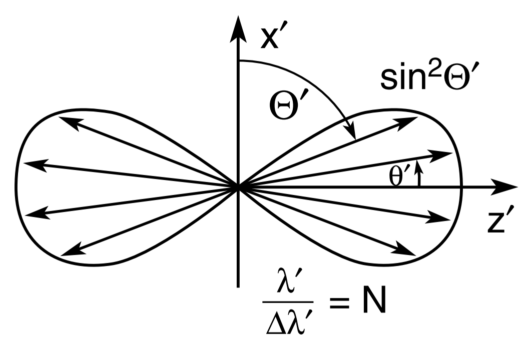
Dipole radiation:

$$\frac{dP'}{d\Omega'} = \frac{e^2 a'^2 \sin^2\Theta'}{16\pi^2\epsilon_0 c^3}$$

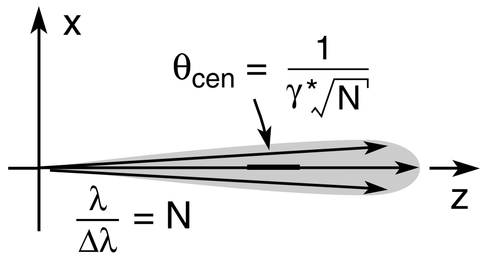
$$\frac{dP'}{d\Omega'} = \frac{e^2 c\gamma^2}{4\epsilon_0\lambda_u^2}\,\frac{K^2}{(1 + K^2/2)^2}\,(1 - \sin^2\theta' \cos^2\phi')\cos^2\omega'_u t'$$



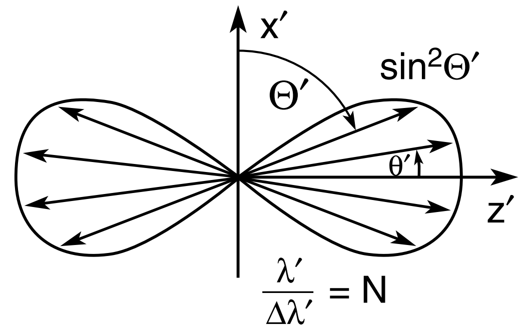
Lorentz transformation



Calculating Power in the Central Radiation Cone: Using the well known "dipole radiation" formula by transforming to the frame of reference moving with the electrons. (cont.)



Lorentz transformation



$$\frac{dP}{d\Omega} = 8\gamma^{*2} \frac{dP'}{d\Omega'}$$

$$\frac{d\bar{P}}{d\Omega} = \frac{e^2 c\gamma^4}{\epsilon_0 \lambda_u^2} \frac{K^2}{(1 + K^2/2)^3} \begin{cases} K \leq 1 \\ \theta \leq \theta_{cen} \end{cases}$$

$$\frac{dP'}{d\Omega'} = \frac{e^2 c\gamma^2}{4\epsilon_0 \lambda_u^2} \frac{K^2}{(1 + K^2/2)^2} (1 - \sin^2\theta' \cos^2\phi') \cos^2\omega_u' t'$$

$$\Delta\Omega_{cen} = \pi\theta_{cen}^2 = \pi/\gamma^{*2} N$$

$$\bar{P}_{cen} = \frac{\pi e^2 c\gamma^2}{\epsilon_0 \lambda_u^2 N} \frac{K^2}{(1 + K^2/2)^2}$$

N_e uncorrelated electrons:

$$N_e = IL/ec, \; L = N\lambda_u$$

$$\boxed{\bar{P}_{cen} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + K^2/2)^2}}$$

Corrections to $\bar{P}_{cen}$ for Finite K

Our formula for calculated power in the central radiation cone ($\theta_{cen} = 1/\gamma^*\sqrt{N}$, $\Delta\lambda/\lambda = 1/N$)

$$\bar{P}_{cen} \simeq \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + K^2/2)^2} \qquad (5.39)$$

is strictly valid for $K << 1$. This restriction is due to our neglect of K^2 terms in the axial velocity v_z. The $\bar{P}_{cen}$ formula, however, indicates a peak power at $K = \sqrt{2}$, suggesting that we explore extension of this very useful analytic result to somewhat higher K values. Kim* has studied undulator radiation for arbitrary K and finds an additional multiplicative factor, $f(K)$, which accounts for energy transfer to higher harmonics:

$$\boxed{\bar{P}_{cen} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + K^2/2)^2} f(K)} \qquad (5.41a)$$

where

$$f(K) = [J_0(x) - J_1(x)]^2 \qquad (5.40a)$$

and

$$x = K^2/4(1 + K^2/2)$$

$$f(K) = 1 - x - \frac{x^2}{4} + \frac{3x^3}{8} + \cdots \qquad (5.40b)$$

K	x	$f(K)$
0	0	1.000
0.5	0.0556	0.944
1.0	0.1667	0.828
$\sqrt{2}$	0.2500	0.740
1.5	0.2647	0.725
2.0	0.3333	0.653
2.5	0.3788	0.606

* K.-J. Kim, "Characteristics of Synchrotron Radiation", pp. 565-632 in *Physics of Particle Accelerators* (AIP, New York, 1989), M. Month and M. Dienes, Editors.
Also see: P.J. Duke, *Synchrotron Radiation* (Oxford Univ. Press, UK, 2000).

Power in the Central Cone

$$\lambda_x = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

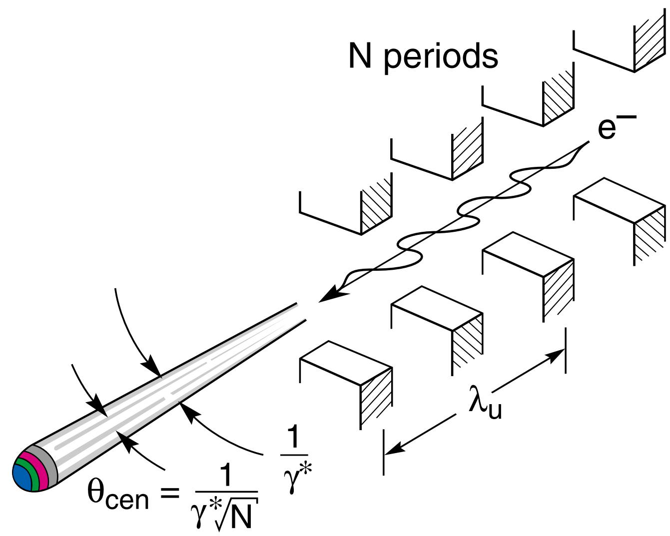
$$\bar{P}_{cen} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{\left(1 + \frac{K^2}{2}\right)^2} f(K)$$

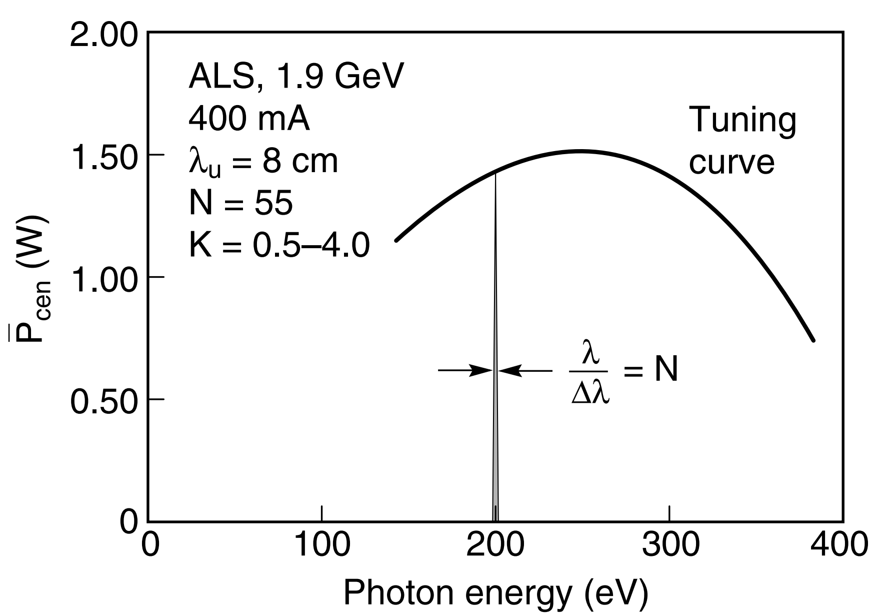
$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{cen} = \frac{1}{N}$$

$$K = \frac{e B_0 \lambda_u}{2\pi m_0 c}$$

$$\gamma^* = \gamma / \sqrt{1 + \frac{K^2}{2}}$$





$\bar{P}_{cen}$ in Terms of Photon Energy

From the undulator equation

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

On axis, $\theta = 0$, and with $f\lambda = c$

$$f = 2\gamma^2 c\,/\lambda_u\left(1 + \frac{K^2}{2}\right)$$

In terms of photon energy (on-axis)

$$\hbar\omega = 4\pi\hbar\gamma^2 c\,/\lambda_u\left(1 + \frac{K^2}{2}\right)$$

We can now replace $K^2/(1 + K^2/2)^2$ in $\bar{P}_{cen}$ by an expression involving $\hbar\omega$. Introducing the limiting photon energy $\hbar\omega_o$, corresponding to $K = 0$,

$$\hbar\omega_o = 4\pi\hbar\gamma^2 c\,/\lambda_u$$

then

$$\boxed{\bar{P}_{cen} = \frac{2\pi e\gamma^2 I}{\epsilon_0\lambda_u}\cdot\frac{\hbar\omega}{\hbar\omega_0}\left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0)} \tag{5.41c}$$

or

$$\bar{P}_{cen} = (1.14\times 10^{-5}\ \text{W})\frac{\gamma^2 I(\text{A})}{\lambda_u(\text{cm})}\cdot\frac{\hbar\omega}{\hbar\omega_0}\left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0) \tag{5.41e}$$

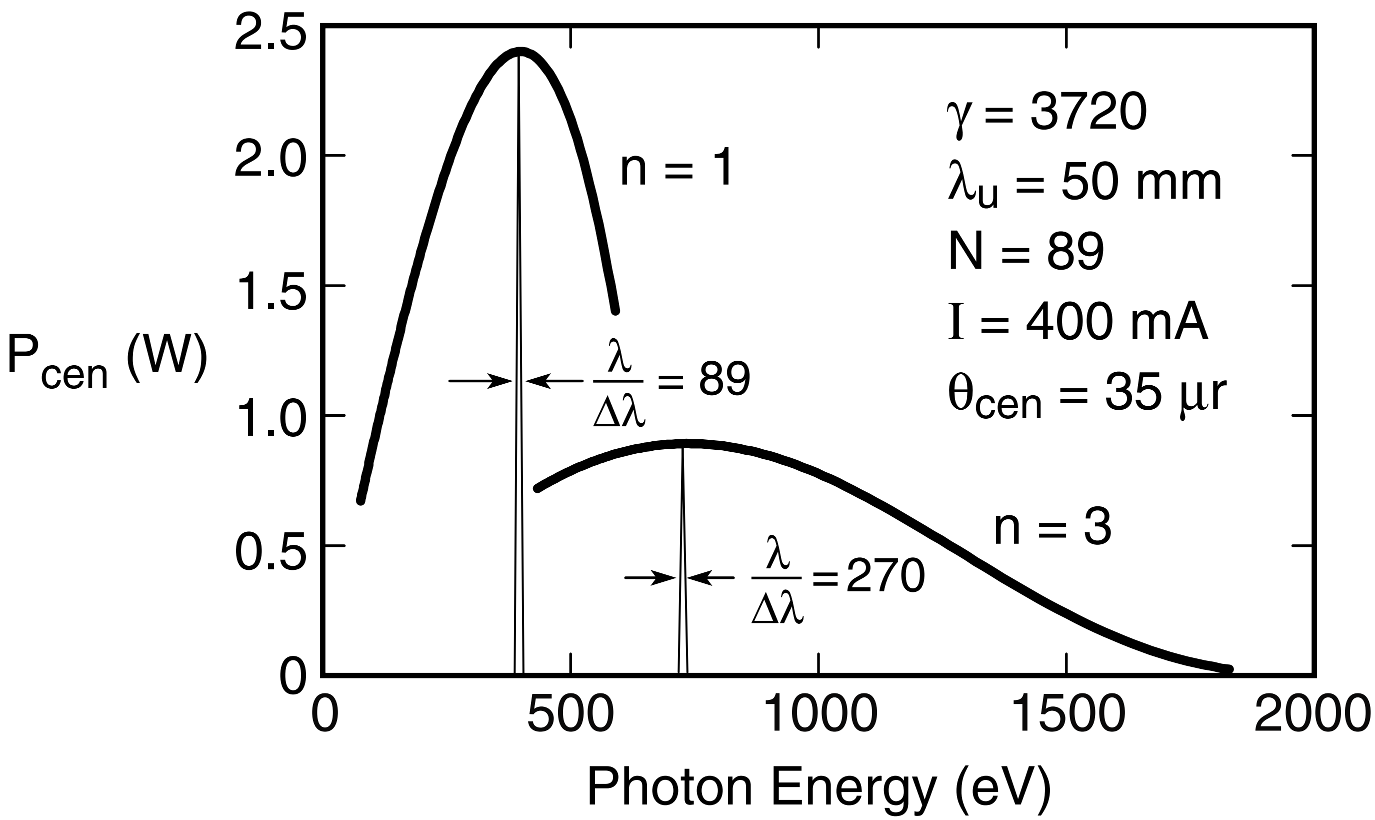
where

$$f(\hbar\omega/\hbar\omega_0) \simeq \frac{7}{16} + \frac{5}{8}\frac{\hbar\omega}{\hbar\omega_0} - \frac{1}{16}\left(\frac{\hbar\omega}{\hbar\omega_0}\right)^2 + \cdots \tag{5.41d}$$

For $\lambda_u = 5.00$ cm and $\gamma = 3720$, $\hbar\omega_o = 686$ eV. For $\lambda_u = 3.30$ cm and $\gamma = 13{,}700$, $\hbar\omega_o = 14.1$ keV

