Temperature Dependence Of The LCLS-II Undulators And Phase Shifters

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April 16, 2020

Abstract
The temperature dependence of the LCLS-II SXR and HXR undulator $K$ values, undulator phase matching, and phase shifter phase integrals is analyzed in this note. The expected temperature dependence is calculated. A series of temperature calibration measurements was performed and this note documents the results of the measurements. Comparisons of the measurements to the expected values are made.

1 Introduction

The LCLS-II project has variable gap hybrid undulators consisting of Vanadium Permendur poles and NdFeB magnets. The variable gap necessitates a phase shifter between each pair of undulators. The phase shifters are of pure permanent magnet design consisting of NdFeB magnets. The NdFeB magnets in the SXR undulators are of type VACODYM 956 DTP and have a remnant field temperature coefficient of\(^2\) \((1/B_r)dB_r/dT \approx -1.0 \times 10^{-3}\) 1/deg C. The NdFeB magnets in the HXR undulators are of type Neorem 776T and have a remnant field temperature coefficient of\(^3\) \((1/B_r)dB_r/dT \approx -0.8 \times 10^{-3}\) 1/deg C. The NdFeNB magnets in the phase shifters are of type VACODYM 983 TP and have a remnant field temperature coefficient of\(^4\) \((1/B_r)dB_r/dT \approx -0.9 \times 10^{-3}\) 1/deg C. In addition to this temperature dependence, the gap of the undulators and phase shifters will change slightly from the encoder values at different temperatures due to thermal expansion of the mechanical structures. In this note we explore how the temperature dependence of the remnant field and the gap affects the undulator $K$ values and phase matching, and the phase shifter phase integrals.

Measurements of the undulator and phase shifter parameters were made at different temperatures. These measurements provide a temperature calibration. The tolerances on the undulator and phase shifter strengths are very tight. The undulators and phase shifters are calibrated at 20.0 deg C, but are used in the tunnel over a temperature range of 19 to 21 deg C. The temperature effects must be accounted for by measuring each device’s temperature in the tunnel and performing corrections based on the temperature calibration measurements. Without the temperature correction, meeting the tolerance on $K$ and the phase integral would not be possible. This note presents the temperature calibration correction factors.

Because of the production measurement schedule, the time available for the temperature calibrations was very limited. This necessitated a two point calibration. Measuring at more temperatures

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\(^1\)Work supported in part by the DOE Contract DE-AC02-76SF00515. This work was performed in support of the LCLS project at SLAC.


\(^3\)Neorem data sheet courtesy of Diego Arbelaez, LBNL.

over a larger range is highly desirable for a more accurate calibration. We believe, however, that the two point calibration is accurate enough to turn on LCLS-II.

Only one type of each device was measured for this test. The SXR undulator was SXU-021, The IXR undulator was IIXU-031. The SXR phase shifter was SXPS-16336, and the IXR phase shifter was HXPS-16319. We assume that the calibration results from these devices apply to all undulators and phase shifters because of the similarity of the units. If further temperature calibrations are done in the future, the validity of this assumption can be tested.

2 Accuracy Requirements

In order to evaluate the effect of temperature changes on the undulators and phase shifters, the accuracy for setting the undulator $K$ values and doing the phase matching, and the accuracy for setting the phase shifter phase integrals must be specified. These quantities have the tightest tolerances and the strongest temperature dependence. Other quantities like field integrals and phase errors have little temperature dependence. The phase matching is temperature dependent, but the correction is derived from the $K$ value temperature correction as will be demonstrated later.

The LCLS-II undulator requirements come from a Physics Requirements Document\textsuperscript{5}. The temperature dependent requirements of interest are:

1. The SXR $K$ value must be set to $\pm 3 \times 10^{-4}$ at all gap settings and all temperatures in the tunnel.

2. The SXR total phase advance in the 4.400000 meter long cell must be known and corrected to a nominal value to $\pm 10$ degrees.

3. The HXR undulator $K$ value must be set to $\pm 2.3 \times 10^{-4}$ at all gap settings and all temperatures in the tunnel.

4. The HXR total phase advance in the 4.012667 meter long cell must be known and corrected to a nominal value to $\pm 5$ degrees.

The LCLS-II phase shifter requirements also come from a Physics Requirements Document\textsuperscript{6}. The temperature dependent requirements are summarized below.

1. The SXR phase change of the phase shifter must be accurate to $5.8^\circ$ at all operational gap settings and temperatures. Equivalently, the phase integral of the phase shifter must be accurate to $3.23 \ T^2\text{mm}^3$.

2. The HXR phase change of the phase shifter must be accurate to $2.9^\circ$ at all operational gap settings and temperatures. Equivalently, the phase integral of the phase shifter must be accurate to $0.67 \ T^2\text{mm}^3$.

3 Temperature Effect Analysis

In this section we perform an analysis of how the strength of the LCLS-II undulators and phase shifters varies with temperature. This lets us understand the temperature dependence, and it lets us compare the measured calibrations to expected values. In the discussion below, a calculation is first performed to find the dependence of the undulator $K$ value on temperature. This is followed by a calculation of how the phase shifter phase integral varies with temperature. We find that the


permanent magnet remnant field and the gap are the parameters of interest. The remnant field temperature dependence is a parameter specified by the magnet manufacturer. The undulator and phase shifter gaps are then discussed and the effect of temperature is analyzed.

3.1 Field In A Hybrid Undulator

Both the SXR and HXR undulators are of hybrid design consisting of permanent magnets and steel poles. The magnetic field in a hybrid undulator depends on the gap, the remnant field in the permanent magnets, and the magnet and pole dimensions. Of these, only the remnant field and the gap vary significantly with temperature. In order to find how the $K$ value varies with temperature, we first find how the $K$ value varies with remnant field and gap. We use a model of the undulator where the field varies sinusoidally along the undulator with only the first harmonic. We derive an expression for the magnetic field in terms of the permanent magnet remnant field and gap. From this, the $K$ value is calculated. Higher harmonic terms in the field are ignored. Saturation in the magnet poles is ignored. As we will see, this model agrees with the measured $K$ values fairly well.

Figure 1 shows a cross section of a hybrid undulator. The blue poles are assumed to have infinite permeability. The magnets have the direction of the remnant field indicated. Dimensions of the magnet and pole lengths along the z-axis, and heights along the y-axis are shown. We assume the magnet assembly is wide enough that variations in the x-direction can be ignored.

![Figure 1: Schematic of the magnet assembly of a hybrid undulator.](image)

The field in the permanent magnets is given by

$$\mathbf{B}_m = \mu_0 (\mathbf{H}_m + \mathbf{M})$$  \hspace{1cm} (1)

where $\mathbf{M}$ is the magnetization density which is assumed constant. Let

$$\mu_0 \mathbf{M} = \mathbf{B}_r$$  \hspace{1cm} (2)

where $\mathbf{B}_r$ is defined to be the remnant field in the magnet. With this definition, the field in the magnet is given by

$$\mathbf{B}_m = \mu_0 \mathbf{H}_m + \mathbf{B}_r$$  \hspace{1cm} (3)

The fields in the magnets are all in the z-direction. We will drop the vector arrow above the symbols and assume each term is the z-component of the vector. We also assume the fields in the magnets are uniform so there is no variation in x, y, or z.
The field in the undulator gap has

\[ \nabla \times \vec{B} = 0 \]  
\[ \nabla \cdot \vec{B} = 0 \]  \hspace{1cm} (4)  
\hspace{1cm} (5)

\[ \vec{B} \] is given by a potential function \( \Phi \) that obeys

\[ \nabla^2 \Phi = 0 \]  \hspace{1cm} (6)

The solution with periodic variation in \( z \) is given by a series of terms. We consider only the dominant first harmonic.

\[ \Phi = \Phi_0 \sinh(k_u y) \cos(k_u z) \]  \hspace{1cm} (7)

where \( y = 0 \) is given by the midplane of the gap and \( z = 0 \) is in the center of a pole pair with a south pole on the top. The fields from this potential are

\[ B_y = B_0 \cosh(k_u y) \cos(k_u z) \]  \hspace{1cm} (8)
\[ B_z = -B_0 \sinh(k_u y) \sin(k_u z) \]  \hspace{1cm} (9)

where \( k_u = 2\pi/\lambda_u \), where \( \lambda_u \) is the undulator period. \( B_0 \) is the peak field in the \( y \)-direction at \( y = 0 \).

