Delta Undulator Magnet Block Sorting Algorithm

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January 28, 2013

Abstract
This note describes the algorithm used to sort the magnet blocks for the Delta undulator. The purpose of the sort is to minimize the amount of tuning required to bring the undulator’s magnetic fields within tolerance. The sort pairs the magnet blocks in such a way that the pairs have smaller error fields than the individual blocks. The pairs are then sorted and placed in the undulator in a way which reduces phase errors. A block placement table is created.

1 Introduction

SLAC is building a Delta undulator which will be placed in the LCLS beam line to produce light with variable polarization. A 1 meter prototype is being constructed and a 3.2 meter full length device will be built after the prototype is successfully tested.

The magnet blocks for the prototype undulator have the shape shown in figure 1. Also shown is a photo of the end of the undulator under construction. Different polarization modes are generated as the rows of magnets are moved longitudinally along the undulator. Figure 2 shows the three types of blocks used in the device. The dimensions of the three types of blocks are the same, only the magnetization directions are different.

Figure 1: This figure shows the shape of the magnet blocks (left) and an end view of the undulator under construction (right). Drawing courtesy of Tim Montagne and photo courtesy of Heinz-Dieter Nuhn.

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1 Work supported in part by the DOE Contract DE-AC02-76SF00515. This work was performed in support of the LCLS project at SLAC.

The magnet blocks for the prototype were all measured in a Helmholtz coil and they were sorted in an effort to minimize magnetic field errors in the assembled device. This note describes the sorting algorithm which was used.

2 Sort Algorithm

2.1 Motivation

The objective of the magnet block sort is to minimize the amount of tuning required for the undulator. The undulator assembly and tuning proceed as follows. The blocks are first loaded into the quadrants, and the quadrants are individually tuned. The quadrants are then assembled to build the undulator. The assembled undulator is measured, but only tuned with difficulty since the undulator must be disassembled to move a block for tuning. Thus most of the tuning is done on the individual quadrants.

The sort matches blocks to reduce field errors. Since the quadrants are tuned individually, the matched blocks are put into individual quadrants and not across quadrants. The matching process pairs blocks to minimize error fields. The pairs are put into periods of the quadrant. Additionally, periods with strong vertical fields are placed adjacent to periods with weak vertical fields in an effort to minimize phase errors. Details of the block pairing and pair placement in the quadrants is given below. This note only discusses the blocks in the central periodic part of the undulator. The end blocks will be discussed in a separate note.

2.2 Vertical Blocks

The vertically magnetized blocks come in two styles: blocks magnetized in the direction from the base to the beam axis (denoted +y blocks), and blocks magnetized in the direction away from the beam axis toward the base (denoted -y blocks). The vertically magnetized blocks can be flipped about the vertical axis. The flip keeps the vertical magnetic moment $m_y$ the same, and it changes the sign of the horizontal magnetic moment $m_x$. Flipping and/or pairing blocks is used to cancel the error component $m_x$, and pairing is used to cancel differences in $m_y$. The nomenclature for the blocks comes from our coordinate system. We use a coordinate system where the z-axis is along the quadrant, the y-axis is from the base of the quadrant toward the beam axis, and the x-axis direction is chosen to make a right handed coordinate system.

The z-component of the magnetic moment only puts a small offset into the beam and is not used in the sort. The y-component of the +y block is matched to the y-component of the -y block so the
The absolute values of the +y-blocks, the reason is that these blocks far from the mean are hard to find accurate matches for.

The sort proceeds as follows. First all blocks with magnetic moment components further than two standard deviations from the means of \(m_x\) and \(m_y\) are considered outliers and are excluded from the sort. This is done for both the +y and -y blocks. The reason is that these blocks far from the mean are hard to find accurate matches for. We use the mean instead of the ideal when considering outliers for practical reasons. We must use the blocks we have to build the undulator and since we are tuning the undulator, remaining errors will be corrected. We assume the blocks are close enough to ideal to build a tunable undulator.