Power in the Central Radiation Cone for an Undulator at the ALS

U5

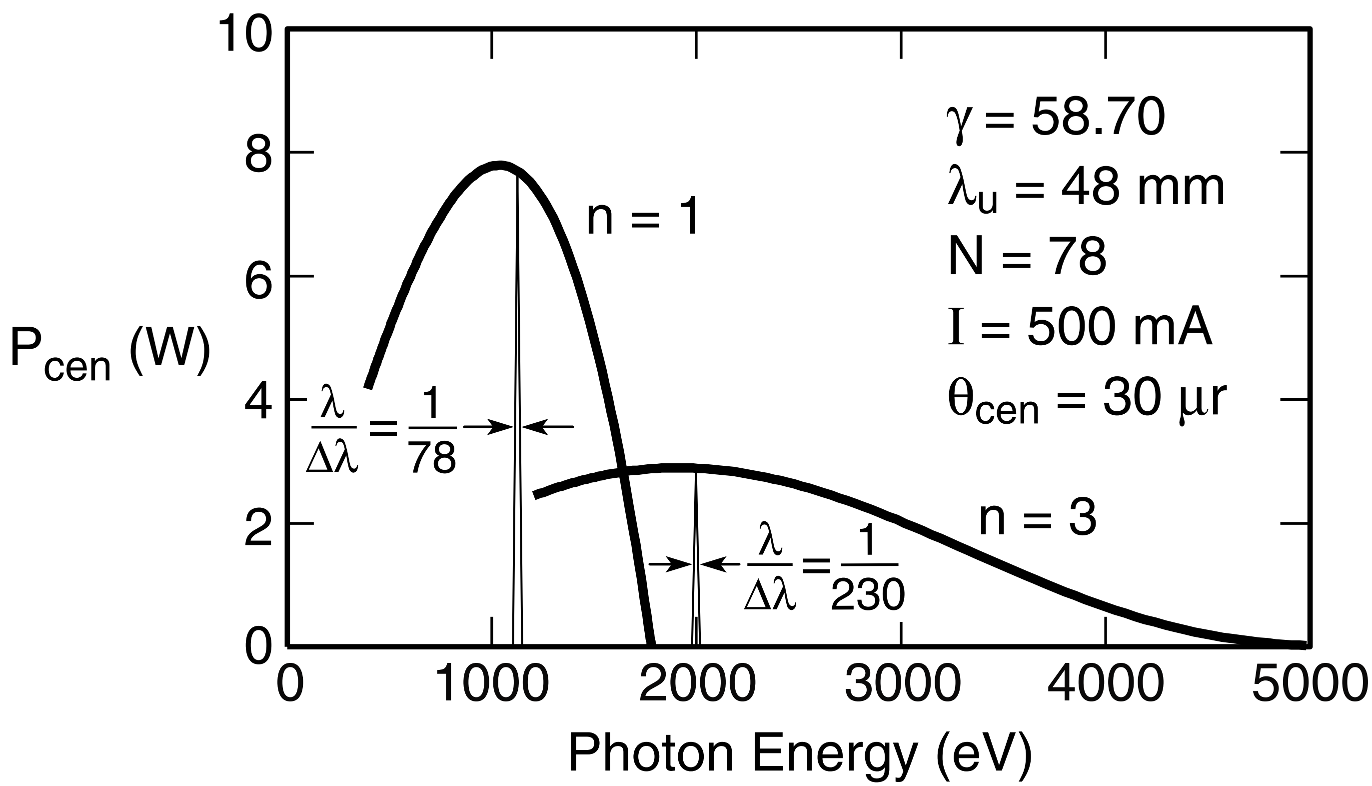


$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left[\frac{\Delta\lambda}{\lambda}\right]_1 = \frac{1}{N}$$

$$\left[\frac{\Delta\lambda}{\lambda}\right]_3 = \frac{1}{3N}$$

Power in the Central Radiation Cone for an Undulator at SSRL



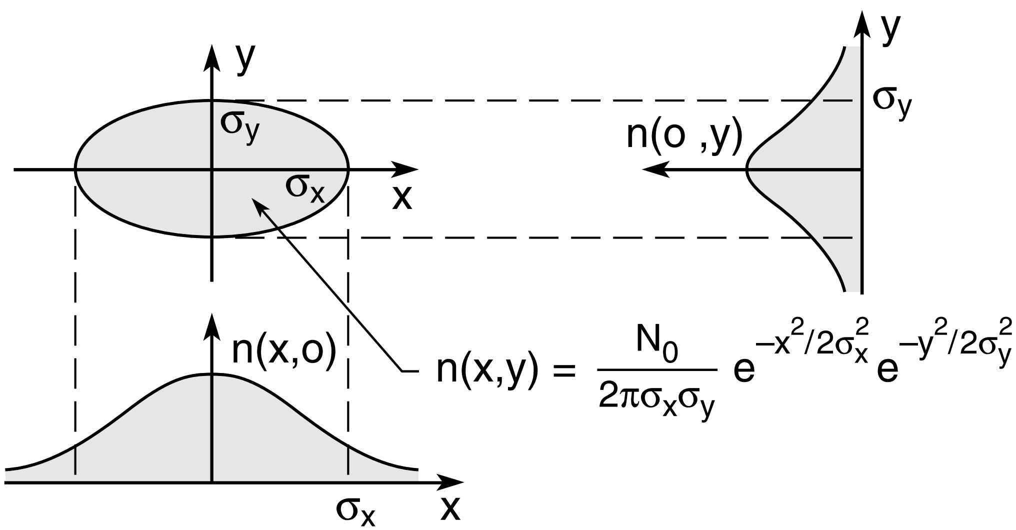
$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$$

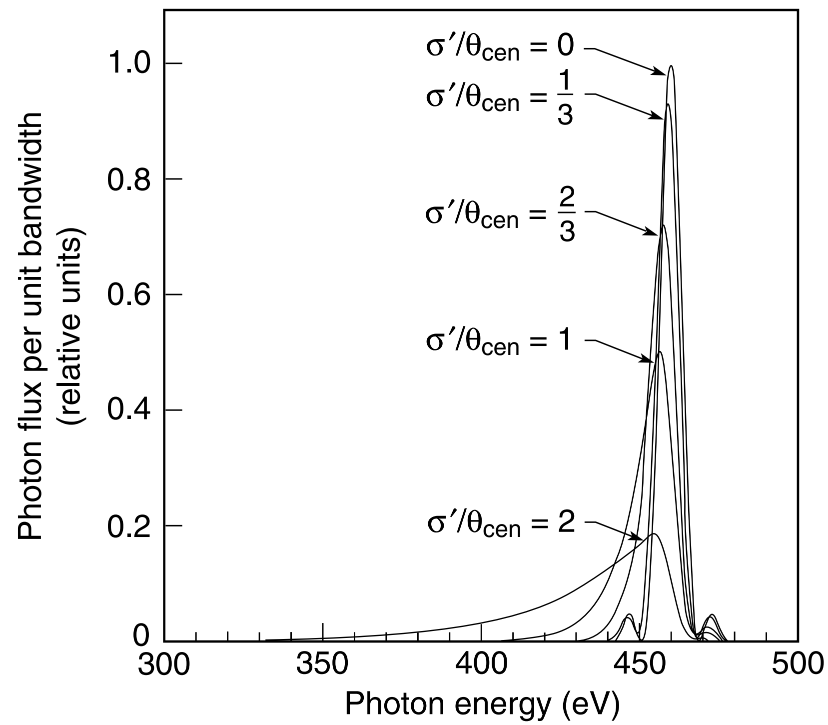
$$\left[\frac{\Delta\lambda}{\lambda}\right]_1 = \frac{1}{N}$$

$$\left[\frac{\Delta\lambda}{\lambda}\right]_3 = \frac{1}{3N}$$

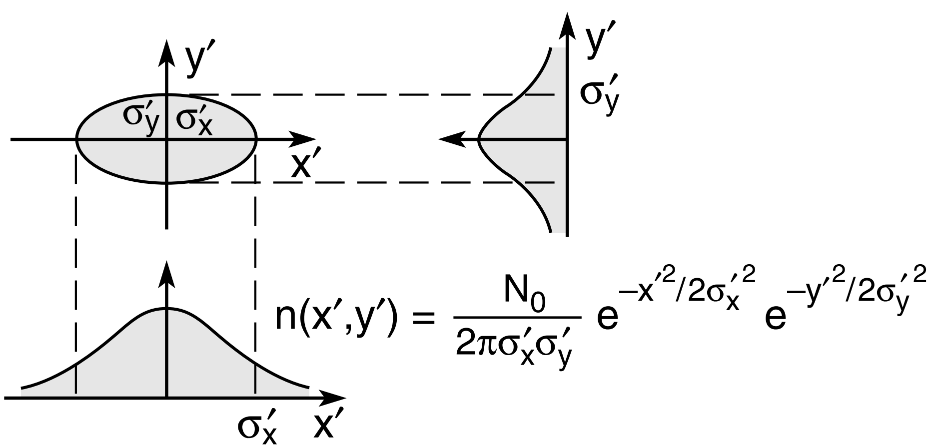
Electron Beam Size and Divergence Affect Brightness and Coherent Power

Beam size: $d = 2\sigma$





Beam angular divergence (σ')



Preserving the spectral line shape of undulator radiation requires

$$\sigma'^2 \ll \theta_{\text{cen}}^2 \qquad (5.55\text{b})$$

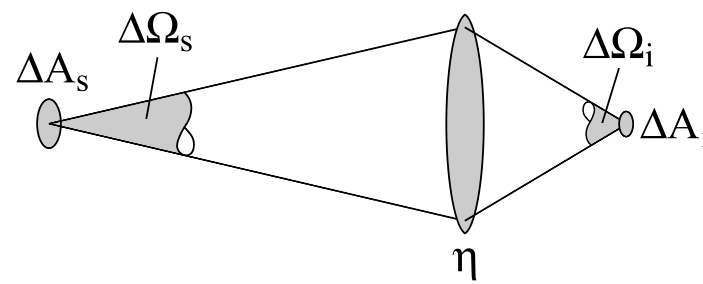
Define effective, or total central cone half-angles

$$\theta_{Tx} = \sqrt{\theta_{\text{cen}}^2 + \sigma_x'^2} \quad \text{and} \quad \theta_{Ty} = \sqrt{\theta_{\text{cen}}^2 + \sigma_y'^2} \qquad (5.56)$$

Brightness and Spectral Brightness

Brightness is defined as radiated power per unit area and per unit solid angle at the source:

$$\boxed{B = \frac{\Delta P}{\Delta A \cdot \Delta \Omega}} \qquad (5.57)$$

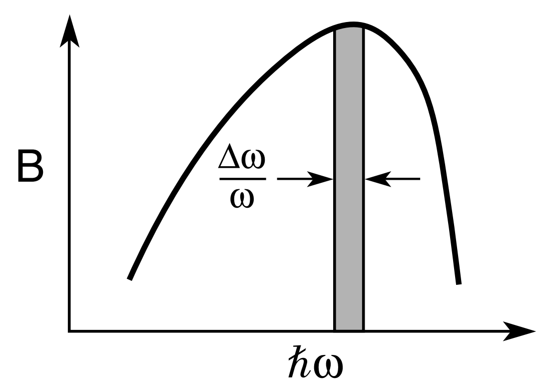


Brightness is a conserved quantity in perfect optical systems, and thus is useful in designing beamlines and synchrotron radiation experiments which involve focusing to small areas.

Perfect optical system:
$\Delta A_s \cdot \Delta\Omega_s = \Delta A_i \cdot \Delta\Omega_i$; $\eta = 100\%$

Spectral brightness is that portion of the brightness lying within a relative spectral bandwidth $\Delta\omega/\omega$:

$$\boxed{B_{\Delta\omega/\omega} = \frac{\Delta P}{\Delta A \cdot \Delta\Omega \cdot \Delta\omega/\omega}} \qquad (5.58)$$



Spectral Brightness of Undulator Radiation

The Synchrotron radiation community prefers to express spectral brightness in units of photons/sec, rather than power, and has standardized on a relative spectral bandwidth of $\Delta\omega/\omega = 10^{-3}$, or 0.1% BW. To obtain a relationship for spectral brightness of undulator radiation we can use our expression for $\bar{P}_{\text{cen}}$, radiated into a solid angle $\Delta\Omega = \pi\theta_{\text{cen}}^2 = \pi\theta_{Tx}\theta_{Ty}$, from an elliptically shaped source area of $\Delta A = \pi\sigma_x\sigma_y$, and within a relative spectral bandwidth $\Delta\omega/\omega = 1/N$. Defining the photon flux in the central radiation cone as

$$\bar{F}_{\text{cen}} = \frac{\bar{P}_{\text{cen}}}{\hbar\omega/\text{photon}} \tag{5.59}$$

$$\bar{B}_{\Delta\omega/\omega} = \frac{\bar{F}_{\text{cen}}}{\Delta A \cdot \Delta\Omega \cdot N^{-1}} = \frac{\bar{F}_{\text{cen}} \cdot (N/1000)}{\Delta A \cdot \Delta\Omega \cdot (0.1\%\text{BW})} \tag{5.60}$$

on-axis

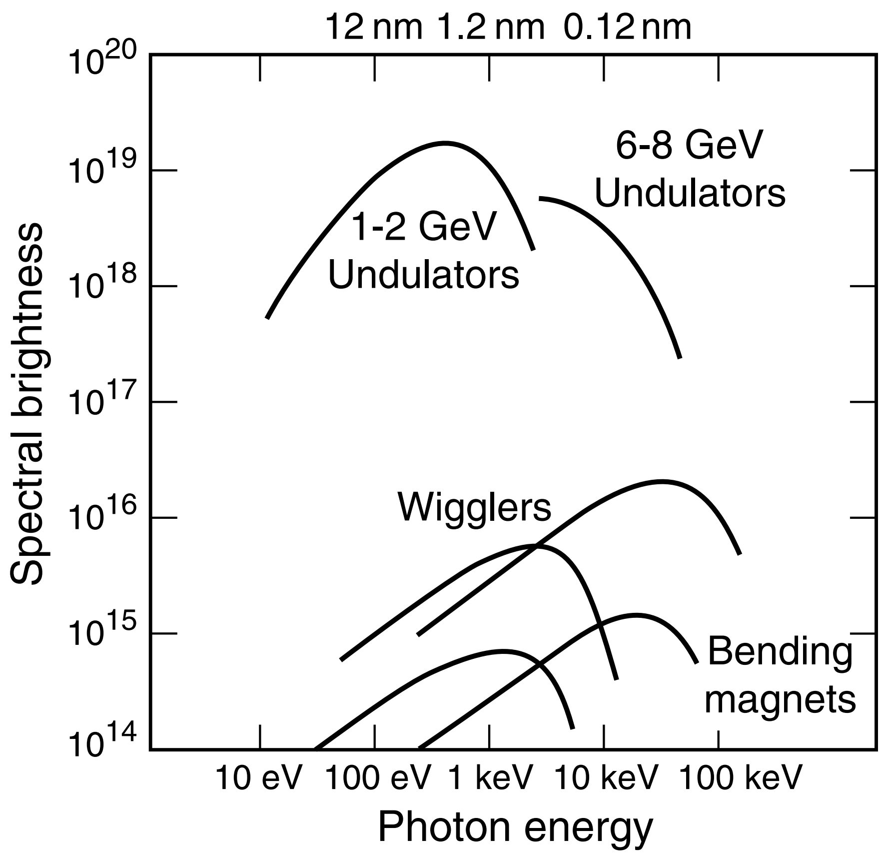
$$\bar{B}_{\Delta\omega/\omega}(0) = \frac{\bar{F}_{\text{cen}} \cdot (N/1000)}{2\pi^2\sigma_x\sigma_y\theta_{Tx}\theta_{Ty}(0.1\%\text{BW})} \tag{5.64}$$

or

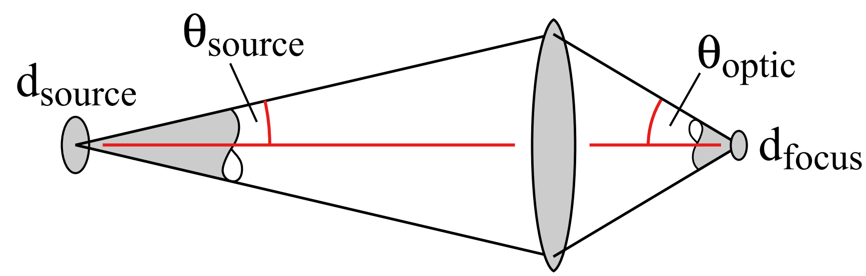
$$\bar{B}_{\Delta\omega/\omega}(0) = \frac{7.25\times 10^6\gamma^2 N^2 I(\text{A})}{\sigma_x(\text{mm})\sigma_y(\text{mm})\left(1+\dfrac{\sigma_x'^2}{\theta_{\text{cen}}^2}\right)^{1/2}\left(1+\dfrac{\sigma_y'^2}{\theta_{\text{cen}}^2}\right)^{1/2}} \cdot \frac{K^2 f(K)}{\left(1+K^2/2\right)^2}\,\frac{\text{photons/s}}{\text{nm}^2\text{mrad}^2(0.1\%\text{BW})} \tag{5.65}$$

Assumes $\sigma'^2 \ll \theta_{\text{cen}}^2$. Note the N^2 factor.