The total flux going into a pole is zero. Consider the right half of the south pole at \( z = 0 \). The flux going into the half pole is

\[ -B_m h_m w + B_0 \cosh(k_u g/2) \frac{l_p}{2} w = 0 \]  \hspace{1cm} (10)

where \( h_m \) is the height of the magnet and pole, \( w \) is the width in the \( x \)-direction, \( l_p \) is the length of the pole in the \( z \)-direction, and \( g \) is the undulator gap. This gives

\[ B_m = B_0 \frac{l_p}{2h_m} \cosh(k_u g/2) \]  \hspace{1cm} (11)

Since there are no free currents,

\[ \nabla \times \vec{H} = 0 \]  \hspace{1cm} (12)

or the line integral of \( \vec{H} \) along the dotted path in figure 1 is zero. The dotted path is horizontal along \( y = 0 \), vertical at \( z = \lambda_u/4 \), horizontal at any point in the magnet and pole, and vertical at \( z = 0 \). \( \vec{H} = 0 \) in the pole because of the assumed infinite permeability. \( H_z = 0 \) along the undulator axis at \( y = 0 \), and \( H_y = 0 \) at \( z = \lambda_u/4 \) from the equations for the fields. The only contributions to the line integral come from the segments marked 1 and 2 in the figure. The line integral gives

\[ -H_m \frac{l_m}{2} - \int_0^{g/2} \frac{B_2}{\mu_0} dy = 0 \]  \hspace{1cm} (13)

Performing the integral gives

\[ \mu_0 H_m = -B_0 \frac{2}{l_m k_u} \sinh(k_u g/2) \]  \hspace{1cm} (14)

Combining the expressions for \( B_m \) and \( H_m \) in the equation

\[ B_m = \mu_0 H_m + B_r \]  \hspace{1cm} (15)

gives

\[ B_0 \frac{l_p}{2h_m} \cosh(k_u g/2) = -B_0 \frac{2}{l_m k_u} \sinh(k_u g/2) + B_r \]  \hspace{1cm} (16)
The peak field on the undulator axis is

\[ B_0 = B_r \frac{1}{\frac{e}{mc} \frac{1}{2k_m} \cosh(k_u \frac{g}{2}) + \frac{2}{k_m} \sinh(k_u \frac{g}{2})} \]  \hspace{1cm} (17)

The \( K \) value for a sinusoidal field is given by

\[ K = \frac{e}{mc} \frac{1}{k_u} B_0 \]  \hspace{1cm} (18)

where \( e \) is the magnitude of the electron charge, \( m \) is the electron mass, and \( c \) is the speed of light.

\[ K = B_r \frac{e}{mc} \frac{1}{k_u} \frac{1}{\frac{e}{mc} \frac{1}{2k_m} \cosh(k_u \frac{g}{2}) + \frac{2}{k_m} \sinh(k_u \frac{g}{2})} \]  \hspace{1cm} (19)

This formula was compared to the SXU-021 measurements. Figure 2 shows the comparison. All magnet and pole dimensional values used in the calculation were actual SXU-021 values. The value for \( B_r \) (1.29 T) came from the magnet manufacturer\(^7\). The agreement is very good at large gaps and differs slightly at small gaps where pole saturation and the effects of additional harmonics become important. We take the good agreement with the measurements as evidence that the form of the function is generally correct. Note that \( K \) is proportional to \( B_r \) at all gaps. This simplifies the analysis of how temperature changes resulting in changes of \( B_r \) affect \( K \).

![Figure 2: Calculated and measured K value for SXU-021.](image)

The temperature effects can now be separated and studied. The change of \( K \) with temperature is given by

\[ \frac{dK}{dT} = \frac{\partial K}{\partial B_r} \frac{dB_r}{dT} + \frac{\partial K}{\partial g} \frac{dg}{dT} \]  \hspace{1cm} (20)

and

\[ \frac{1}{K} \frac{dK}{dT} = \frac{1}{B_r} \frac{dK}{dT} + \frac{1}{K} \frac{\partial K}{\partial g} \frac{dg}{dT} \]  \hspace{1cm} (21)

\(^7\)Value from www.vacuumschmelze.com.
From the measurements of SXU-021 and HXU-031 used in this note, we have the following maximum values.

\[
\frac{1}{K} \frac{\partial K}{\partial g} = -1.14 \times 10^{-4} \text{ } 1/\mu\text{m} \quad \text{SXR} \tag{22}
\]

\[
\frac{1}{K} \frac{\partial K}{\partial g} = -1.58 \times 10^{-4} \text{ } 1/\mu\text{m} \quad \text{HXR} \tag{23}
\]

Since the formula for the gap dependence of the \( K \) value is complicated, we will use these values to make worst case estimates of the effect of temperature dependent gap changes.

### 3.2 Field In A PPM Phase Shifter

Both the SXR and HXR phase shifters are of a pure permanent magnet design. As noted above, the field inside a magnet is given by

\[
\vec{B}_m = \mu_0 \vec{H}_m + \vec{B}_r \tag{24}
\]

where the remnant field \( \vec{B}_r \) is assumed constant. The fields obey the equations

\[
\nabla \times \vec{H}_m = 0 \tag{25}
\]

\[
\nabla \cdot \vec{B}_m = 0 \tag{26}
\]

Inserting equation 24 into equation 26, we find

\[
\nabla \cdot \vec{H}_m = -\frac{1}{\mu_0} \nabla \cdot \vec{B}_r \tag{27}
\]

We define the magnetic volume charge density as \( \rho_m = -\frac{1}{\mu_0} \nabla \cdot \vec{B}_r \). Equations 25 and 27 become

\[
\nabla \times \vec{H}_m = 0 \tag{28}
\]

\[
\nabla \cdot \vec{H}_m = \rho_m \tag{29}
\]

The fields outside the permanent magnets have this same form with \( \rho_m = 0 \). So in general, the fields inside the magnets and outside can be written

\[
\nabla \times \vec{H} = 0 \tag{30}
\]

\[
\nabla \cdot \vec{H} = \rho_m \tag{31}
\]

The solution for \( \vec{H} \) is derived in an analogous manner to electrostatics by using the magnetic scalar potential \( \phi_m \), where \( \vec{H} = -\nabla \phi_m \).

\[
\nabla^2 \phi_m = -\rho_m \tag{32}
\]

Using standard techniques, one solves for \( \phi_m \).

\[
\phi_m(\vec{x}) = \frac{1}{4\pi} \int \frac{\rho_m(\vec{x}')}{|\vec{x}' - \vec{x}|} dV' \tag{33}
\]

In this expression, care must be used at the boundaries of a permanent magnet block where \( \rho_m \) is singular. We introduce a surface charge density to deal with the boundaries. The magnetic
surface charge density is derived from the magnetic volume charge density using the definition of divergence:

\[ \nabla \cdot \vec{B}_r = \lim_{V \to 0} \frac{\int \vec{B}_r \cdot \vec{d}A}{V} \]  

(34)

where the integral is over the surface of the volume and \( \vec{d}A \) is an area element oriented toward the outer normal to the surface. Consider a small volume at the surface of a block. The magnetic charge in the volume is \( \rho_m dV \). The definition of divergence lets us rewrite the charge at the surface in terms of a surface charge density as follows.

\[
\begin{align*}
\rho_m dV &= -\frac{1}{\mu_0} \nabla \cdot \vec{B}_r \ dV \\
&= -\frac{1}{\mu_0} \lim_{V \to 0} \int \vec{B}_r \cdot \vec{d}A \\
&= \frac{1}{\mu_0} \vec{B}_r \cdot \vec{d}A \\
&= \sigma_m \ dA
\end{align*}
\]

(35) (36) (37) (38)

where \( \sigma_m = \frac{1}{\mu_0} \vec{B}_r \cdot \hat{n} \), where \( \hat{n} \) is the unit vector normal to the block, pointing out of the block.