Of the remaining +y blocks after outliers are excluded, a quantity

\[ r = \sqrt{(m_x - \langle m_x \rangle)^2 + (m_y - \langle m_y \rangle)^2} \]

is calculated for each block. \(r\) is a measure of how far the block is from the mean. We start with the +y block with the largest \(r\) value and we denote its magnetic moment components by \(m_{x-1}\) and \(m_{y-1}\), since this is the first block in the pair. We calculate a quantity \(d\) for all -y blocks which are not outliers and are not already matched. \(d\) is defined by

\[ d = \sqrt{(|m_x| - |m_{x-1}|)^2 + (m_y + m_{y-1})^2} \]

The absolute values of \(m_x\) account for the fact that the y-block can be flipped. The \(m_y\) term matches the opposite sign y component of the magnetic moment. The -y block with the minimum \(d\) value is the match for the given +y block we are trying to match. The paired -y block’s magnetic moments are \(m_{x-2}\) and \(m_{y-2}\). This procedure is repeated for all the +y blocks going from the largest \(r\) value to the smallest \(r\) value.

The results for the pairing are illustrated in figure 3. The top left plot is a scatter plot of the \(m_x\) and \(m_y\) values of all the +y blocks for the 1 meter prototype undulator. The bottom left plot is the \(m_x, m_y\) scatter plot for the -y blocks. The plot on the right is the net \(m_x, m_y\) scatter plot of all the matched pairs. The scales of all the plots have the same range. The width of the pair distribution is reduced by a factor of approximately 5.5 for \(m_x\) and 2.5 for \(m_y\) compared to the width of the +y or -y distributions. A table of the results for the 1 meter prototype is given below.

<table>
<thead>
<tr>
<th>#</th>
<th>Outliers</th>
<th>(m_x) ave (Tm³)</th>
<th>(m_x) rms (Tm³)</th>
<th>(m_y) ave (Tm³)</th>
<th>(m_y) rms (Tm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+y</td>
<td>143</td>
<td>1.149 × 10^{-6}</td>
<td>1.992 × 10^{-6}</td>
<td>1.704 × 10^{-6}</td>
<td>9.293 × 10^{-9}</td>
</tr>
<tr>
<td>-y</td>
<td>141</td>
<td>-7.123 × 10^{-9}</td>
<td>2.129 × 10^{-8}</td>
<td>-1.705 × 10^{-6}</td>
<td>7.788 × 10^{-9}</td>
</tr>
<tr>
<td>Pairs</td>
<td>125</td>
<td>2.874 × 10^{-9}</td>
<td>3.562 × 10^{-9}</td>
<td>-1.352 × 10^{-9}</td>
<td>3.441 × 10^{-9}</td>
</tr>
</tbody>
</table>

### 2.3 Horizontal Blocks

The horizontal blocks have one style. This means that all pairs come from the same population. One block in each pair is flipped about the y-axis to reverse the direction of \(m_x\). This is required since each pair goes into one period of an undulator quadrant.

The sort for the horizontal blocks proceeds in a similar manner to the vertical blocks, but with all blocks coming from the same set of blocks. First all blocks further than two standard deviations from the means of \(m_x\) and \(m_y\) are considered outliers and are excluded from the sort. As for the y-blocks, the reason is that these blocks far from the mean are hard to find accurate matches for.

After excluding outliers, the quantity

\[ r = \sqrt{(m_x - \langle m_x \rangle)^2 + (m_y - \langle m_y \rangle)^2} \]

is calculated for each block.
is calculated for each block. $r$ is a measure of how far the block is from the mean. We start with the block with the largest $r$ value and we denote its magnetic moment components by $m_{x,1}$ and $m_{y,1}$, since this is the first block in the pair. We calculate a quantity $d$ for all other blocks which are not outliers or are not already matched. $d$ is defined by

$$d = \sqrt{(-m_x + m_{x,1})^2 + (m_y + m_{y,1})^2}$$  

$m_{x,1}$ and $m_{y,1}$ are the moments of the block we are trying to match. Since the block which will be the match will be flipped about the y-axis, $d$ finds the block with similar $m_x$, which is reversed upon the flip, and opposite $m_y$. The block with the smallest $d$ value is the match for the block we are trying to match. The pair block’s magnetic moments are $m_{x,2}$ and $m_{y,2}$. This procedure is repeated for all the horizontal blocks going from the largest $r$ value to the smallest $r$ value.