Spectral Brightness is Useful for Experiments that Involve Spatially Resolved Studies



- Brightness is conserved (in lossless optical systems)



$$d_{source} \cdot \theta_{source} = d_{focus} \cdot \theta_{optic}$$

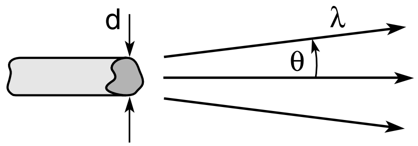
Smaller after focus

Large in a focusing optic

- Starting with many photons in a small source area and solid angle, permits high photon flux in an even smaller area

Ch05_F24VG_6.05.ai

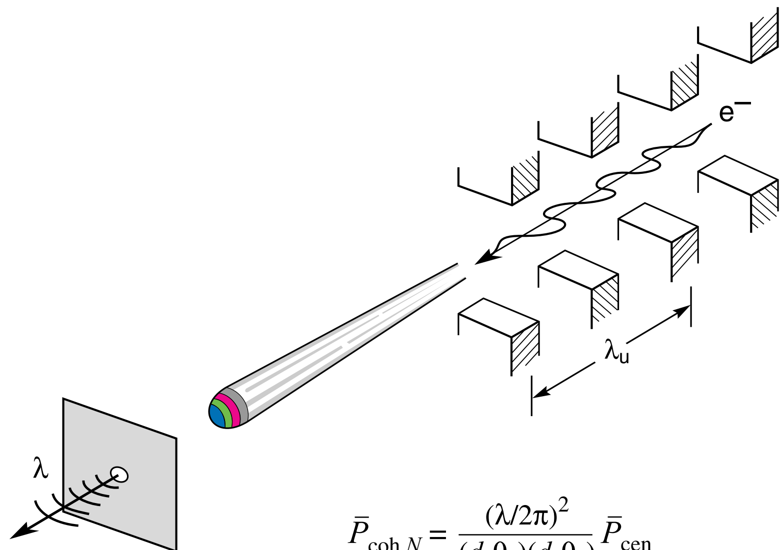
COHERENCE AT SHORT WAVELENGTHS



$$l_{coh} = \lambda^2/2\Delta\lambda \ \{\text{temporal (longitudinal) coherence}\} \qquad (8.3)$$

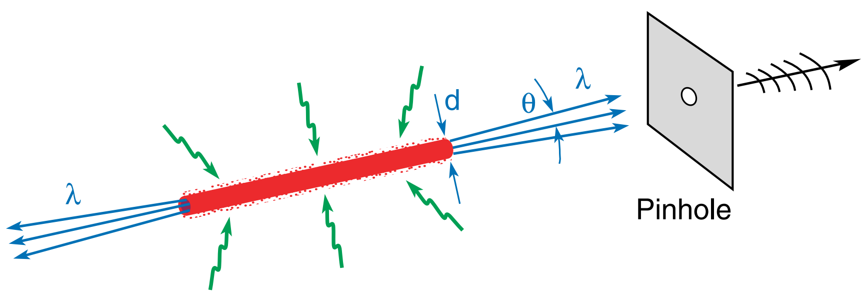
$$d \cdot \theta = \lambda/2\pi \ \{\text{spatial (transverse) coherence}\} \qquad (8.5)$$

$$\text{or} \ \ d \cdot 2\theta|_{FWHM} = 0.44\,\lambda \qquad (8.5^*)$$



$$\bar{P}_{coh,N} = \frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y\theta_y)}\bar{P}_{cen} \qquad (8.6)$$

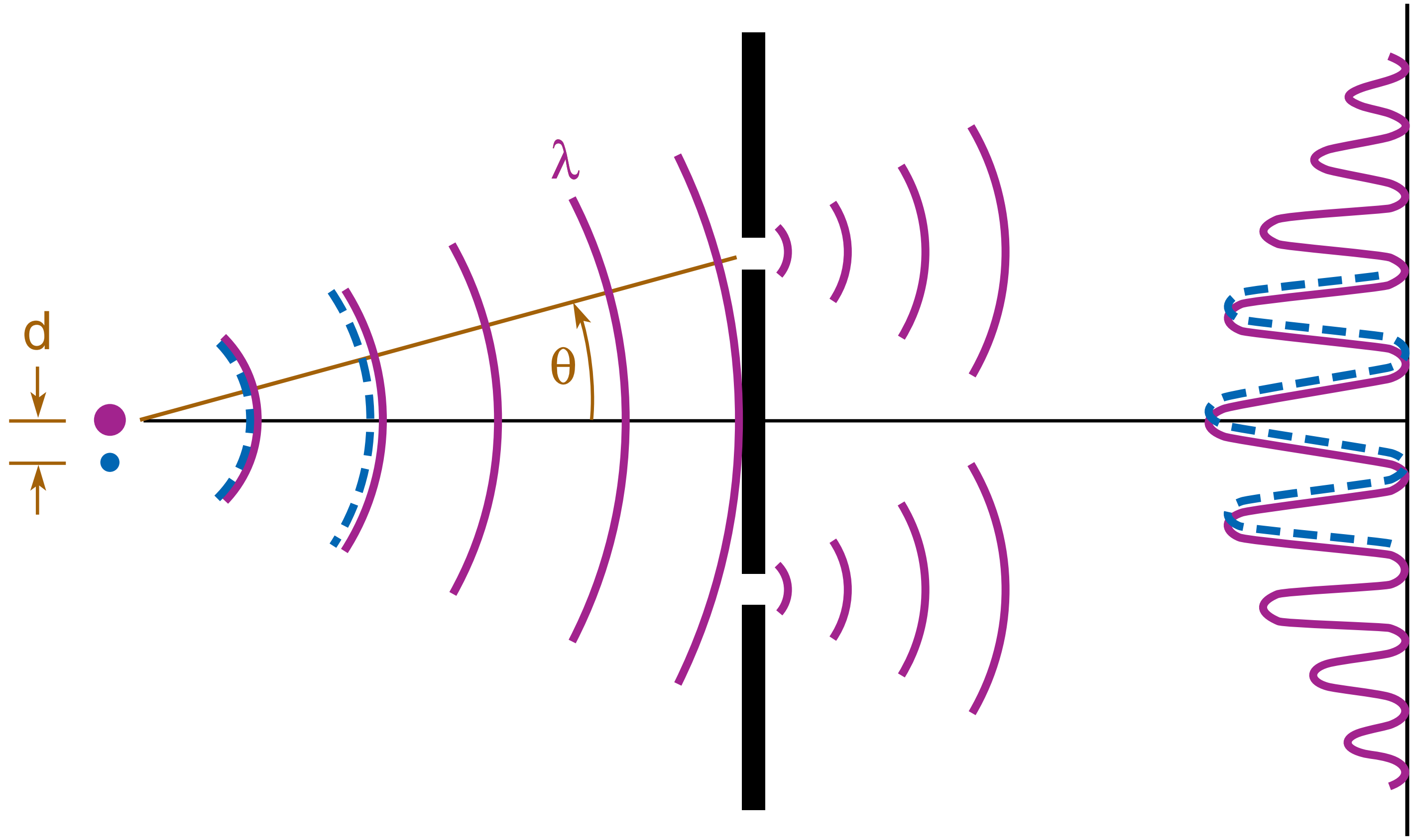
$$\bar{P}_{coh,\lambda/\Delta\lambda} = \frac{e\lambda_u I\eta(\Delta\lambda/\lambda)N^2}{8\pi\epsilon_0 d_x d_y} \cdot \left[1-\frac{\hbar\omega}{\hbar\omega_0}\right] f(K) \qquad (8.9)$$



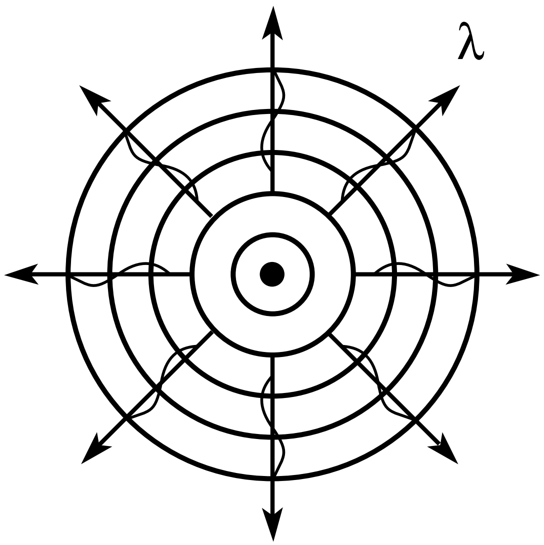
$$P_{coh} = \frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y\theta_y)} P_{laser} \qquad (8.11)$$

Young's Double Slit Experiment: Spatial Coherence and the Persistence of Fringes

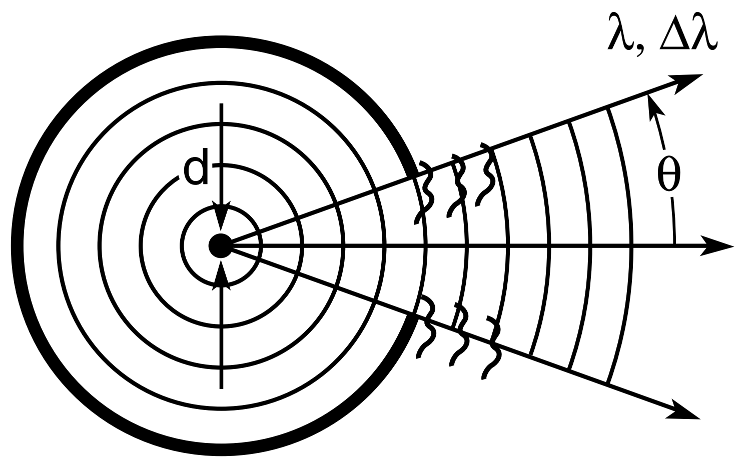
Persistence of fringes as the source grows from a point source to finite size.



Coherence, Partial Coherence, and Incoherence



Point source oscillator
$-\infty < t < \infty$



Source of finite size,
divergence, and duration

Spatial and Temporal Coherence

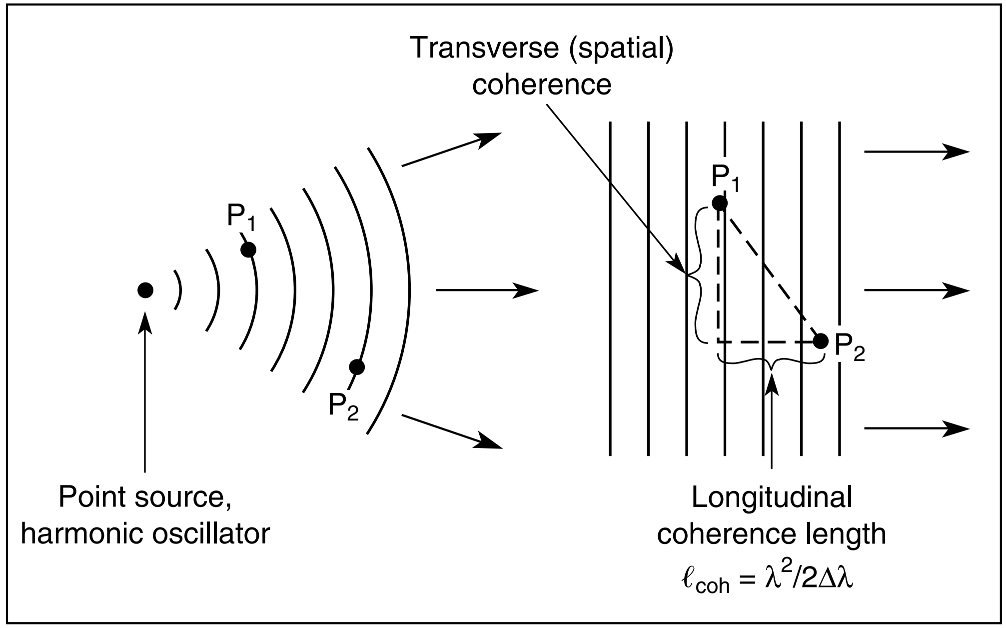
Mutual coherence factor

$$\Gamma_{12}(\tau) \equiv \langle E_1(t+\tau)E_2^*(t)\rangle \qquad (8.1)$$

Normalize degree of spatial coherence (complex coherence factor)

$$\mu_{12} = \frac{\langle E_1(t)E_2^*(t)\rangle}{\sqrt{\langle |E_1|^2\rangle}\sqrt{\langle |E_2|^2\rangle}} \qquad (8.12)$$

A high degree of coherence ($\mu \to 1$) implies an ability to form a high contrast interference (fringe) pattern. A low degree of coherence ($\mu \to 0$) implies an absence of interference, except with great care. In general radiation is partially coherent.