Using the volume charge density inside the blocks and the surface charge density on the boundaries, the solution for \( \phi_m \) is

\[
\phi_m(\vec{x}) = \frac{1}{4\pi} \int \frac{\rho_m(\vec{x}')}{|\vec{x}' - \vec{x}|^3} dV' + \frac{1}{4\pi} \int \frac{\sigma_m(\vec{x}')}{|\vec{x}' - \vec{x}|^3} dA'
\]

(39)

The field \( \vec{H} = -\nabla \phi_m \) has the following form.

\[
\vec{H}(\vec{x}) = \frac{1}{4\pi} \int \frac{\rho_m(\vec{x}') \vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3} dV' - \frac{1}{4\pi} \int \frac{\sigma_m(\vec{x}') \vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3} dA'
\]

(40)

or

\[
\vec{H}(\vec{x}) = \frac{1}{4\pi} \int \frac{\rho_m(\vec{x}') \vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3} dV' + \frac{1}{4\pi} \int \frac{\sigma_m(\vec{x}') \vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3} dA'
\]

(41)

If we assume the blocks have uniform magnetization, then \( \rho_m = 0 \) inside the blocks. In this case the field is

\[
\vec{H}(\vec{x}) = \frac{1}{4\pi} \int \frac{\sigma_m(\vec{x}') \vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3} dA'
\]

(42)

or

\[
\vec{H}(\vec{x}) = \frac{1}{4\pi} \int \frac{1}{\mu_0} \left[ \vec{B}_r(\vec{x}') \cdot \hat{n}(\vec{x}') \right] \frac{\vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3} dA'
\]

(43)

\[
\vec{B}(\vec{x}) = \frac{1}{4\pi} \int \left[ \vec{B}_r(\vec{x}') \cdot \hat{n}(\vec{x}') \right] \frac{\vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3} dA'
\]

(44)

These are the formulas which are used to calculate the fields from the PPM phase shifter blocks. The field \( \vec{B}(\vec{x}) \) is calculated numerically by dividing the surface of each magnet block into area elements. The manufacturer specifies \( \vec{B}_r \) for the block. \( \vec{B}_r \cdot \hat{n} \) is calculated at each area element and multiplied by the area of the element to get the equivalent magnetic charge.

\[
q_i = \vec{B}_r \cdot \hat{n} \ dA_i
\]

(45)
The charges are multiplied by the spatial factor in equation 44 and summed to calculate the field.

\[
\vec{B}(\vec{x}) = \frac{1}{4\pi} \sum_i q_i \frac{\vec{x} - \vec{x}_i}{|\vec{x} - \vec{x}_i|^3}
\]  

(46)

The sum is over all charges on each block, and all blocks in the phase shifter.

This formula was numerically compared to the SXPS-16336 measurements. Figure 3 shows the comparison. All magnet dimensional values used in the calculation were actual SXPS-16336 values. The value for \( B_r \) (1.22 T) came from the magnet manufacturer\(^8\). The agreement is very good. The model accurately predicts the behavior of the phase shifter.

![Figure 3: Calculated and measured SXR phase shifter phase integral vs gap.](image)

The model predicts that the magnetic field on the axis is given by a function of the form

\[
B_y = B_r f'(z, g)
\]

(47)

where \( B_r = |\vec{B}_r| \). Since all magnets have the same \( B_r \), that term can be taken out of the integral. \( f'(z, g) \) is a function of position \( z \) and the phase shifter gap \( g \), and does not depend on \( B_r \). This lets us calculate the temperature effects of \( B_r \) and the gap independently.

The phase integral for a \( B_y \) field is given by

\[
PI = \int \left( \int z B_y dz' \right)^2 dz
\]

(48)

This gives

\[
PI = B_r^2 \int \left( \int f'(z', g) dz' \right)^2 dz
\]

(49)

or

\[
PI = B_r^2 f(g)
\]

\(^8\)Value from www.vacuumschmelze.com.
The temperature effects can now be separated and studied. The change of $PI$ with temperature is given by

$$\frac{dPI}{dT} = \frac{\partial PI}{\partial B_r} \frac{dB_r}{dT} + \frac{\partial PI}{\partial g} \frac{dg}{dT}$$

(51)

and

$$\frac{1}{PI} \frac{dPI}{dT} = 2 \frac{1}{B_r} \frac{dB_r}{dT} + \frac{1}{PI} \frac{dg}{dT}$$

(52)

From the measurements of SXPS-16336 and HXPS-16319 used in this note, we have the following maximum values.

$$\frac{1}{PI} \frac{\partial PI}{\partial g} = -8.73 \times 10^{-5} \text{ 1/} \mu \text{m  SXR}$$

(54)

$$\frac{1}{PI} \frac{\partial PI}{\partial g} = -1.24 \times 10^{-4} \text{ 1/} \mu \text{m  HXR}$$

(55)

Since the formula for the gap dependence of the phase integral is complicated, we will use these values to make worst case estimates of the effect of temperature dependent gap changes.

### 3.3 Remnant Field Change With Temperature

The magnet manufacturers give a specification for how the remnant field of a given magnet material changes with temperature. As noted earlier, the NdFeB magnets in the SXR undulators are of type VACODYM 956 DTP and have a specified remnant field temperature coefficient of

$$\frac{1}{B_r} \frac{dB_r}{dT} = -1.0 \times 10^{-3} \text{ 1/} \text{deg C}$$

(56)

This value is gap independent in the undulators and is a constant describing the permanent magnet material property. From the undulator analysis above, a change in $B_r$ gives a $K$ change of

$$\frac{1}{K} \frac{dK}{dT} \bigg|_{B_r} = \frac{1}{B_r} \frac{dB_r}{dT} = -1.0 \times 10^{-3} \text{ 1/} \text{deg C}$$

(57)

The NdFeB magnets in the HXR undulators are of type Neorem 776T and have a specified remnant field temperature coefficient of

$$\frac{1}{B_r} \frac{dB_r}{dT} = -0.8 \times 10^{-3} \text{ 1/} \text{deg C}$$

(58)

From the undulator analysis above, a change in $B_r$ gives a $K$ change, independent of gap, of

$$\frac{1}{K} \frac{dK}{dT} \bigg|_{B_r} = \frac{1}{B_r} \frac{dB_r}{dT} = -0.8 \times 10^{-3} \text{ 1/} \text{deg C}$$

(59)

The NdFeNB magnets in the phase shifters are of type VACODYM 983 TP and have a specified remnant field temperature coefficient of

$$\frac{1}{B_r} \frac{dB_r}{dT} = -0.9 \times 10^{-3} \text{ 1/} \text{deg C}$$

(60)

From the phase shifter analysis above, a $B_r$ change gives a $PI$ change of

$$\frac{1}{PI} \frac{dPI}{dT} \bigg|_{B_r} = 2 \frac{1}{B_r} \frac{dB_r}{dT} = -1.8 \times 10^{-3} \text{ 1/} \text{deg C}$$

(61)
3.4 Gap Change With Temperature

As the temperature of the undulators and phase shifters change, their gap changes due to thermal expansion of the materials used to make them. A control system is used to set the gap, but the control is also affected by thermal expansion of materials. We distinguish between the gap given by the control system and the actual physical gap by using capital $G$ for the physical gap in this section of the note. In other sections the distinction is either not important, or when discussing the measurements, gap will always refer to the control system’s measurement of the gap. We now consider how the undulator and phase shifter physical gaps change with temperature.