The results for the pairing are illustrated in figure 4. The left plot is a scatter plot of the $m_x$ and $m_y$ values of all the horizontal blocks for the 1 meter prototype undulator. Outliers are indicated in red. The plot on the right is the net $m_x$, $m_y$ scatter plot of all the matched pairs. The scales of both plots have the same range. The width of the pair distribution is reduced by a factor of approximately 4.7 for $m_x$ and 3.7 for $m_y$ compared to the width of the individual block distribution. A table of the results for the 1 meter prototype is given below.

Figure 3: Scatter plots of $(m_x, m_y)$ for the +y blocks (upper left) and -y blocks (lower left), and the block pairs (right). All plots have the same scale ranges.
Figure 4: Scatter plots of \((m_x, m_y)\) for the horizontally magnetized blocks (left), and the block pairs (right). All plots have the same scale ranges.

<table>
<thead>
<tr>
<th></th>
<th>#</th>
<th>Outliers</th>
<th>(m_x) ave (Tm³)</th>
<th>(m_x) rms (Tm³)</th>
<th>(m_y) ave (Tm³)</th>
<th>(m_y) rms (Tm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocks</td>
<td>284</td>
<td>36</td>
<td>(-3.325 \times 10^{-8})</td>
<td>(3.324 \times 10^{-8})</td>
<td>(1.524 \times 10^{-9})</td>
<td>(2.177 \times 10^{-8})</td>
</tr>
<tr>
<td>Pairs</td>
<td>124</td>
<td>-</td>
<td>(-1.608 \times 10^{-8})</td>
<td>(7.075 \times 10^{-9})</td>
<td>(5.322 \times 10^{-9})</td>
<td>(6.369 \times 10^{-9})</td>
</tr>
</tbody>
</table>

### 2.4 Pair Placement

Once the pairs are chosen to minimize the net \(m_x\) and \(m_y\) of the pair, the pairs are put into periods of the quadrants of the undulator. The pair placement is chosen in an effort to minimize phase errors. The average \(m_y\) of each y-pair is calculated

\[
m_y\text{ ave} = \frac{1}{2}(m_{y1} - m_{y2})
\]  

\(m_y\) ave is sorted from strongest to weakest. The pair with the largest \(m_y\) ave is put into period 1 of quadrant 1. The pair with the smallest \(m_y\) ave is put into period 2 of quadrant 1. The idea is that a period with large slippage is followed by a period with small slippage. The horizontal block pairs are chosen in the order they come in the program, which is essentially random, but sorted according to \(r\) value. The second largest \(m_y\) ave pair is put in period 1 of quadrant 2. The second smallest \(m_y\) ave pair is put in period 2 of quadrant 2. This procedure is followed until all the quadrants are populated. A listing is made of which block is put into each slot of each quadrant as shown in figure 5.

### 2.5 Effect Of The Number Of Blocks

A study was performed to see if having more blocks would reduce the spread of the net magnetic moments of the pairs. Indeed, the effect is dramatic. Figure 6 shows how the ratio of the standard deviation of the pair distribution divided by the standard deviation of the individual block distributions varies with the number of blocks. The study was performed by generating fake magnetic moment data with the parameters of the measured block distributions. As intuition would indicate, having more blocks allows better matching for the pairs.
3 Conclusion

A computer program was written to sort the magnet blocks for the Delta undulator based on the measured magnetic moments. Blocks were first paired to cancel error fields within each period of a quadrant of the undulator. The periods were then arranged to minimize phase errors by placing a period with strong blocks next to a period with weak blocks. The effectiveness of the sorting increases with a larger number of magnet blocks.

Acknowledgements

I am grateful to Heinz-Dieter Nuhn and Yurii Levashov for many discussions about this work.
Figure 6: The width of the pair $m_x$ distribution gets smaller compared to the width of the individual block $m_x$ distribution as the number of blocks increases.