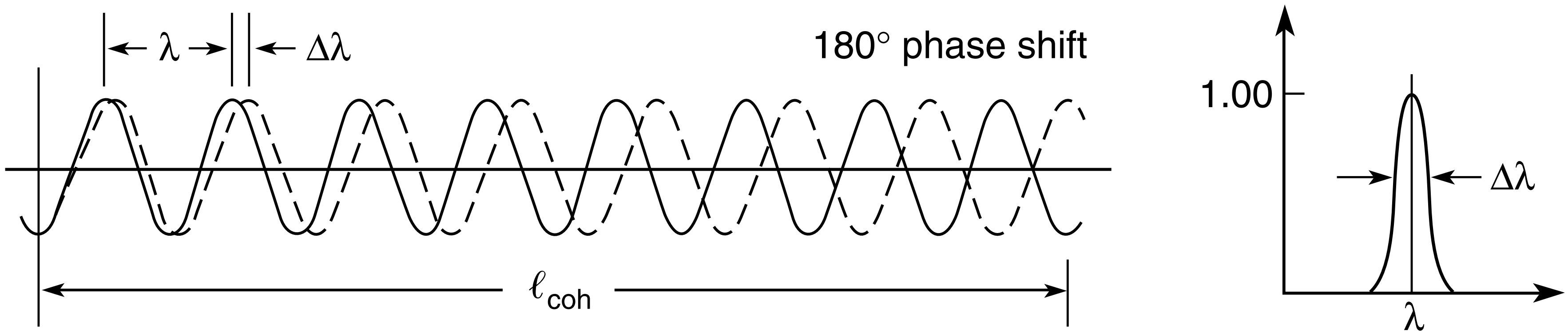
Longitudinal (temporal) coherence length

$$\boxed{\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda}} \qquad (8.3)$$

Full spatial (transverse) coherence

$$\boxed{d \cdot \theta = \lambda/2\pi} \qquad (8.5)$$

Spectral Bandwidth and Longitudinal Coherence Length



Define a coherence length ℓ_{coh} as the distance of propagation over which radiation of spectral width $\Delta\lambda$ becomes 180° out of phase. For a wavelength λ propagating through N cycles

$$\ell_{coh} = N\lambda$$

and for a wavelength $\lambda + \Delta\lambda$, a half cycle less $(N - \frac{1}{2})$

$$\ell_{coh} = (N - \tfrac{1}{2})\,(\lambda + \Delta\lambda)$$

Equating the two

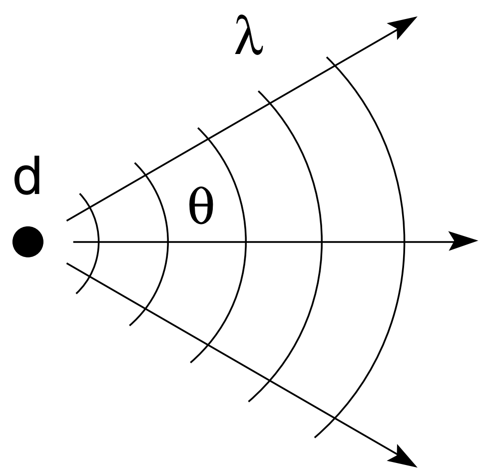
$$N = \lambda/2\Delta\lambda$$

so that

$$\boxed{\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda}} \qquad (8.3)$$

A Practical Interpretation of Spatial Coherence

- **Associate spatial coherence with a spherical wavefront.**
- **A spherical wavefront implies a point source.**
- **How small is a "point source"?**



From Heisenberg's Uncertainty Principle ($\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$), the smallest source size "d" you can resolve, with wavelength λ and half angle θ, is

$$\mathbf{d} \cdot \theta = \frac{\lambda}{2\pi}$$

Partially Coherent Radiation Approaches Uncertainty Principle Limits

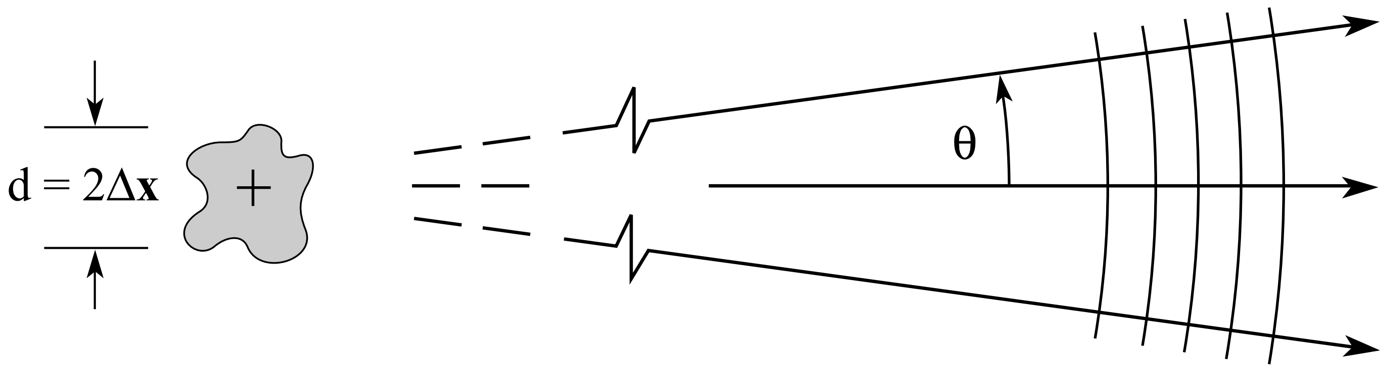
$$\Delta \mathbf{x} \cdot \Delta \mathbf{p} \geq \hbar/2 \qquad (8.4)$$

$$\Delta \mathbf{x} \cdot \hbar \Delta \mathbf{k} \geq \hbar/2$$

$$\Delta \mathbf{x} \cdot \mathbf{k} \Delta \theta \geq 1/2$$

$$2\Delta \mathbf{x} \cdot \Delta \theta \geq \lambda/2\pi$$

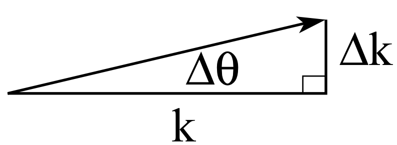
Standard deviations of Gaussian distributed functions (Tipler, 1978, pp. 174-189)



Spherical wavefronts occur in the limiting case

Note:

$$\Delta \mathbf{p} = \hbar \Delta k$$
$$\Delta k = k \Delta \theta$$

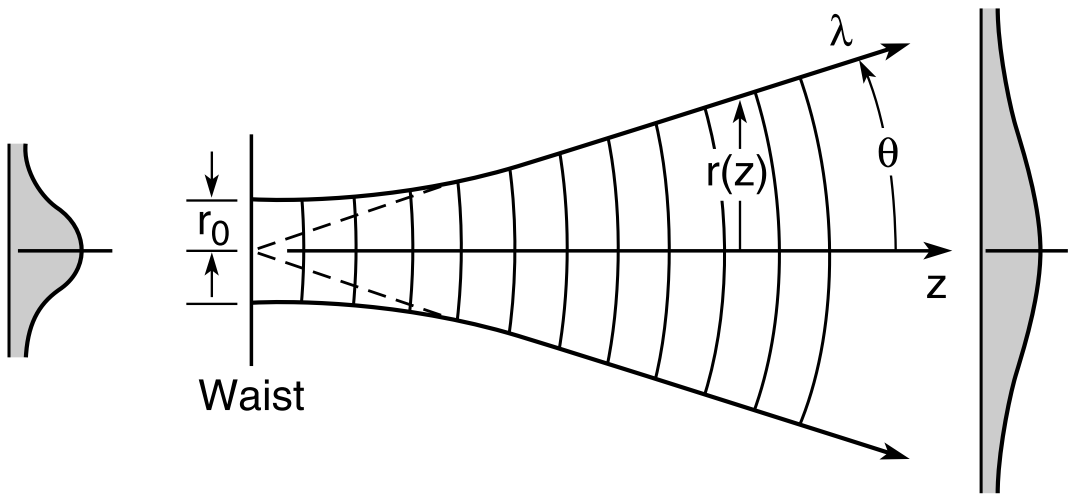


$$\left.\begin{array}{c} \boxed{d \cdot \theta = \lambda/2\pi} \\ \text{(spatially coherent)} \end{array}\right\} \frac{1}{\sqrt{e}} \text{ quantities}$$

or

$$(d \cdot 2\theta)_{FWHM} \simeq \lambda/2 \;\} \text{ FWHM quantities}$$

Propagation of a Gaussian Beam



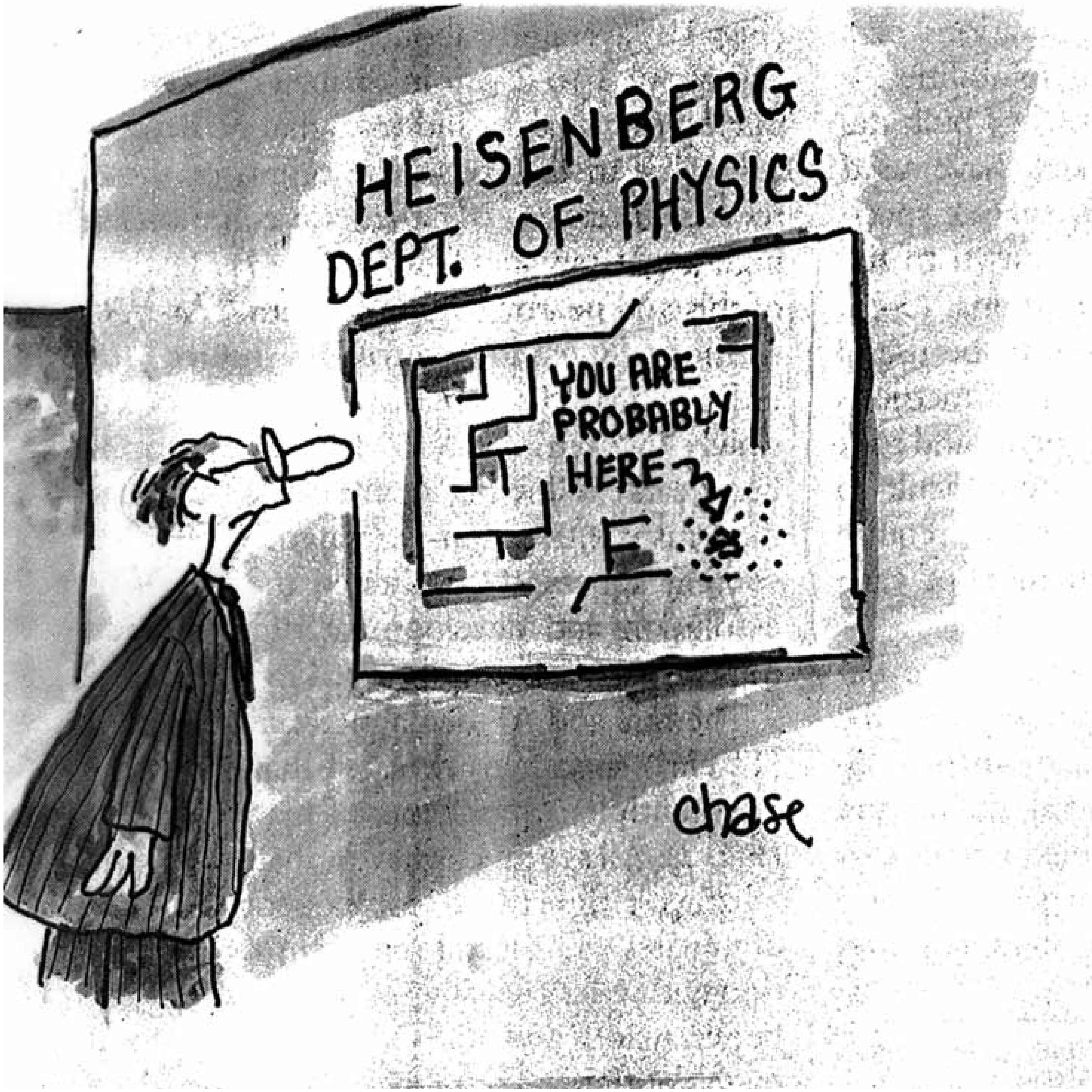
For a spherical wave propagating with a Gaussian intensity distribution, $I/I_0 = \exp(-r^2/2r_0)$, where r_0 is the $1/\sqrt{e}$ waist radius at the origin ($z = 0$), the intensity distribution grows with a $1/\sqrt{e}$ radius given by (Siegman, Lasers)

$$r(z) = r_0\sqrt{1 + \left(\frac{\lambda z}{4\pi r_0^2}\right)^2}$$

In the far field, where $z \gg 4\pi r_0^2/\lambda$, the $1/\sqrt{e}$ divergence half angle is

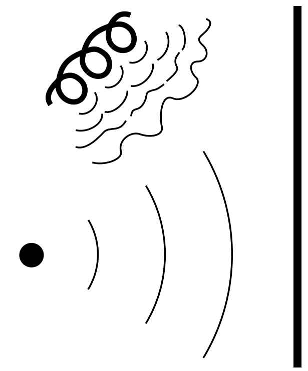
$$\theta \equiv \frac{r(z)}{z} = \frac{\lambda}{4\pi r_0}$$

with waist diameter $d = 2r_0$, we have TEM_{00} radiation with $d \cdot \theta = \lambda/2\pi$

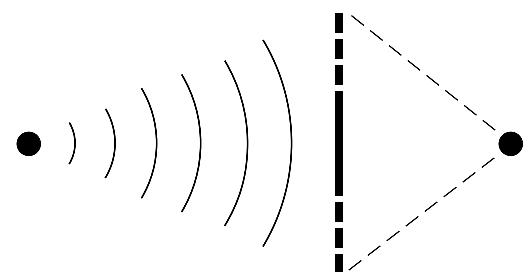

HEISENBERG
DEPT. OF PHYSICS
YOU ARE
PROBABLY
HERE
chase

Spherical Waves and Spatial Coherence

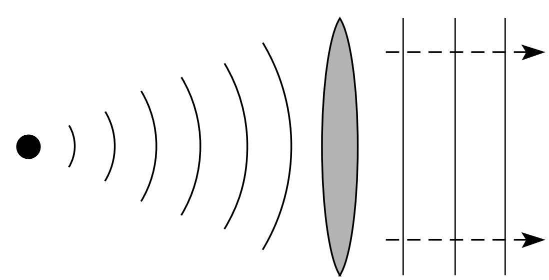
- **A spherical wave provides a reference with which to encode complex wavefronts, as in holography.**



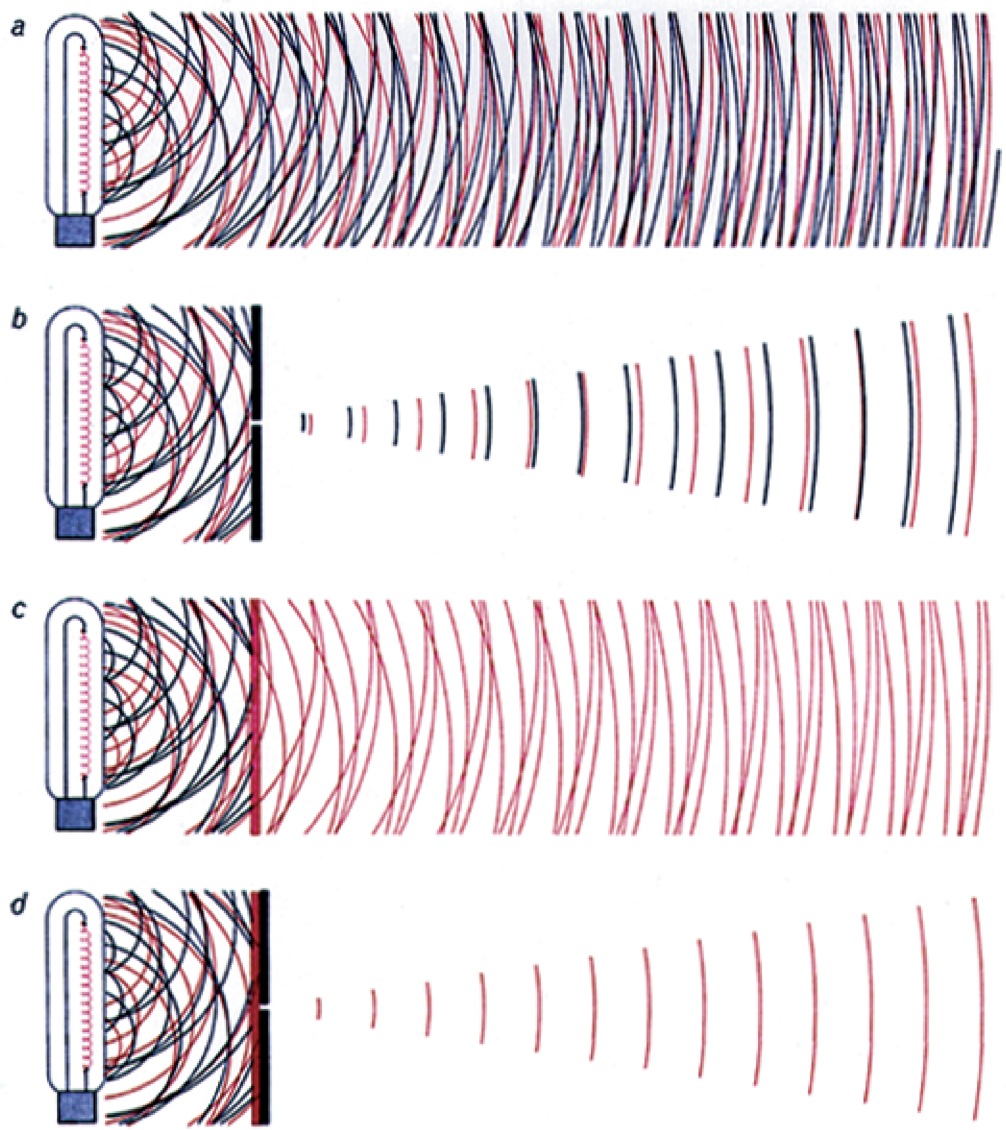
- **A spherical wave can be focused to the smallest possible spot size, as in scanning microscopy.**