As an example, consider an SXR undulator shown schematically in figure 4. The same analysis applies to the IIIXR undulator and both the SXR and IIIXR phase shifters. Aluminum strongbacks hold magnet/pole assemblies. A control system reads a scale on the upper strongback in order to set the gap. The read head of the scale is positioned by an arm that is attached to the lower strongback. The control system guarantees a specified reading on the linear scale. Thermal expansion of materials, however, can change the actual undulator gap.

![Diagram of undulator gap](image)

Figure 4: The undulator gap is set by the control system and thermal expansion of the various materials.

In the figure, the following dimensions are identified:

- $L_1$ = length of arm positioning read head of the scale
- $L_2$ = length of strongback from fixed scale position to steel pole
- $L_3$ = length of steel pole
- $L_4$ = length of strongback from arm attachment point to steel pole
- $G$ = gap height

Let the position reference be the uppermost attachment point of the arm on the bottom strongback. The arm holds the read head of the scale. At a given gap setting, when the temperature changes, the top strongback moves with the arm such that the read head is always at the specified line on the scale. The control system guarantees the scale reading by using a servo loop when the gap is set. A brake is then applied and temperature may change the gap afterward as discussed...
below, but for now we are focusing only on setting the gap. From the figure, the gap height is given by

\[ G = L_1 - L_2 - 2L_3 - L_4 \]  

(62)

When the undulator is at a different temperature, thermal expansion makes the various lengths change, and the gap change is given by

\[ \delta G = \delta L_1 - \delta L_2 - 2\delta L_3 - \delta L_4 \]  

(63)

The lengths change by the thermal expansion coefficient multiplied by the length multiplied by the temperature change. In order to study temperature fluctuations later, we consider the general case where the arm may be at a different temperature than the strongback. Let \( \delta T_{\text{arm}} \) be the temperature change of the arm, and let \( \delta T \) be the temperature change of the strongbacks and magnet poles. We also consider the case where the arm material is slightly different than the strongback material. Let \( C_{\text{arm}} \) be the thermal expansion coefficient of the aluminum arm, and let \( C_a \) be the thermal expansion coefficient of the aluminum strongbacks. Let \( C_s \) be the thermal expansion coefficient of the steel poles. In this case

\[ \delta G = C_{\text{arm}}L_1\delta T_{\text{arm}} - [C_aL_2 + 2C_sL_3 + C_aL_4] \delta T \]  

(64)

Using equation 62, we can rewrite this expression as

\[ \delta G = C_{\text{arm}}L_1\delta T_{\text{arm}} - [C_a(L_1 - 2L_3 - G) + 2C_sL_3] \delta T \]  

(65)

Combining terms, we find

\[ \delta G = C_aG\delta T + [C_{\text{arm}}L_1\delta T_{\text{arm}} - C_aL_1\delta T] + [(C_a - C_s)(2L_3) \delta T] \]  

(66)

The second term can be further factored into a term depending on the difference in the thermal expansion coefficients between the arm and the strongbacks, and a term that depends on the difference in temperature between the arm and the strongbacks.

\[ C_{\text{arm}}L_1\delta T_{\text{arm}} - C_aL_1\delta T = C_{\text{arm}}L_1\delta T_{\text{arm}} - C_aL_1\delta T_{\text{arm}} + C_aL_1\delta T_{\text{arm}} - C_aL_1\delta T \]

\[ = (C_{\text{arm}} - C_a)L_1\delta T_{\text{arm}} + C_aL_1(\delta T_{\text{arm}} - \delta T) \]  

(67)

Combining terms, the change in the gap due to temperature changes is

\[ \delta G = C_aG\delta T + (C_{\text{arm}} - C_a)L_1\delta T_{\text{arm}} + C_aL_1(\delta T_{\text{arm}} - \delta T) + (C_a - C_s)(2L_3) \delta T \]  

(68)

This equation indicates that the gap changes due to temperature come from four sources. The first term shows that the control system servo loop makes the gap change with the thermal expansion coefficient of aluminum over the length of the gap in the aluminum arm. The second term shows that the gap changes due to the arm being made of a different material than the strongbacks. The third term shows that the gap changes if the arm is at a different temperature than the strongbacks as might be the case if the temperature is changing and equilibrium in the strongbacks has not been reached. The fourth term shows that the gap changes because of the difference in thermal expansion of the aluminum arm and the steel poles over the length of the steel poles. This term is absent in the phase shifters.

The undulator is calibrated in the MMF at 20 deg C. In the tunnel, the temperature can be different by up to 1 deg C. We wish to know the effect of the 1 deg C temperature change on the undulator K value due to mechanical gap changes.

We begin by estimating the size of each term in equation 68. We use the following parameters.

\[ C_a = 20 \mu m/m/\text{deg C} \]  

(69)
\[ C_{\text{arm}} = 20.2 \ \mu m/m/\text{deg C} \quad (70) \]
\[ C_s = 10 \ \mu m/m/\text{deg C} \quad (71) \]
\[ G = 0.02 \ m \quad (72) \]
\[ L_1 = 0.5 \ m \quad (73) \]
\[ L_3 = 0.01 \ m \quad (74) \]
\[ \delta T = 1 \ \text{deg C} \quad (75) \]
\[ \delta T_{\text{arm}} = 1.1 \ \text{deg C} \]

The thermal expansion coefficient of the arm is taken to be 1% different than the thermal expansion coefficient of the strongbacks. The temperature of the arm is taken to be 0.1 deg C different than the strongbacks due to a small rapid change in the ambient temperature.

With these parameters, the first term in equation 68 is
\[ \delta G_1 = C_a G \delta T \quad (76) \]
\[ = (20 \ \mu m/m/\text{deg C}) (0.02 \ m) (1 \ \text{deg C}) \]
\[ = 0.4 \ \mu m \]

This term is relatively small, but significant. The larger the gap, the greater the effect of the thermal expansion of the aluminum arm over the length of the gap.

The second term in equation 68 is
\[ \delta G_2 = (C_{\text{arm}} - C_s) L_1 \delta T_{\text{arm}} \quad (77) \]
\[ = (20.2 \ \mu m/m/\text{deg C} - 20 \ \mu m/m/\text{deg C}) (0.5 \ m)(1.1 \ \text{deg C}) \]
\[ = 0.1 \ \mu m \]

This term is fairly small. The difference in material properties between the arm and the strongbacks is reasonable, but it is not a major source of error.

The third term in equation 68 is
\[ \delta G_3 = C_a L_1 (\delta T_{\text{arm}} - \delta T) \quad (78) \]
\[ = (20 \ \mu m/m/\text{deg C})(0.5 \ m)(1.1 \ \text{deg C} - 1.0 \ \text{deg C}) \]
\[ = 1.0 \ \mu m \]

This term is fairly large because of the large thermal expansion coefficient of aluminum and the large length of the arm. We are sensitive to changes in air temperature that are fast compared to the time it takes the strongbacks to reach equilibrium.

The fourth term in equation 68 is
\[ \delta G_4 = (C_a - C_s) (2L_3) \delta T \quad (79) \]
\[ = (20 \ \mu m/m/\text{deg C} - 10 \ \mu m/m/\text{deg C}) 2(0.01 \ m) (1 \ \text{deg C}) \]
\[ = 0.2 \ \mu m \]

This term is relatively small.