- **A spherical wave can be collimated with minimal divergence.**

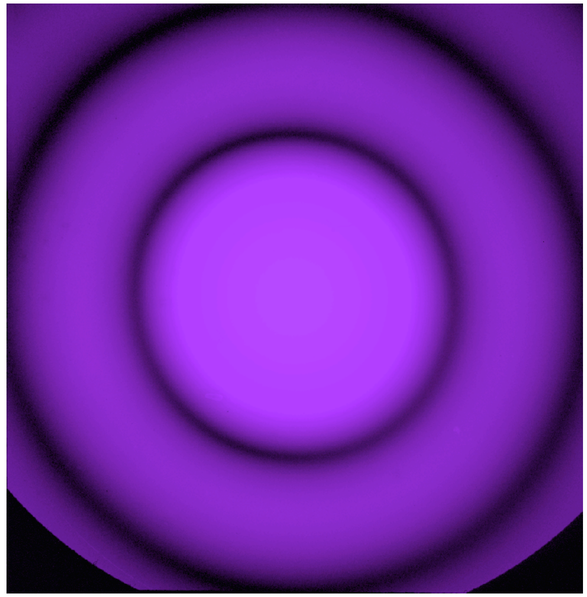


Spatial and Spectral Filtering to Produce Coherent Radiation

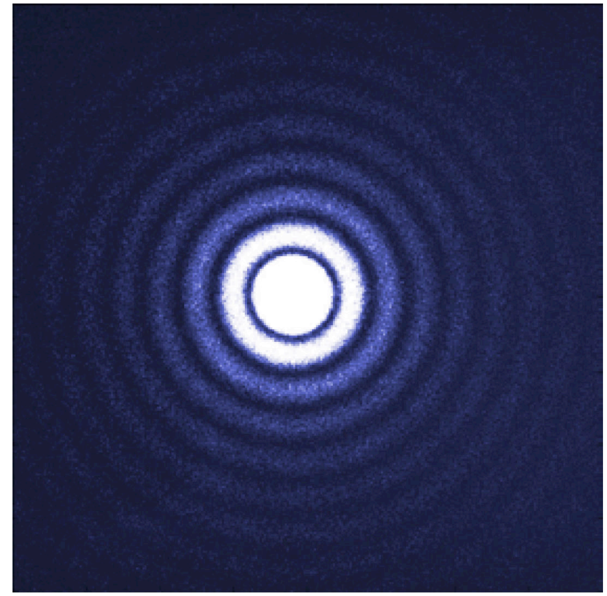


Courtesy of A. Schawlow, Stanford.

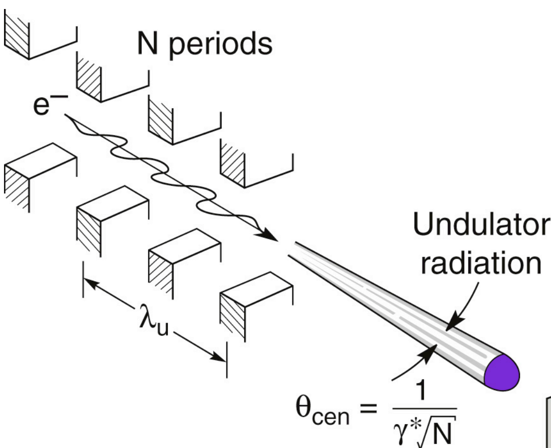
Spatially Coherent Undulator Radiation

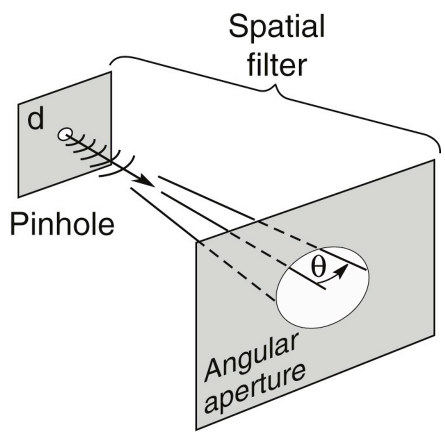


$\lambda = 13.4$ nm



$\lambda = 2.5$ nm





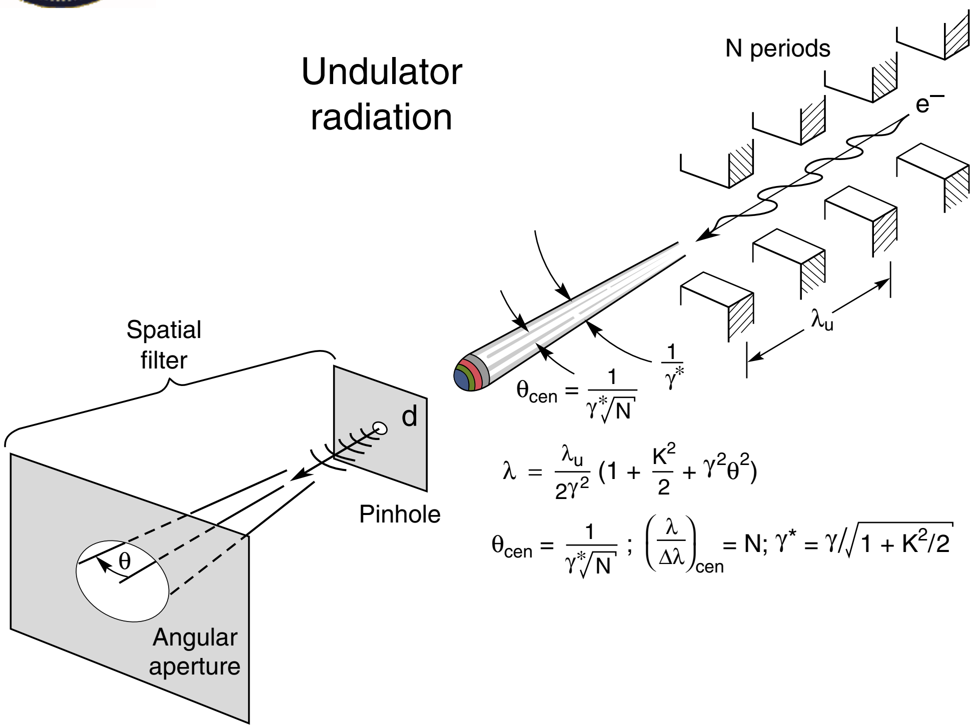
1 μm^D pinhole

25 mm wide CCD at 410 mm

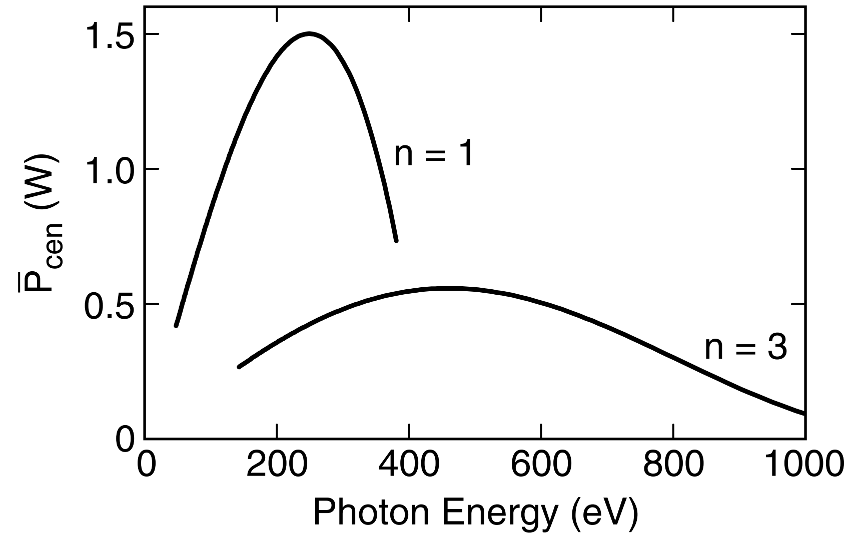
$$\boxed{d \cdot \theta = \frac{\lambda}{2\pi}}$$

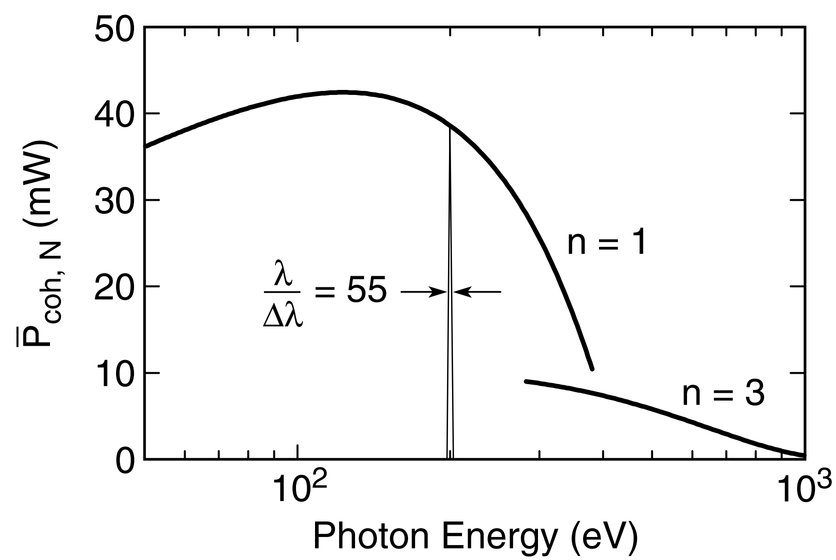
Courtesy of Patrick Naulleau, LBNL / Kris Rosfjord, UCB and LBNL

Spatially Filtered Undulator Radiation



ALS, U8





Using a pinhole-aperture spatial filter, passing only radiation that satisfies $d \cdot \theta = \lambda/2\pi$

$$\boxed{\bar{P}_{coh,N} = \left(\frac{\lambda/2\pi}{d_x\theta_x}\right)\left(\frac{\lambda/2\pi}{d_y\theta_y}\right)\bar{P}_{cen}} \qquad (8.6)$$

$$\bar{P}_{coh,N} = \frac{e\lambda_u I N}{8\pi\epsilon_0 d_x d_y}\left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0) \qquad (8.9)$$

for $d_x = 2\sigma_x$, $d_y = 2\sigma_y$, $\theta_{Tx} \rightarrow \theta_x$, $\theta_{Ty} \rightarrow \theta_y$, and $\sigma'^2 << \theta^2_{cen}$.

Spatial and Spectral Filtering of Undulater Radiation

In addition to the pinhole – angular aperture for spatial filtering and spatial coherence, add a monochromator for narrowed bandwidth and increased temporal coherence:

$$\bar{P}_{coh,\lambda/\Delta\lambda} = \underbrace{\eta}_{\text{beamline efficiency}} \underbrace{\frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y,\theta_y)}}_{\text{spatial filtering}} \cdot \underbrace{N\frac{\Delta\lambda}{\lambda}}_{\text{spectral filtering}} \cdot \bar{P}_{cen} \qquad (8.10a)$$

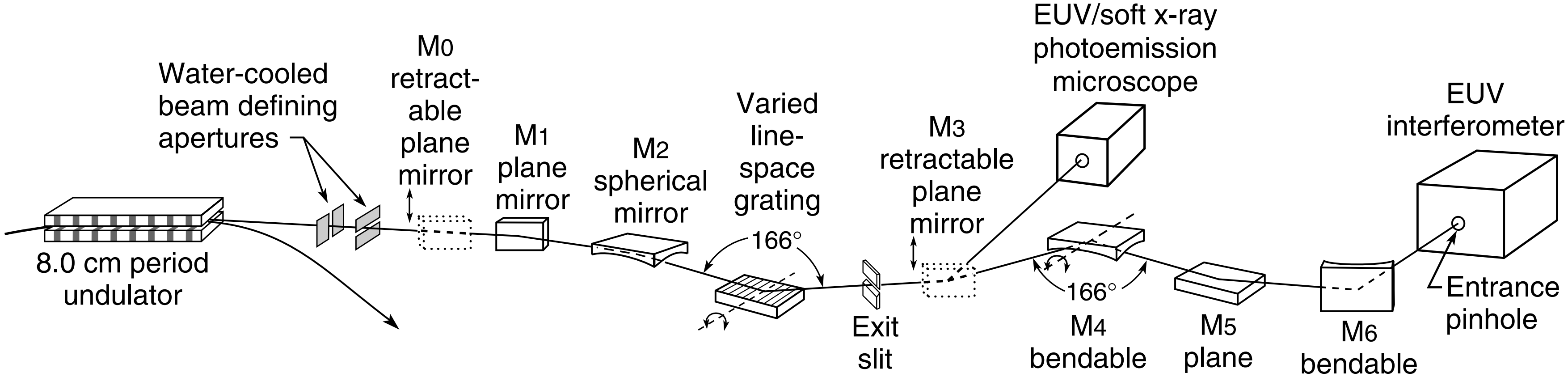
which for $\sigma'^2_{x,y} << \theta^2_{cen}$ (the undulator condition) gives the spatially and temporally coherent power ($d \cdot \theta = \lambda/2\pi$; $l_{coh} = \dfrac{\lambda^2}{2\,\Delta\lambda}$)

$$\boxed{\bar{P}_{coh,\lambda/\Delta\lambda} = \frac{e\lambda_u I\eta(\Delta\lambda/\lambda)N^2}{8\pi\epsilon_0 d_x d_y} \cdot \left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0)} \qquad (8.10c)$$

which we note scales as N^2.