If we assume that the tunnel temperature is very stable, we can ignore the third term. In this case, the 1 deg C temperature change between the MMF and the tunnel causes the gap to increase by 0.7 \ \mu m. This gives
\[ \frac{dG}{dT} = 0.7 \ \mu m/\text{deg C} \quad (80) \]
As derived above, the $K$ value varies with temperature according to
\[
\frac{1}{K} \frac{dK}{dT} = \frac{1}{B_r} \frac{dB_r}{dT} + \frac{1}{K} \frac{dG}{dT} \tag{81}
\]
and the gap dependent term goes as
\[
\frac{1}{K} \frac{dK}{dT}|_G = \frac{1}{K} \frac{dG}{dT} \tag{82}
\]
We use the following maximum values derived from the measurements in order to make a worst case estimate.
\[
\frac{1}{K} \frac{dK}{dG} = -1.14 \times 10^{-4} \text{ 1/\mu m  SXR} \tag{83}
\]
\[
\frac{1}{K} \frac{dK}{dG} = -1.58 \times 10^{-4} \text{ 1/\mu m  HXR} \tag{84}
\]
Using the 0.7 \mu m gap change per deg C, we find for the SXR undulators
\[
\left. \frac{1}{K} \frac{dK}{dT} \right|_G = \left( -1.14 \times 10^{-4} \text{ 1/\mu m} \right) \left( 0.7 \mu m / \text{deg C} \right) \tag{85}
\]
\[
= -8.0 \times 10^{-5} \text{ 1/deg C  SXU} \tag{86}
\]
This effect is significantly smaller than the effect from the variation of remnant field of the magnets with temperature and is within the tolerance.

For the HXR undulators we use the same 0.7 \mu m/deg C gap change to find
\[
\left. \frac{1}{K} \frac{dK}{dT} \right|_G = \left( -1.58 \times 10^{-4} \text{ 1/\mu m} \right) \left( 0.7 \mu m / \text{deg C} \right) \tag{87}
\]
\[
= -1.1 \times 10^{-4} \text{ 1/deg C  HXU} \tag{88}
\]
The effect of the gap change with temperature for the HXU is also within the tolerance.

For the phase shifters, we derived above
\[
\left. \frac{1}{PI} \frac{dPI}{dT} \right|_G = \frac{1}{PI} \frac{dPI}{dG} \frac{dG}{dT} \tag{89}
\]
with
\[
\frac{1}{PI} \frac{dPI}{dG} = -8.73 \times 10^{-5} \text{ 1/\mu m  SXR} \tag{90}
\]
\[
\frac{1}{PI} \frac{dPI}{dG} = -1.24 \times 10^{-4} \text{ 1/\mu m  HXR} \tag{91}
\]
Assuming the 0.7 \mu m gap change per deg C, this gives
\[
\left. \frac{1}{PI} \frac{dPI}{dT} \right|_G = -6.1 \times 10^{-5} \text{ 1/deg C  SXU} \tag{92}
\]
\[
\left. \frac{1}{PI} \frac{dPI}{dT} \right|_G = -8.7 \times 10^{-5} \text{ 1/deg C  HXU} \tag{93}
\]
Putting in maximal values of $PI = 4153 \text{ T}^2 \text{mm}^3$ for SXPS-16336, and $PI = 614 \text{ T}^2 \text{mm}^3$ for HXPS-16319, we find maximal values
\[
\left. \frac{dPI}{dT} \right|_G = -0.25 \text{ T}^2 \text{mm}^3 / \text{deg C  SXU} \tag{94}
\]
\[
\left. \frac{dPI}{dT} \right|_G = -0.053 \text{ T}^2 \text{mm}^3 / \text{deg C  HXU} \tag{95}
\]
These changes for a 1 deg C temperature change are within the tolerance.
3.5 Effect Of Temperature Fluctuations

As an aside, suppose now that the air conditioner in the tunnel fluctuates by 0.1 deg C. In this case, the arm temperature will change more rapidly than the strongback temperature. The $\delta G_3$ term given above will now be important. The gap will change by 1.0 $\mu$m with the 0.1 deg C temperature rise. The $K$ value will change by

$$\frac{1}{K} \frac{dK}{dT} |_G = -8.0 \times 10^{-8} \text{ per 0.1 deg C} \quad \text{SXU}$$  \hfill (96)

$$\frac{1}{K} \frac{dK}{dT} |_G = -1.2 \times 10^{-4} \text{ per 0.1 deg C} \quad \text{HXU}$$

These changes are within tolerance but they exemplify the sensitivity to temperature changes of the arm.

3.6 Effect Of Motor Brakes

Another thing to note is that after the undulator gap is set, the control system locks the brakes on the motors and does not constantly reset the gap. For the case of the SXR undulators, the situation is shown in figure 5. Ignoring the height of the steel poles, the gap is given by

![Diagram of gap control](image)

Figure 5: After the gap is set, the motor brakes are applied and the gap is controlled by the thermal expansion of the undulator frame and the strongbacks.

$$G = D_1 - 2D_2$$  \hfill (97)

where $D_1$ is the height of the steel undulator frame between support points on the strongbacks, and $D_2$ is the height of the aluminum strongback from the support point to the gap. The change in gap with temperature is given by

$$\delta G = \delta D_1 - 2\delta D_2$$

$$\approx (C_s - C_a) D_1 \delta T$$  \hfill (98)
In this expression we ignored the small gap dimension compared to the overall undulator dimensions. For \( D_1 = 1.5 \) m and a temperature change of 0.1 deg C, the gap changes by

\[
\delta G = (10 \ \mu m/m/\text{deg C} - 20 \ \mu m/m/\text{deg C}) (1.5 \text{ m}) (0.1 \text{ deg C}) = -1.5 \ \mu m
\]  

(99)

The effect on K will be

\[
\frac{\delta K}{K} = (-1.14 \times 10^{-4} \ 1/\mu m) (-1.5 \mu m) = 1.7 \times 10^{-4}
\]  

(100)

If the time of the temperature rise lasts long enough for the magnets to also change temperature by 0.1 deg C, their effect on the K value change will be

\[
\left(\frac{\delta K}{K}\right)_{\text{mag}} = -10^{-4}
\]  

(101)

The combined effect will be a relative K value change of \( +0.7 \times 10^{-4} \). The K value will increase with temperature when the motor brakes are on. The control system can periodically check the undulator gap and correct it.

### 3.7 Temperature Effect Summary

In this section we calculated how the undulator K values and the phase shifter phase integrals change with temperature. The phase matching effects are calculated from the change in K and will be derived later. We noted that the gaps of the undulators and phase shifters are set by having the control system go to a certain encoder reading. After an analysis, we found that temperature changes had an effect on the actual gap when the encoder reading was held constant, but the dominant change in the K value and phase integral came from the change in the permanent magnet remnant field with temperature. Our result is that the expected temperature dependence of the SXR undulators should approximately be given by

\[
\frac{1}{K} \frac{dK}{dT} \simeq -1.0 \times 10^{-3} \text{ 1/deg C  SXU}
\]  

(102)

The temperature dependence of the HXR undulators should be given by

\[
\frac{1}{K} \frac{dK}{dT} \simeq -0.8 \times 10^{-3} \text{ 1/deg C  HXU}
\]  

(103)

The temperature dependence of both the SXR and HXR phase shifters should be given by

\[
\frac{1}{PI} \frac{dPI}{dT} \simeq -1.8 \times 10^{-3} \text{ 1/deg C}
\]  

(104)

These are expected values and we can compare them to the actual measured values, which we discuss next.
4 Temperature Calibration Plan

The plan we used to calibrate the SXR and HXR undulators and phase shifters for temperature changes was as follows:

1. Before December, 2019, set up the spare SXR undulator (SXU-021) at the SXR Kugler bench. Tune and calibrate the undulator.

2. In December, 2019, stop production measurements of the HXR undulators. Bring in a new HXR undulator (HXU-031) and tune and calibrate it.

3. In December, 2019, set up one calibrated HXR phase shifter (HIXPS-16319) at the upstream end of SXU-021, and one calibrated SXR phase shifter (SXPS-16336) at the downstream end of HIXU-021. (The HXR phase shifter, instead of being set up at the HIXR bench with the vertical gap, was set up at the SXR bench with the horizontal gap and horizontal probes so that it was calibrated in the same orientation that it is used.) The phase shifters are shown in figure 6.

4. To begin the temperature calibration, perform "final dataset" measurements on each undulator. The SXR and HXR measurements can be done in parallel. The phase shifters at the SXR bench should have their gap set to 15 mm in order to minimize stray fields. These measurements are to be done at 20.0 ± 0.1 deg C.