Spatially and Spectrally Filtered Undulator Radiation

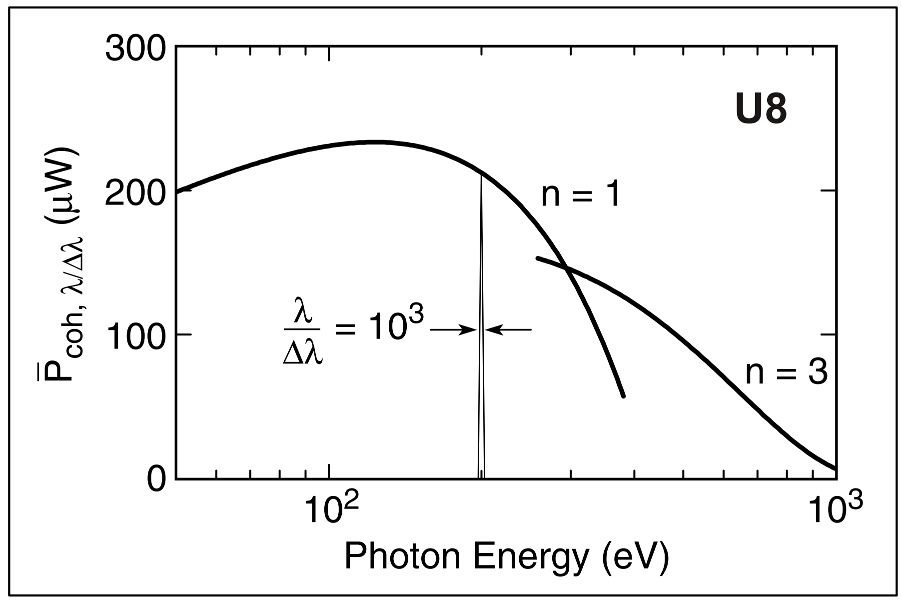
- Pinhole filtering for full spatial coherence
- Monochromator for spectral filtering to $\lambda/\Delta\lambda > N$



$$\bar{P}_{\mathrm{coh},\lambda/\Delta\lambda} = \underbrace{\eta}_{\substack{\text{beamline}\\ \text{efficiency}}} \underbrace{\frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y\theta_y)}}_{\substack{\text{spatial}\\ \text{filtering}}} \cdot \underbrace{N\frac{\Delta\lambda}{\lambda}}_{\substack{\text{spectral}\\ \text{filtering}}} \cdot \bar{P}_{\mathrm{cen}} \qquad (8.10a)$$

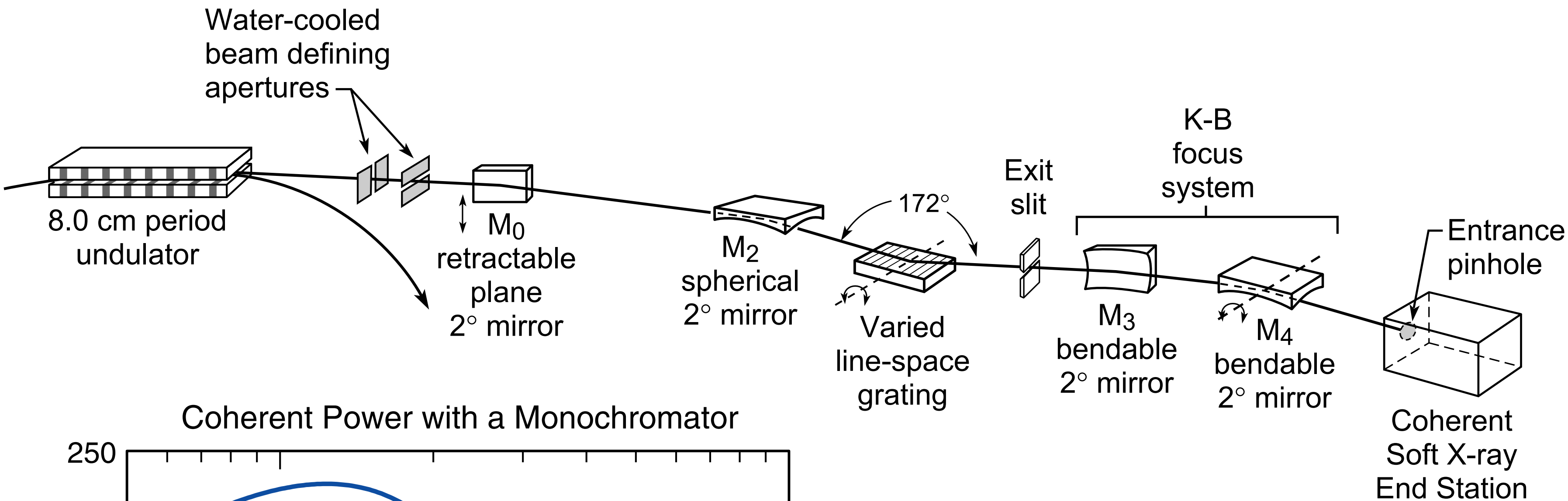
$$\bar{P}_{\mathrm{coh},\lambda/\Delta\lambda} = \frac{e\lambda_u I\eta(\Delta\lambda/\lambda)N^2}{8\pi\epsilon_0 d_x d_y} \cdot \left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0)$$

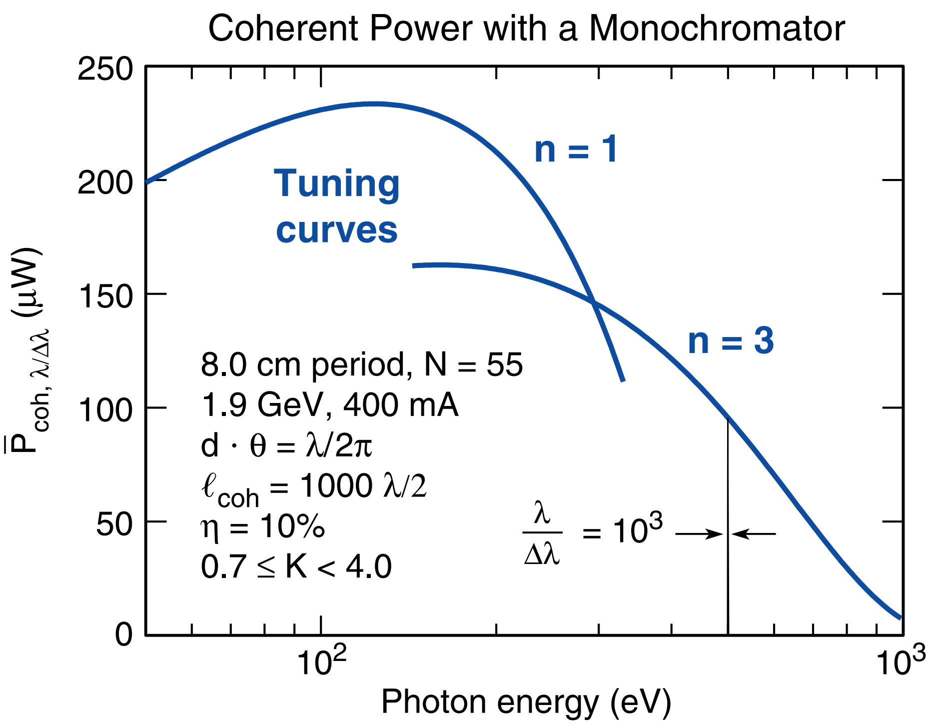
$$(\sigma'^2 \ll \theta_{\mathrm{cen}}^2) \qquad (8.10c)$$



Ch08_F11VG_ALS_Nov04.ai

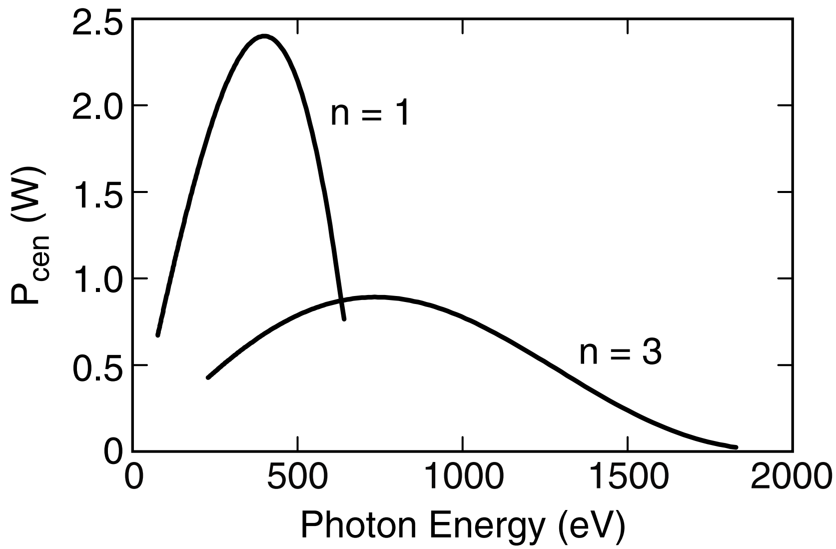
Coherent Soft X-Ray Beamline: Use of a Higher Harmonic (n = 3) to Access Shorter Wavelengths



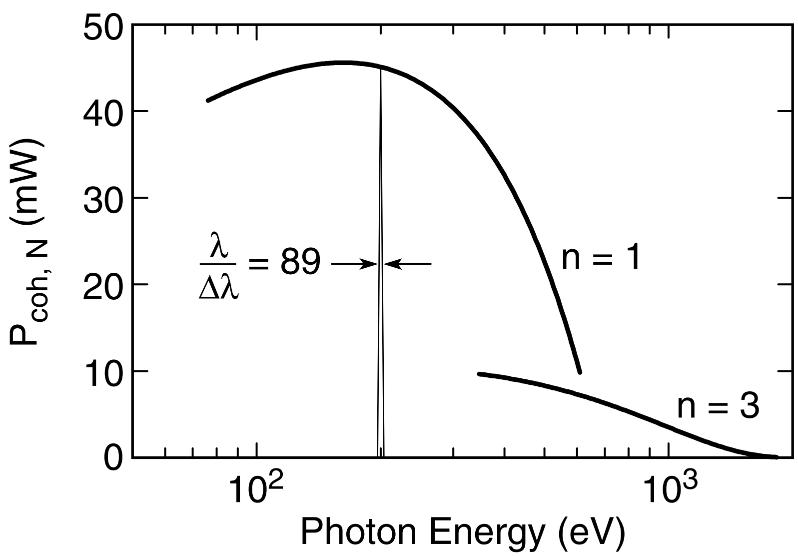


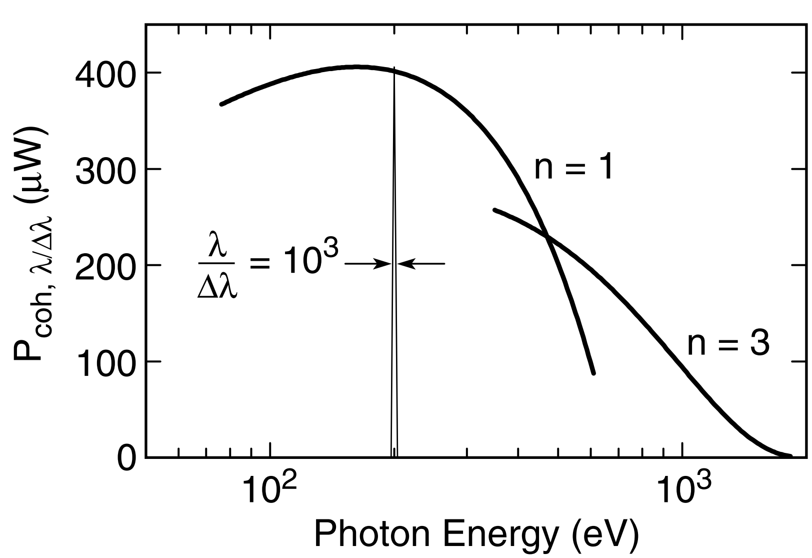
Coherent Power at the ALS

U5

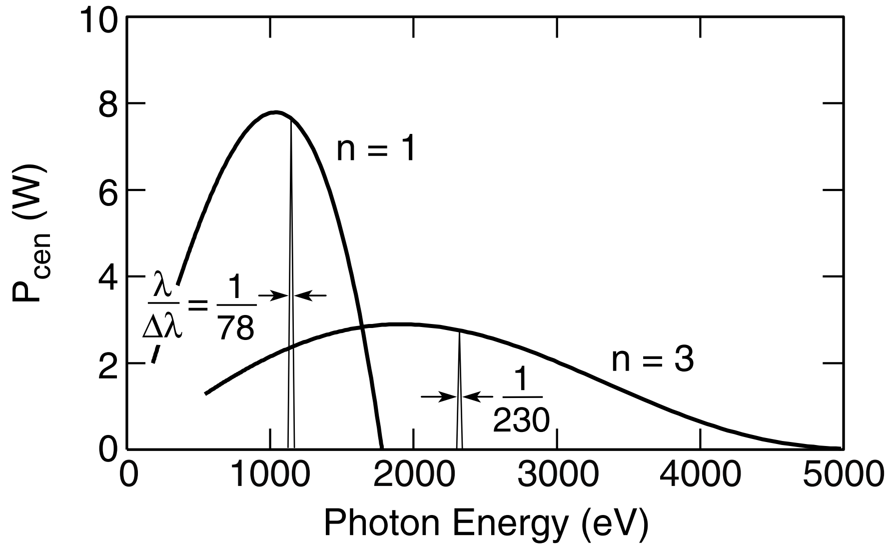


1.9 GeV, 400 mA
λ_u = 50 mm
N = 89
$0.5 \leq K \leq 4.0$
σ_x = 260 μm
σ_x' = 23 μr
σ_y = 16 μm
σ_y' = 3.9 μr
θ_{cen} = 35 μr