5. Perform "final dataset" measurements of each phase shifter with the SXR undulator gap at the 10 mm tuning gap to minimize stray fields. These measurements are to be done at 20.0 ± 0.1 deg C.

6. Verify that the Hall probe scans can be analyzed to reproduce the original calibrations of each phase shifter and undulator. The scans at the SXR bench must be segmented in order to isolate the undulator and each phase shifter.

7. At the end of December, 2019, before the holiday break, adjust the laboratory temperature set point to 21.0 deg C. The lab came to temperature equilibrium over the holiday break.

8. After the holiday break, in January, 2020, realign the Kugler benches at the new temperature 21.0 deg C. Calibrate the SXR and HXR Hall probes at 21.0 deg C.

9. Repeat the "final dataset" measurements of steps 4 and 5, only now at 21.0 deg C.

10. Analyze the measurement data to extract the undulator and phase shifter parameters at 21.0 deg C.

11. Set the laboratory temperature back to 20.0 deg C. Wait one week for the temperature of all items in the lab to come to equilibrium.

12. Realign the Kugler benches at 20.0 deg C. Calibrate the SXR and HXR Hall probes at 20.0 deg C.

13. Repeat the "final dataset" measurements of steps 4 and 5 at 20.0 deg C.

14. Verify that the original results of the 20.0 deg C measurements are obtained.

15. Calculate the temperature coefficients of the undulator and phase shifter parameters.
5 Temperature Calibrations

In the previous analysis we calculated how we expect the undulator $K$ values and phase shifter phase integrals to vary with temperature. We now proceed to the measurements.

5.1 SXR Undulator Calibration

As noted in the test plan, a "final dataset" was performed at 20 deg C, 21 deg C, and again at 20 deg C. This gave two temperature changes: the first from 20 deg C to 21 deg C, and the second from 21 deg C to 20 deg C. Each temperature change allows us to measure the change in $K$ and thus determine the temperature coefficient. In addition, each "final dataset" has a set of measurements as the gap is opening, and another set of measurements as the gap is closing. This gives two measurements of the change in $K$ for each of the two temperature changes, resulting in four determinations of the temperature coefficient. In this section we give the results of the four measurements of the SXR undulator temperature coefficient.

The SXR undulator had the phase shifter magnetic fields included in each measurement record. In order to find the $K$ value, the region of the phase shifters in the measurement was replaced in software by the background field value at the end of the measurement. After the replacement, the analysis progressed as normal. The $K$ values obtained by this method in the first 20 deg C data set agreed with measurements before the phase shifters were installed.

Figure 7 shows the temperature of SXU-021 at 20 deg C, 21 deg C, and again at 20 deg C as a function of gap during the measurements. The temperatures as the gap is opening for the $K$ measurements and as the gap is closing are shown individually. Note that when the temperature of the laboratory was brought back to 20 deg C for the final measurements, it did not return to the original value. The differences in the temperature coefficients used in the temperature coefficient calculations are shown in figure 8.

Figure 9 shows the differences in $K$ value as a function of gap. The tolerance on the $K$ value is also shown as the dashed lines. Without correction, the $K$ value at 21 deg C is far out of tolerance compared to the calibration value at 20 deg C. At small gaps, the effect of pole saturation is evident.

Figure 10 shows the temperature coefficient $(1/K)dK/dT$ as a function of gap. The mean value
of the four measurements is \((1/K)dK/dT = -1.006 \times 10^{-3}\) 1/deg C. The temperature coefficient is fairly constant with gap except at small gaps where pole saturation effects are evident. From the plot, for a 1 deg C temperature difference between the tunnel and the calibration temperature, the maximum deviation of the temperature coefficient from the mean gives \(dK/K \simeq 2. \times 10^{-4}\) which is within the \(3 \times 10^{-4}\) tolerance on setting the \(K\) value of the SXR undulators. Thus using only the mean value of the temperature coefficient should be sufficient for all gaps. The mean temperature coefficient agrees well with the expected value.
Figure 8: Temperature differences used in calculating the temperature coefficient.

Figure 9: The difference in $K$ values $K(21 \text{ deg C}) - K(20 \text{ deg C})$ as a function of gap. The dashed lines show the tolerance on setting the value of $K$. 
Figure 10: SXU temperature coefficient as a function of gap.
5.2 HXR Undulator Calibration

Figure 11 shows the temperature of HXU-031 at 20 deg C, at 21 deg C, and again at 20 deg C as a function of gap during the measurements. Figure 12 shows the differences in the temperatures used in the temperature coefficient calculations.

Figure 13 shows the difference in $K$ values as a function of gap. The tolerance on the $K$ value is also shown in the dashed lines. Without correction, the $K$ value at 21 deg C is far out of tolerance compared to the calibration value at 20 deg C.

Figure 14 shows the temperature coefficient $(1/K)dK/dT$ as a function of gap. The mean value is $(1/K)dK/dT = -0.7648 \times 10^{-3}$ 1/deg C. The temperature coefficient is fairly constant over a gap range of 7.2 mm to 20 mm, which covers the operating range of the undulator. From the plot, for a 1 deg C temperature difference between the tunnel and the calibration temperature, the maximum deviation of the temperature coefficient from the mean gives $dK/K \approx 1.5 \times 10^{-4}$ which is within the $2.3 \times 10^{-4}$ tolerance on setting the $K$ value of the HXR undulators. Thus using the mean value of the temperature coefficient should be sufficient for all gaps. The mean temperature coefficient agrees well with the expected value.
Figure 12: Temperature differences used in the temperature coefficient calculations.

Figure 13: The difference in $K$ values $K_{21 \text{ deg C}} - K_{20 \text{ deg C}}$ as a function of gap. The dashed lines show the tolerance on setting the value of $K$. 
Temperature Calibration Constant 1/K dK/dT

Mean = -7.648e-04 1/degC

Figure 14: HXU temperature coefficient as a function of gap.
5.3 SXR Phase Shifter Calibration

After "final dataset" measurements were made for SXU-021 at different gaps, the undulator gap was fixed and "final dataset" measurements were performed for the phase shifters. This was repeated at 20 deg C, 21 deg C, and again at 20 deg C. Like the undulators, the phase shifters had two temperature changes: the first from 20 deg C to 21 deg C, and the second from 21 deg C to 20 deg C. Each temperature change allows us to measure the change in phase integral and thus determine the temperature coefficient. In addition, each "final dataset" has a set of measurements as the phase shifter gap is opening, and another set of measurements as the gap is closing. This gives two measurements of the change in phase integral for each of the two temperature changes, resulting in four determinations of the temperature coefficient. In this section we give the results of the four measurements of the SXR phase shifter temperature coefficient.

Each measurement contained field values from both phase shifters, SXPS and HXPS, and the SXR undulator. The measurements around each phase shifter were cut out of the record and independently analyzed.

The phase integral values for the first 20 deg C measurements did not agree with the values previously obtained in the phase shifter calibration. At this point, we realized that there was a problem, and we found that the phase shifters were misaligned at the bench. Subsequent measurements at 21 deg C and 20 deg C were made on the phase shifter axis, but the schedule did not permit repeating the initial 20 deg C measurements before the temperature was changed to 21 deg C. In the following plots, the initial 20 deg C measurements are shown, but they are not used to determine the calibration constant.

Figure 15 shows the temperature of SXPS-16336 at 20 deg C, at 21 deg C, and again at 20 deg C as a function of gap during the measurements. The initial 20 deg C measurements were made under severe time constraints before the Christmas shutdown, and the phase shifter had not reached equilibrium when the measurements started. The power to the phase shifter was turned off during the shutdown and turned back on when the measurements at 21 deg C resumed. The phase shifter gets fairly warm because the encoder read head is mounted on a magnet jaw, and it takes a long time to come to equilibrium. The phase shifter was not in equilibrium during the initial 21 deg C measurements. The effect on the temperature coefficient is acceptable, however, and this data is used below in the calculations. Figure 16 shows the differences in the temperatures used in the temperature coefficient calculations.