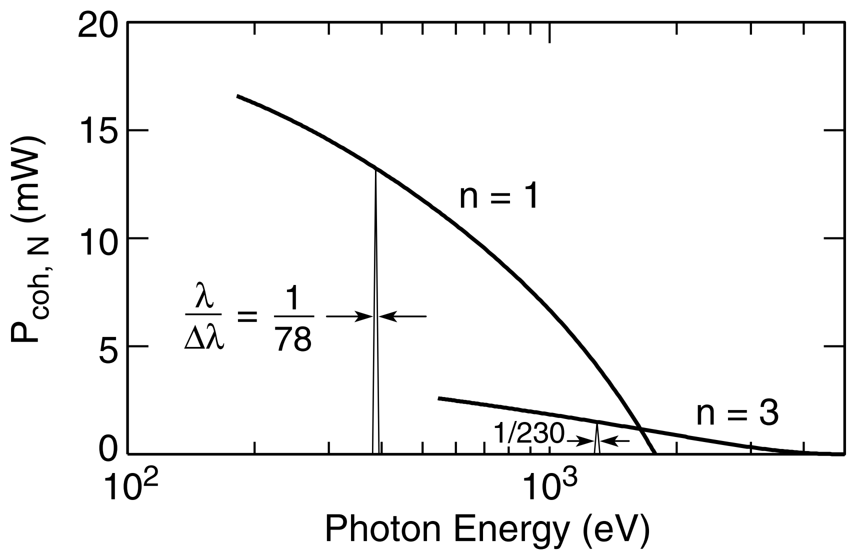


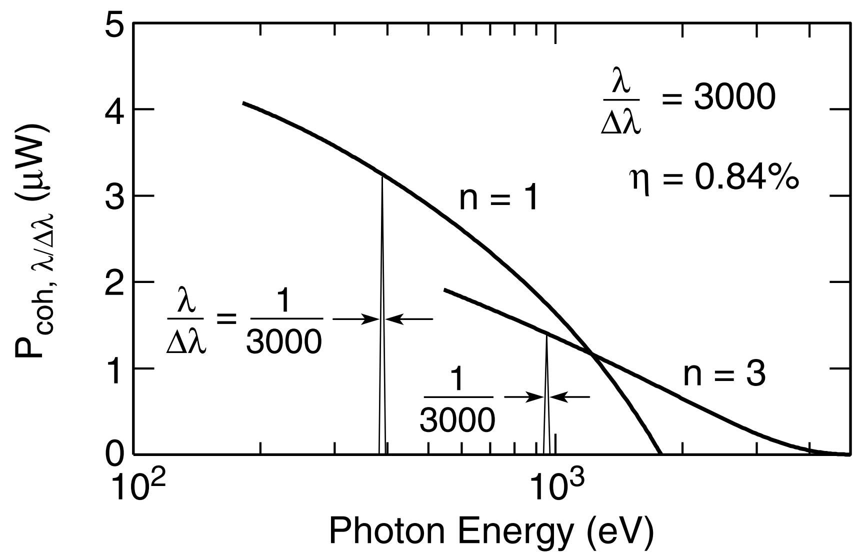


Power Curves for the Stanford EPU



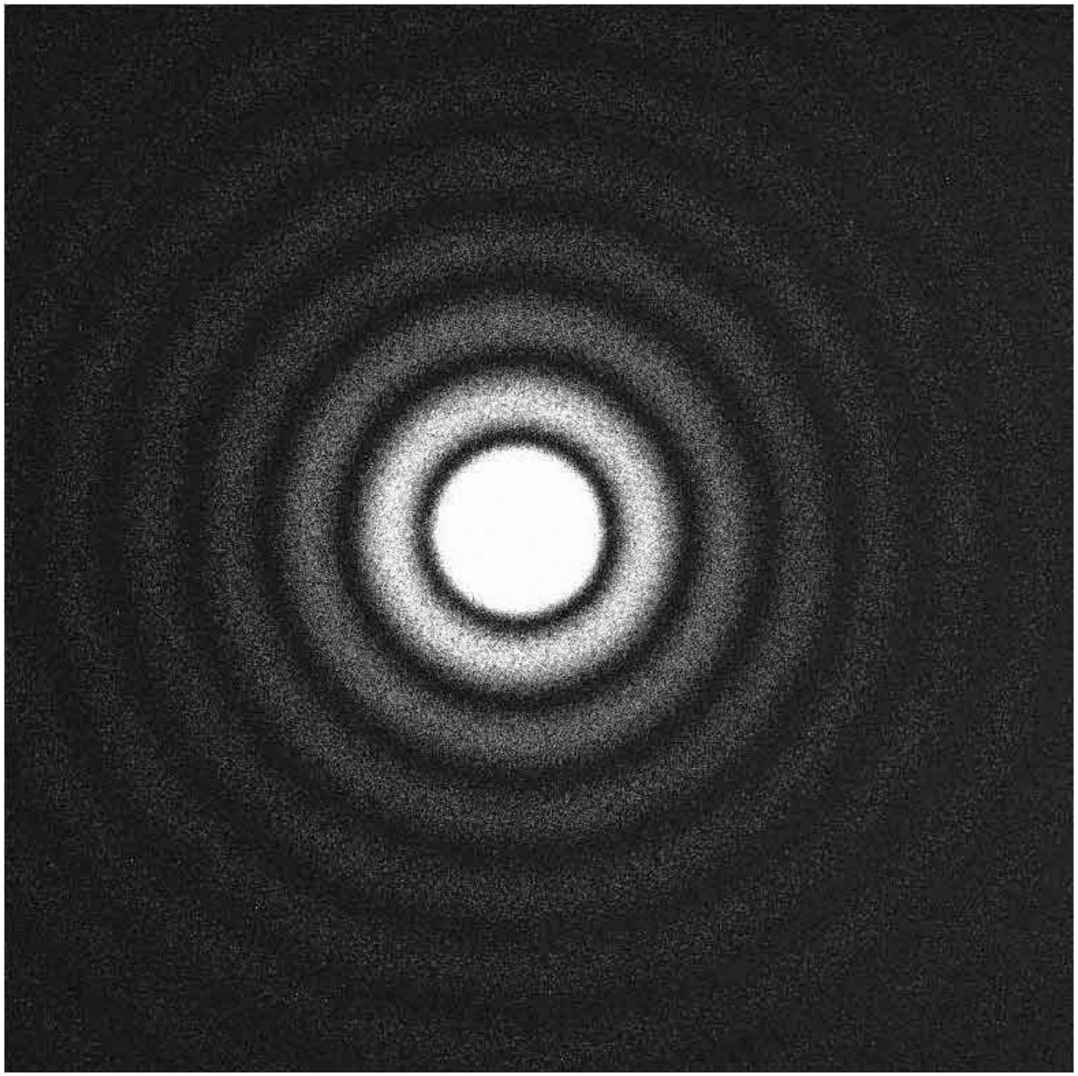
3 GeV, 500 mA
λ_u = 48 mm
N = 78
K $\leq$ 4.2
σ_h = 420 μm
σ_v = 42 μm
σ_h' = 42 μr
σ_v' = 6 μr
θ_{cen} = 30 μr

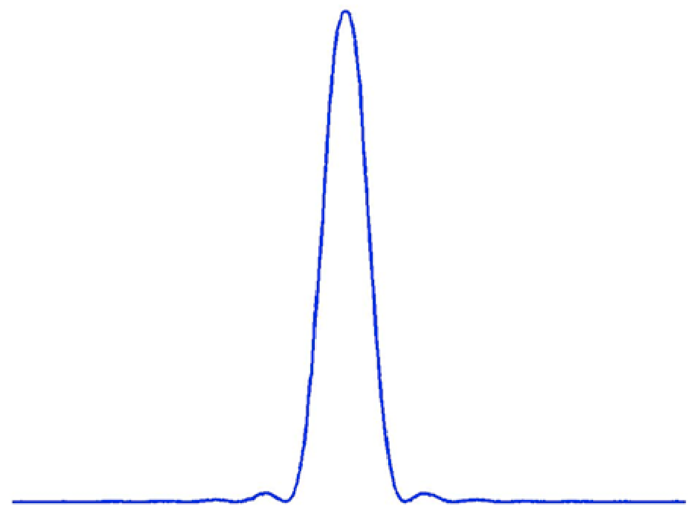




StanfordEPUpwr_Curves.ai

Spatially Coherent Soft X-Rays With Pinhole Spatial Filtering: Airy Patterns at 500 eV

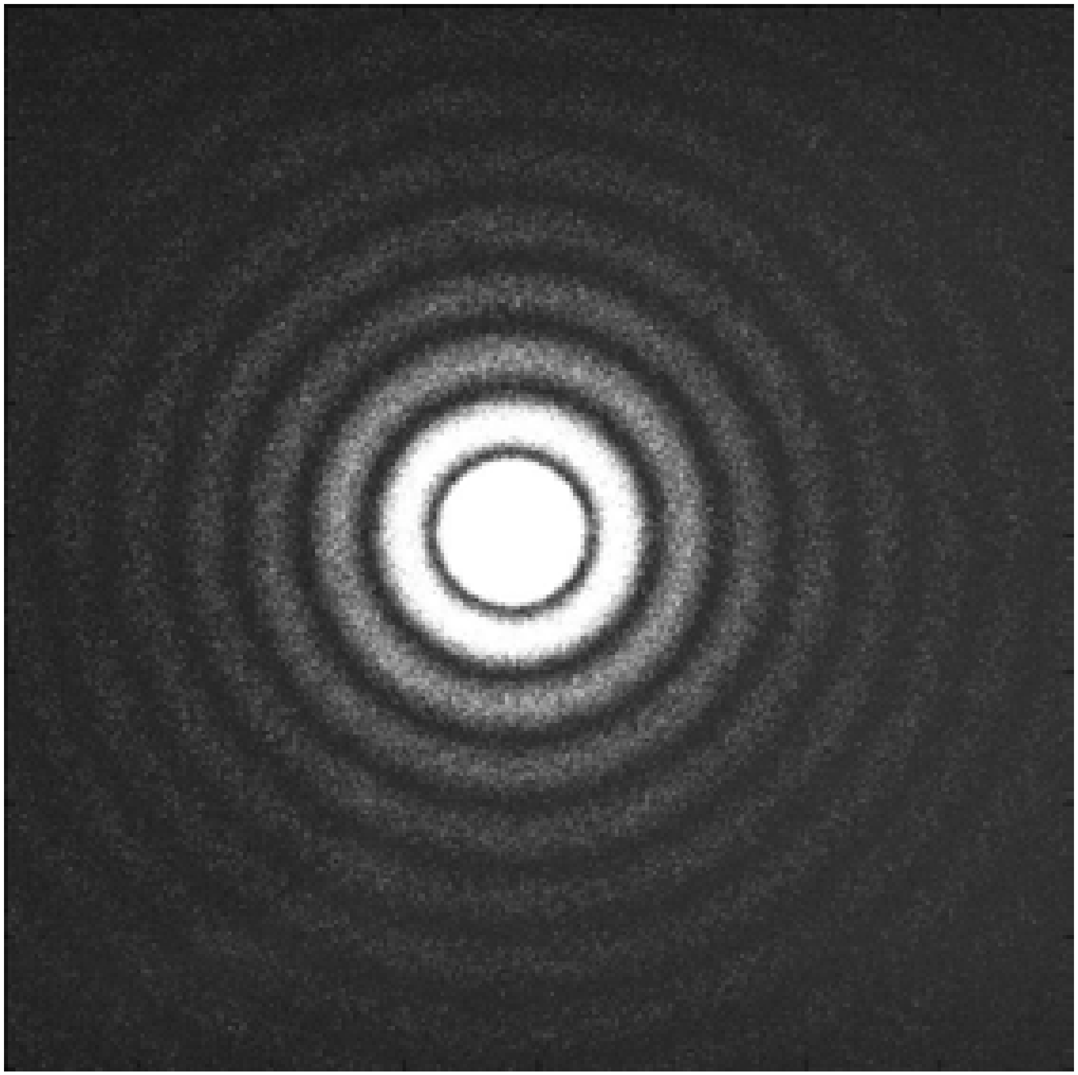


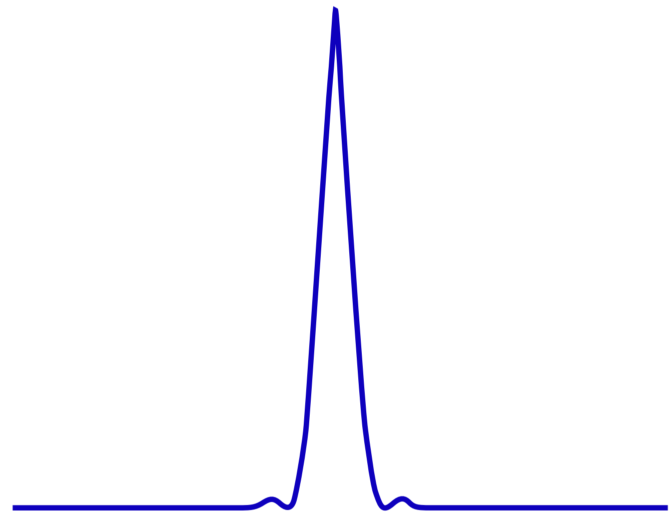


λ = 2.48 nm (500 eV)
d = 2.5 μm
t = 200 msec
ALS beamline 12.0.2
λ_u = 80 mm, N = 55, n = 3

Courtesy of Kristine Rosfjord, UC Berkeley and LBNL
IEEE JSTQE 10, 1405 (Nov/Dec 2004)

Spatially Coherent Soft X-Rays With Pinhole Spatial Filtering: Airy Patterns at 600 eV



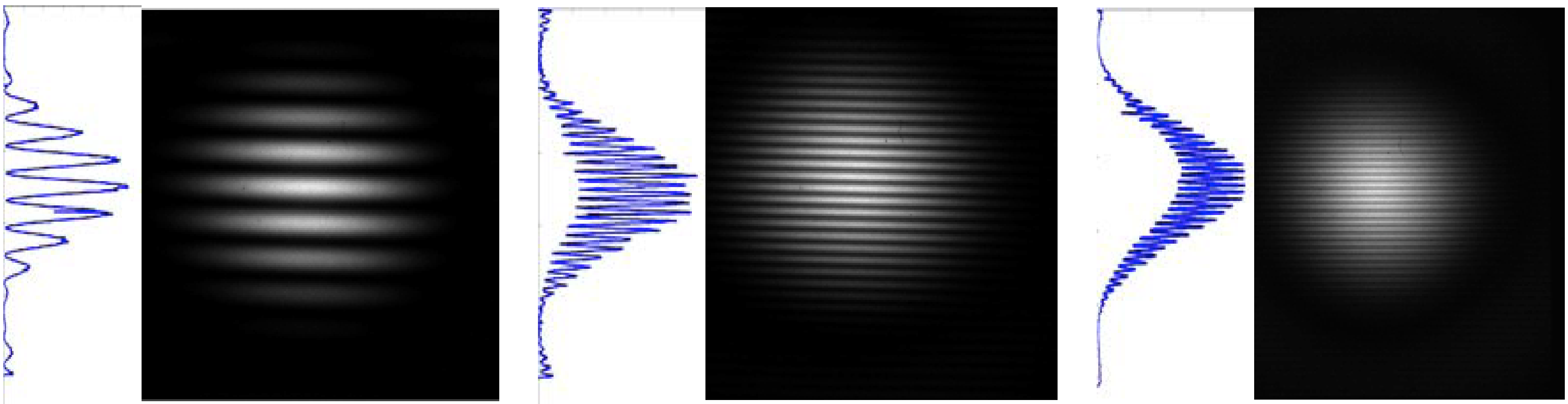


λ = 2.48 nm (600 eV)
d = 2.5 μm
t = 200 msec
ALS beamline 12.0.2
λ_u = 80 mm, N = 55, n = 3

Courtesy of Kristine Rosfjord, UC Berkeley and LBNL
IEEE JSTQE 10, 1405 (Nov/Dec 2004)

AiryPattrn600eV.ai

Measuring the Spatial Coherence of Undulator Radiation: Double Pinhole Experimental Results



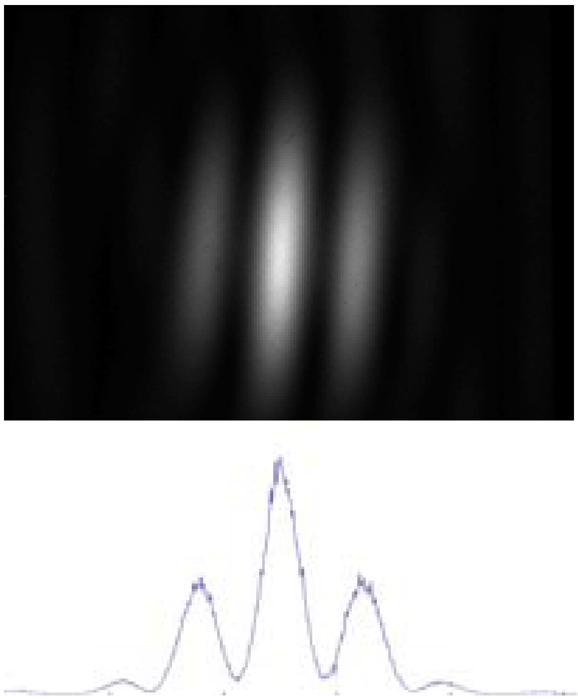
2 μm vertical separation | 6 μm vertical separation | 8 μm vertical separation

λ = 2.48 nm (500 eV)
λ_u = 80 mm, N = 55, n = 3
60 μm $\times$ 9.4 μm focal spot
d = 500 nm, ALS 12.0.2