Figure 17 shows the difference in phase integral values as a function of gap. The tolerance on the phase integral is also shown in the dashed lines. Without correction, the $P_I$ value at 21 deg C is out of tolerance compared to the calibration value at 20 deg C. The corresponding change in phase from the temperature change is shown in figure 18.

Figure 19 shows the temperature coefficient $(1/P_I)dP_I/dT$ as a function of gap. The mean value excluding the initial 20 deg C measurements is $(1/P_I)dP_I/dT = -1.645 \times 10^{-3}$ 1/deg C. This agrees fairly well with the expected value. The temperature coefficient using the 21 -> 20 deg C change is fairly constant over a gap range of 10 mm to 30 mm, which covers the operating range of the phase shifter.
Figure 15: SXPS-16336 temperature as a function of gap during the measurements.

Figure 16: Temperature differences used in the temperature coefficient calculations.
Figure 17: The difference in phase integral $PI(21 \text{ deg C}) - PI(20 \text{ deg C})$ as a function of gap. The dashed lines show the tolerance on setting the value of $PI$.

Figure 18: Change in phase from the phase shifter as the temperature is changed by 1 deg C.
Figure 19: SXPS temperature coefficient as a function of gap.
5.4 HXR Phase Shifter Calibration

The HXR phase shifter measurements were done in the same scans as the SXR phase shifter measurements. The same comments about alignment errors in the first 20 deg C measurements applies, and these measurements are not used in determining the temperature coefficient.

Figure 20 shows the temperature of HXPS-16319 at 20 deg C, at 21 deg C, and again at 20 deg C as a function of gap during the measurements. As noted above, the initial 20 deg C measurements were made under severe time constraints before the Christmas shutdown, and the phase shifter had not reached equilibrium when the measurements started. The power to the phase shifter was turned off during the shutdown and turned back on when the measurements at 21 deg C resumed. The phase shifter gets fairly warm because the encoder read head is mounted on a magnet jaw, and it takes a long time to come to equilibrium. The phase shifter was not in equilibrium during the initial 21 deg C measurements. The effect on the temperature coefficient is acceptable, however, and this data is used below in the calculations. Figure 21 shows the differences in the temperatures used in the temperature coefficient calculations.

Figure 22 shows the difference in phase integral values as a function of gap. The tolerance on the phase integral is also shown in the dashed lines. Without correction, the $P_I$ value at 21 deg C is out of tolerance compared to the calibration value at 20 deg C. The corresponding change in phase from the temperature change is shown in figure 23.

Figure 24 shows the temperature coefficient $(1/P_I) dP_I/dT$ as a function of gap. The mean value excluding the initial 20 deg C measurements is $(1/P_I) dP_I/dT = -1.626 \times 10^{-3}$ 1/deg C. This agrees well with the SXR phase shifter temperature coefficient, and also with the expected value. The temperature coefficient using the 21 -> 20 deg C change is fairly constant over a gap range of 10 mm to 30 mm, which covers the operating range of the phase shifter.
Figure 21: Temperature differences used in the temperature coefficient calculations.

Figure 22: The difference in phase integral $PI(21 \, \text{deg} \, C) - PI(20 \, \text{deg} \, C)$ as a function of gap. The dashed lines show the tolerance on setting the value of $PI$. 

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Figure 23: Change in phase from the phase shifter as the temperature is changed by 1 deg C.

Figure 24: HXPS temperature coefficient as a function of gap.
5.5 Temperature Calibration Summary

A summary of the measurements in this section is given in the following table. The values given are largely independent of gap.

<table>
<thead>
<tr>
<th>Device</th>
<th>Calibration Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>SXR Undulator</td>
<td>((1/K)dK/dT = -1.006 \times 10^{-6}) (1/\text{deg C})</td>
</tr>
<tr>
<td>HXR Undulator</td>
<td>((1/K)dK/dT = -0.7648 \times 10^{-3}) (1/\text{deg C})</td>
</tr>
<tr>
<td>SXR Phase Shifter</td>
<td>((1/PI)dPI/dT = -1.645 \times 10^{-3}) (1/\text{deg C})</td>
</tr>
<tr>
<td>HXR Phase Shifter</td>
<td>((1/PI)dPI/dT = -1.626 \times 10^{-4}) (1/\text{deg C})</td>
</tr>
</tbody>
</table>

6 Temperature Corrections

In this section we describe how to use the measurements of the previous section to set the undulators and phase shifters in the tunnel.

For this section, we emphasize that the undulators and phase shifters are set to a certain encoder reading. We use the term gap to mean the encoder reading after it has been offset and scaled to correspond to the physical gap. When we measured the temperature calibration factors \((1/K)dK/dT\) and \((1/PI)dPI/dT\), we set the encoder reading to the same value at different temperatures. We use the fact that as the temperature is changed, the control system keeps the encoder reading the same in the discussion that follows.

6.1 Set K

The undulators are calibrated at temperature \(T_c\) which is close to 20 deg C. A calibration file is produced giving the undulator gap (the offset and scaled encoder reading) and \(K\) value at a range of settings at \(T_c\). Spline fits to the data in the file are used to give the \(K\) value at any gap, or the gap at any \(K\) value, all at \(T_c\).

We desire to set the undulator gap to give a \(K\) value, \(K_t\), at tunnel temperature \(T_t\). In order to use the \(K\) vs gap spline file to find the gap, we must first use the temperature calibration from this note to find the \(K\) value at the calibration temperature, \(K_c\). The change in \(K\) at fixed gap going from \(T_t\) to \(T_c\) is

\[
\delta K = [(1/K)dK/dT](K_t)(T_c - T_t)
\]  

(105)

where the term in the brackets is the constant calibration factor. The \(K\) value at fixed gap at the calibration temperature is

\[
K_c = K_t + \delta K
\]  

(106)

The spline fit to the \(K\) vs gap calibration data gives the undulator gap \(g_c\) which gives \(K_c\) at \(T_c\). The undulator gap must be set to \(g_c\).

As a consistency check, apply the temperature calibration at fixed \(g_c\) going from \(T_c\) to \(T_t\). The change in \(K\) is \(-\delta K\) given above. The \(K\) value at the tunnel temperature is

\[
K_t = K_c + (-\delta K)
\]  

(107)

which is the desired value. So using the temperature calibration to find \(K_c\) and setting the gap to \(g_c\) from the spline file, gives the desired \(K_t\) in the tunnel.

6.2 Set PI

Setting the phase integral is completely analogous to setting the \(K\) value. The same procedure as discussed above is used, but with \(K\) replaced by \(PI\).
6.3 Phase Matching

We must determine the value of the phase integral that we want to set the phase shifter to. As part of the undulator calibration, we make files containing the required phase integral at many undulator gaps. A spline fit gives the phase integral at any gap. This is all at the calibration temperature $T_c$. For a different temperature, we do not have a phase matching temperature calibration factor as we do for setting the $K$ value or the phase shifter phase integral. Rather, we calculate how the phase matching varies with temperature.

The phase advance in the core of an undulator is $\pi$ per pole as given by the resonance condition. Let $\phi_{\text{enter}}$ be the phase from the entrance cell boundary to the first undulator core pole, which is the first pole after the $N_e$ end poles. There is a similar term $\phi_{\text{exit}}$ for the phase advance from the last core pole to the exit cell boundary. The total phase in the cell is

$$\phi_{\text{cell}} = \phi_{\text{enter}} + (N_p - 2N_e - 1)\pi + \phi_{\text{exit}} \tag{108}$$

where $N_p$ is the number of poles in the undulator. A solution that gives phase matching is for $\phi_{\text{cell}} = 2\pi M$, for some integer $M$, and $\phi_{\text{enter}} = \phi_{\text{exit}} = \phi_{pm}$. The phase matching phase $\phi_{pm}$ is then given by

$$\phi_{pm} = \frac{2\pi M - (N_p - 2N_e - 1)\pi}{2} \tag{109}$$

We compare $\phi_{\text{enter}}$ to $\phi_{pm}$ and it is the job of the phase shifter to add phase $\phi_{ps}$ so that

$$\phi_{\text{enter}} + \phi_{ps} = \phi_{pm} + 2\pi n \tag{110}$$
i.e. the entrance phase plus the phase shifter phase must give the phase matching solution within a multiple of $2\pi$. The phase shifter correction is then

$$\phi_{ps} = 2\pi n - (\phi_{\text{enter}} - \phi_{pm}) \tag{111}$$

The phase shifter phase integral required to make this phase correction is

$$PL_{\text{enter}} = \frac{\phi_{ps}}{2\pi} \left( \frac{me}{e} \right)^2 \lambda_u \left( 1 + \frac{1}{2}K^2 \right) \tag{112}$$

The phase matching spline file contains values of $PL_{\text{enter}}$ at many undulator gaps, all at the calibration temperature. When the temperature changes, the value of $PL_{\text{enter}}$ must be corrected because of the change in $K$ and the change in $\phi_{\text{enter}}$ which depends on $K$.

When the temperature changes, the change in $PL_{\text{enter}}$ is given by

$$\frac{dPL_{\text{enter}}}{dT} = \frac{\phi_{ps}}{2\pi} \left( \frac{me}{e} \right)^2 \lambda_u K \frac{dK}{dT} + \frac{d\phi_{ps}}{dT} \frac{1}{2\pi} \left( \frac{me}{e} \right)^2 \lambda_u \left( 1 + \frac{1}{2}K^2 \right) \tag{113}$$

We can rewrite this as

$$\frac{dPL_{\text{enter}}}{dT} = PL_{\text{enter}} \frac{K}{(1 + \frac{1}{2}K^2)} \frac{dK}{dT} + \frac{1}{\phi_{ps}} \frac{d\phi_{ps}}{dT} PL_{\text{enter}} \tag{114}$$

In terms of the temperature coefficient for $K$, this becomes

$$\frac{dPL_{\text{enter}}}{dT} = PL_{\text{enter}} \left[ \frac{K^2}{(1 + \frac{1}{2}K^2)} + \frac{1}{\phi_{ps}} \frac{d\phi_{ps}}{dT} K \right] \frac{1}{K} \frac{dK}{dT} \tag{115}$$

The $K$ dependence of $\phi_{ps}$ comes from $\phi_{\text{enter}}$. The phase advance through the entrance comes from the slippage in the drift region before the undulator, plus terms with a weak $K$ dependence
from the first couple poles that launch the trajectory in the undulator, plus terms going as π per pole up to the first core pole. We approximate this as

\[ \phi_{\text{enter}} \simeq 2\pi \frac{L}{\lambda_u} \left( \frac{1}{1 + \frac{1}{2} K^2} \right) + \text{const} \]  \hspace{1cm} (116)

where \( L \) is the drift distance. The change in \( \phi_{ps} \) with \( K \) is then given by

\[ \frac{d\phi_{ps}}{dK} = -\frac{d\phi_{\text{enter}}}{dK} = 2\pi \frac{L}{\lambda_u} \frac{K}{(1 + \frac{1}{2} K^2)^2} \]  \hspace{1cm} (117)

Using this expression, we find

\[ \frac{dP_{I_{\text{enter}}}}{dT} = P_{I_{\text{enter}}} \left[ \frac{K^2}{(1 + \frac{1}{2} K^2)} + \frac{1}{\phi_{ps}} \frac{2\pi L}{\lambda_u} \frac{K^2}{(1 + \frac{1}{2} K^2)^2} \right] \frac{1}{K} \frac{dK}{dT} \]  \hspace{1cm} (118)

which simplifies to

\[ \frac{dP_{I_{\text{enter}}}}{dT} = P_{I_{\text{enter}}} \left[ 1 + \frac{1}{\phi_{ps}} 2\pi \frac{L}{\lambda_u} \frac{1}{(1 + \frac{1}{2} K^2)} \right] \frac{K^2}{(1 + \frac{1}{2} K^2)} \frac{1}{K} \frac{dK}{dT} \]  \hspace{1cm} (119)

\[ = \left[ P_{I_{\text{enter}}} + L \left( \frac{mc}{e} \right)^2 \right] \frac{K^2}{(1 + \frac{1}{2} K^2)} \frac{1}{K} \frac{dK}{dT} \]  \hspace{1cm} (120)

The procedure for finding the phase integral contribution that the phase shifter must add to do the phase matching for an undulator entrance is as follows. At tunnel temperature \( T_t \), choose the desired \( K \) value, \( K_t \). Using the correction procedure given above, determine the \( K \) value at the calibration temperature, \( K_c \). From the calibration spline files, determine the undulator gap, \( G_c \). Use the phase matching spline files at given \( G_c \) to find the phase integral the phase shifter should add at the calibration temperature, \( P_{I_{\text{enter}}}^c \). Find the change in the phase integral required to go to the tunnel temperature.

\[ \delta P_{I_{\text{enter}}} = \left[ P_{I_{\text{enter}}}^c + L \left( \frac{mc}{e} \right)^2 \right] \frac{K_t^2}{(1 + \frac{1}{2} K_t^2)} \frac{1}{K_t} \frac{dK}{dT} \]  \hspace{1cm} (121)

The phase integral to add at temperature \( T_t \) is then

\[ P_{I_{\text{enter}}}^t = P_{I_{\text{enter}}}^c + \delta P_{I_{\text{enter}}} \]  \hspace{1cm} (122)

This procedure was tested during the temperature calibration measurements. Consider the HXU-031 data which does not have saturation effects of the undulator poles. The measured difference in the \( P_{I_{\text{enter}}} \) values, \( P_{I_{\text{enter}}} (20 \, \text{deg C end}) - P_{I_{\text{enter}}} (21 \, \text{deg C}) \), is shown in figure 25. The \( P_{I_{\text{enter}}} \) values at 20 deg C were corrected to 21 deg C using the above procedure. The difference between the corrected values and the measured values at 21 deg C is shown in figure 26. The corrected \( P_{I_{\text{enter}}} \) values agree fairly well with the measured values verifying the correction procedure.
Figure 25: $PI_{enter}$ measured during the final 20 deg C measurements minus $PI_{enter}$ measured during the 21 deg C measurements.

Figure 26: $PI_{enter}$ measured at 20 deg C and corrected to 21 deg C minus the measured values at 21 deg C.
7 Summary

Temperature corrections are required for LCLS-II undulators and phase shifters which are operated in the tunnel at different temperatures than their calibrations were performed at. The temperature corrections can be done with a single temperature calibration factor for the undulator $K$ values and a single calibration factor for the phase shifter phase integrals. The temperature calibration factor was obtained on one each SXR undulator, HXR undulator, SXR phase shifter, and HXR phase shifter, but is assumed to be the same for all of each type of device. The temperature calibration factors obtained in the measurements detailed in this note are:

<table>
<thead>
<tr>
<th>Device</th>
<th>Calibration Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>SXR Undulator</td>
<td>$(1/K) dK/dT = -1.006 \times 10^{-4}$ 1/deg C</td>
</tr>
<tr>
<td>HXR Undulator</td>
<td>$(1/K) dK/dT = -0.7648 \times 10^{-4}$ 1/deg C</td>
</tr>
<tr>
<td>SXR Phase Shifter</td>
<td>$(1/P) dP/dT = -1.645 \times 10^{-3}$ 1/deg C</td>
</tr>
<tr>
<td>HXR Phase Shifter</td>
<td>$(1/P) dP/dT = -1.626 \times 10^{-3}$ 1/deg C</td>
</tr>
</tbody>
</table>

Acknowledgements
We are grateful to Heinz-Dieter Nuhn for discussions about this work.