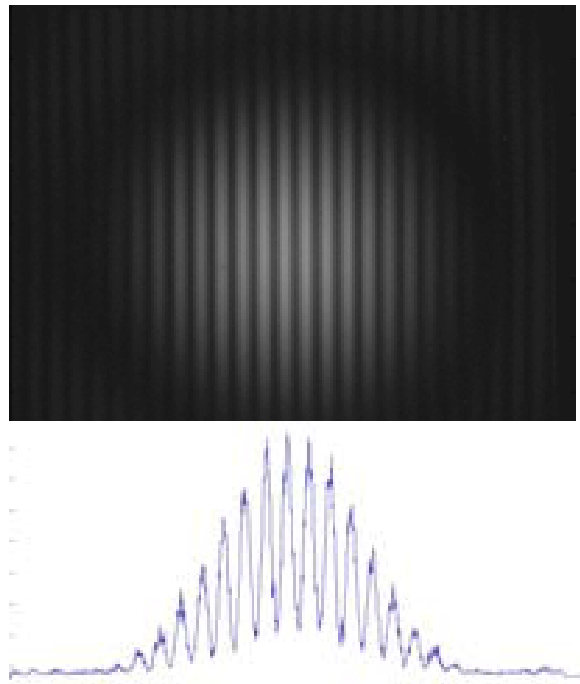
Courtesy of Kristine Rosfjord, UC Berkeley and LBNL
IEEE JSTQE 10, 1405 (Nov/Dec 2004)

Rosfjord_IEEE_F.8.ai

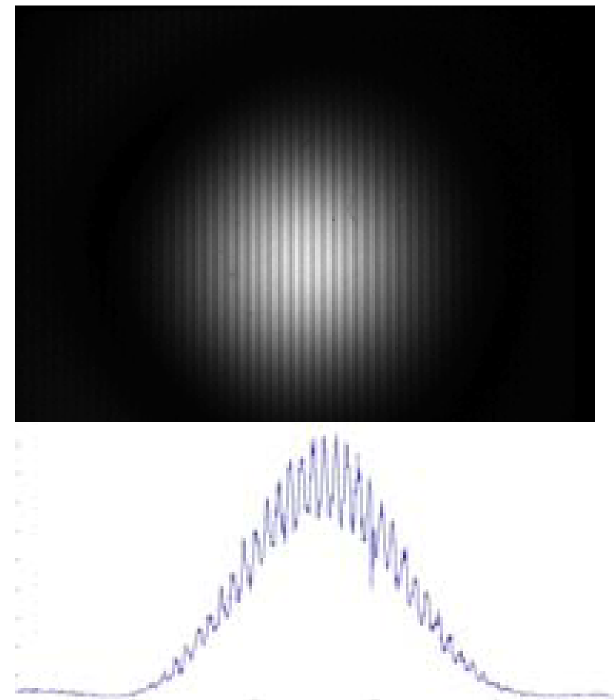
Measuring the Spatial Coherence of Undulator Radiation: Double Pinhole Experimental Results



1 μm horizontal separation



4 μm horizontal separation



8 μm horizontal separation

λ = 2.48 nm (500 eV)
λ_u = 80 mm, N = 55, n = 3
60 μm × 9.4 μm focal spot
d = 500 nm, ALS 12.0.2

Courtesy of Kristine Rosfjord, UC Berkeley and LBNL
IEEE JSTQE 10, 1405 (Nov/Dec 2004)

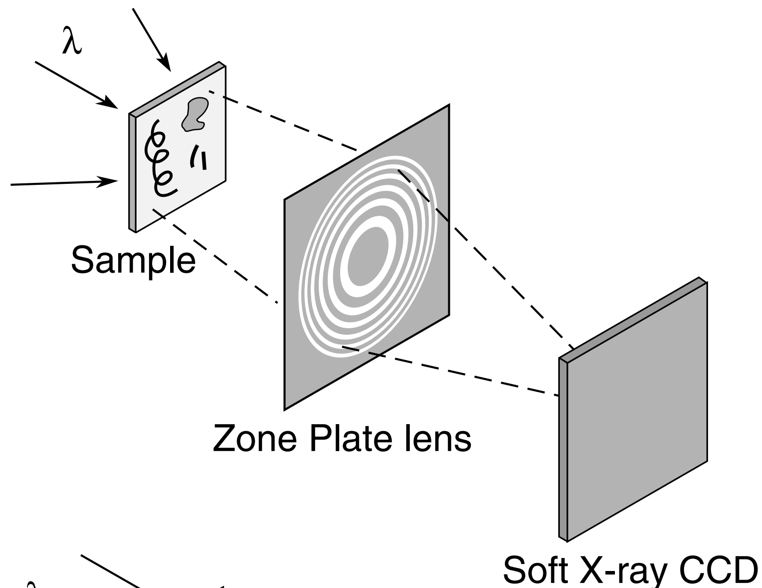
Rosfjord_IEEE_F.10.ai

Some Experiments that Utilize Coherent Soft X-Rays and EUV

- Scanning x-ray microscopy
- Coherent scattering
- X-ray holography
- Interferometry of optics and materials

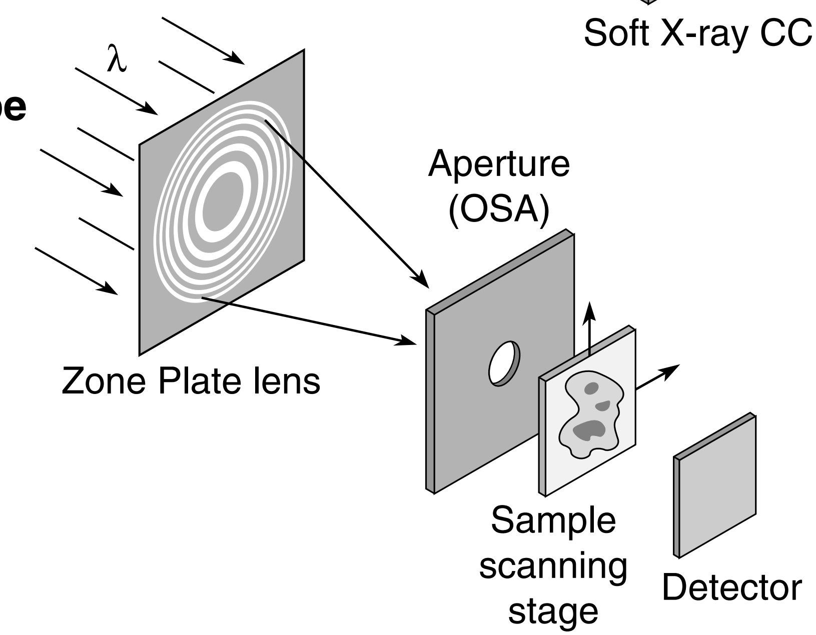
Two Common Soft X-Ray Microscopes

Full-Field Microscope



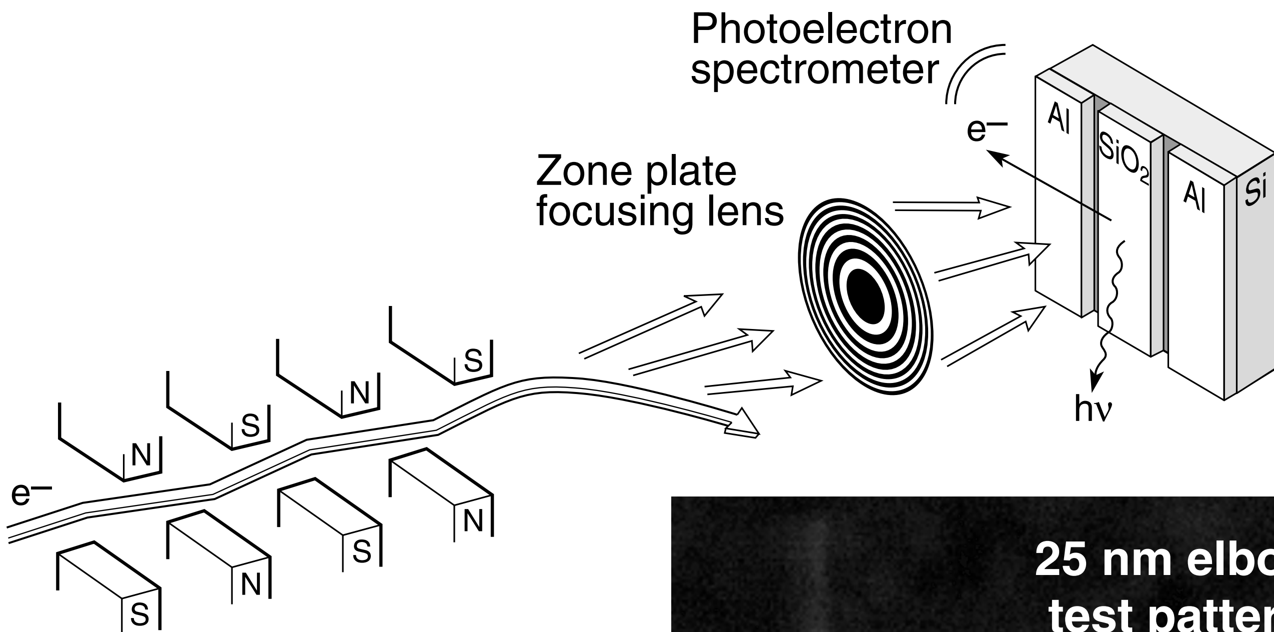
- Best spatial resolution
- Modest spectral resolution
- Shortest exposure time
- Bending magnet radiation
- Higher radiation dose
- Cryofixation
- Molecular labeling

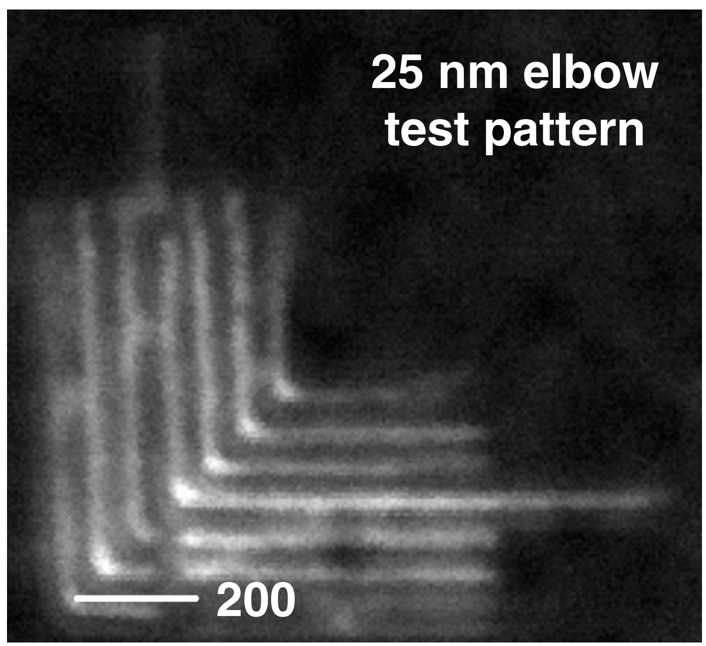
Scanning Microscope



- Least radiation dose
- Good spatial resolution
- Best spectral resolution
- Requires spatially coherent radiation
- Long exposure time
- Molecular labeling
- Cryofixation
- Photoemission and fluorescence imaging possible

Spectromicroscopy: High Spatial and High Spectral Resolution Studies of Surfaces and Thin Films





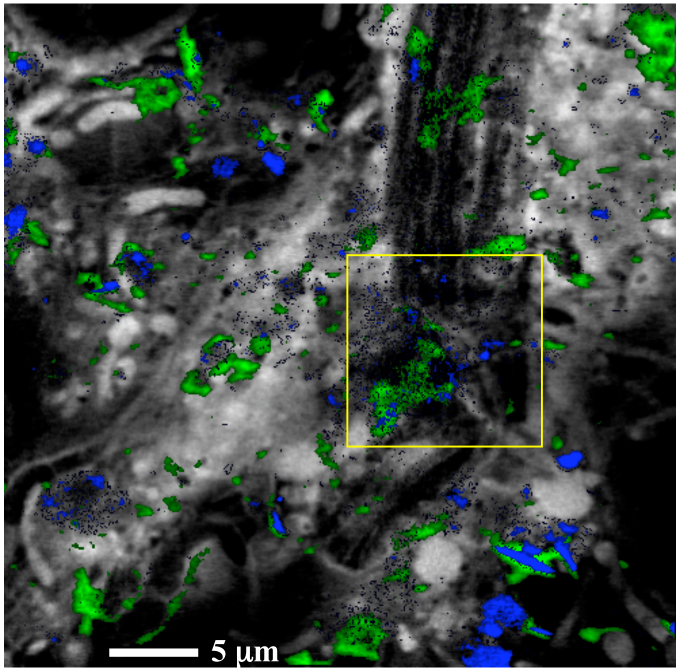
Scanning Soft X-Ray Microscope

ALS beamline 11.0.2
395 eV; $\lambda/\Delta\lambda \simeq 6000$
240×240 pixels
$1.2\ \mu m \times 1.2\ \mu m$
2 ms dwell time

Courtesy of Tolek Tyliszczak (Dec. 2003)

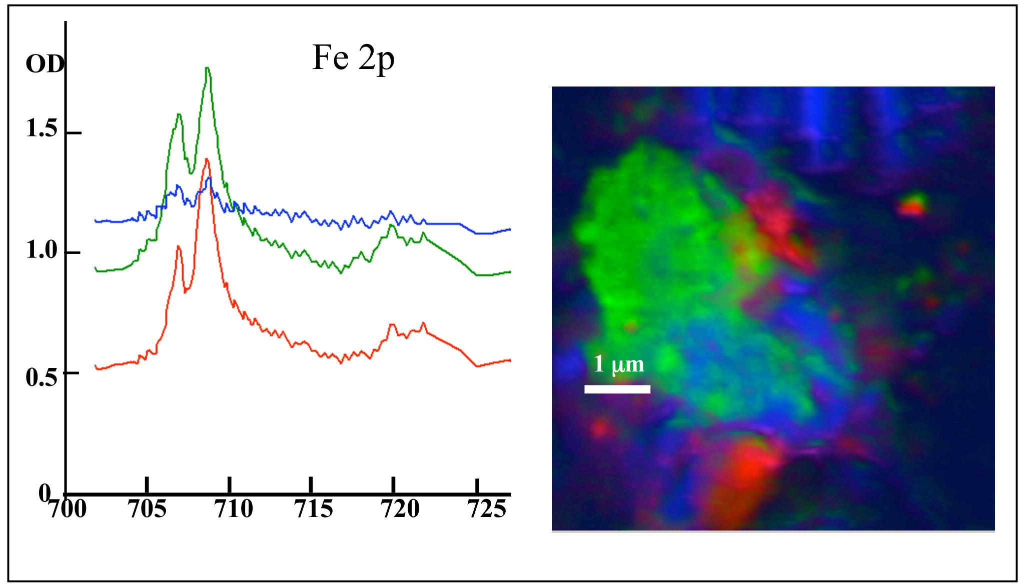
Biofilm from Saskatoon river

ALS-MES 11.0.2



Protein (gray), Ca, K

RESULTS
•Ni, Fe, Mn, Ca, K, O, C elemental map,
(there was no sign of Cr.)
•Different oxidation states for Fe and Ni



Different oxidation states (minerals) found for Fe & Ni

Tohru Araki, Adam Hitchcock (McMaster University)
Tolek Tyliszczak, LBNL
Sample from: John Lawrence, George Swerhone (NWRI-Saskatoon), *Gary Leppard* (NWRI-CCIW)