A Phase Matching Test Of The LCLS-II SXR Undulators

Zachary Wolf, Yuri Levashov
Stanford Linear Accelerator Center
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Abstract

A test of phase matching was performed in the MMF using an LCLS-II SXR undulator and two SXR phase shifters. Measurements of the three combined devices were made. The cross calibration of the undulator and phase shifter measurements was tested. The algorithm used to set the phase shifters was also tested. Results of the tests are presented.

1 Introduction

The LCLS-II soft x-ray (SXR) undulator line requires phase shifters between the adjustable gap undulators in order to keep the proper phase relation between the electron beam and a previously emitted light wave. The undulators and phase shifters must be set in unison since the phase advance required of the phase shifters depends on the undulator $K$ value. Since the undulators were calibrated in one laboratory and the phase shifters were calibrated in a different laboratory, a test is required to guarantee that the measurements were cross calibrated. The software which used the calibration data to set the undulators and phase shifters also requires testing. This note summarizes the results of our tests.

2 Experimental Setup

Soft x-ray undulator SXU-001 was used in the test, along with upstream phase shifter SXPS-16349 and downstream phase shifter SXPS-16345. Figure 1 shows phase shifter SXPS-16349 at the upstream end of the undulator in the laboratory, and figure 2 shows phase shifter SXPS-16345 at the downstream end of the undulator.

The undulator and phase shifters had been previously calibrated. Files containing undulator spline data for giving the undulator K value vs gap, phase matching error at the entrance vs gap, phase matching error at the exit vs gap, and centerline height vs gap were obtained from the V-Drive folder containing the Controls Data. Similarly, the calibration files for the phase shifters giving spline data for phase integral vs gap were obtained from the V-Drive Controls Data folders. The calibration data used for this test is the same data that will be used to set the undulators and phase shifters in the tunnel.

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3 Phase Matching Calculations

In this section we give a brief summary of the method used to set the phase shifters. Further details can be found in other technical notes\textsuperscript{2,3,4}.

3.1 Phase Advance Into The Undulator

The undulator line is divided into independent cells. In order for the cells to be independent, the phase advance in a cell must be a multiple of $2\pi$. Also, in order for the phase to be correct in the core of an undulator, the phase from the cell boundary to the core must have an exact value. In this section, we explore the phase advance from the cell boundary to the core of an undulator.

First consider the drift distance from the cell boundary to the edge of the first undulator pole. The slippage in the drift over a length $L$ is given by

$$S = \frac{1}{2\gamma^2}L$$  \hspace{1cm} (1)

where $\gamma$ is the electron Lorentz factor and $L$ is the drift length. The phase advance in the

\hspace{1cm} \textsuperscript{2}Z. Wolf, "A PPM Phase Shifter Design", LCLS-TN-11-2, July, 2011.
\hspace{1cm} \textsuperscript{3}Z. Wolf, "Phase Matching The LCLS-II Undulators", LCLS-TN-16-3, July, 2016.
\hspace{1cm} \textsuperscript{4}Z. Wolf, "Setting the LCLS-II Phase Shifters", LCLS-TN-17-3-Rev2, February, 2018.
Figure 2: Phase shifter SXPS-16345 at the downstream end of undulator SXU-001.

drift is given by

\[ \phi = 2\pi \frac{S}{\lambda_r} \]  

(2)

where \( \lambda_r \) is the radiation wavelength. \( \lambda_r \) is given by

\[ \lambda_r = \frac{\lambda_u}{2\pi^2} \left( 1 + \frac{1}{2} K^2 \right) \]  

(3)

where \( \lambda_u \) is the undulator period and \( K \) is the undulator parameter. \( K \) is proportional to the magnetic field in the undulator, which changes with gap. Combining factors, we find that the phase advance in a drift section is given by

\[ \phi_d = 2\pi \frac{L}{\lambda_u} \frac{1}{(1 + \frac{1}{2} K^2)} \]  

(4)

When the undulator gap changes, the \( K \) value changes, and the phase advance in the drift section changes. Without correction, the electrons would enter the undulator with the wrong phase for enhancing the FEL process. A phase shifter compensates for this changing phase advance.

In order to illustrate how the phase advance into the core of an undulator depends on the \( K \) value, we wish to compare measurements taken on SXU-001 to a model of the phase advance from the entrance cell boundary to the first core pole after the undulator end poles. In the model, the end of the undulator consists of a zero potential pole (as in SXU-001),
a quarter strength pole, a three quarter strength pole, and then full strength poles. The
slippage in the magnetic field of the undulator (neglecting the small $y'$ term) is given by

$$S(z) = \int_{z_0}^{z} \left( \frac{1}{2\gamma_0^2} + \frac{1}{2} x'^2 \right) dz \quad (5)$$

where $x'$ is the slope of the trajectory. In a sinusoidal magnetic field, the trajectory slope is given by

$$x' = \alpha \frac{K}{\gamma} \sin(\frac{2\pi z}{\lambda_u}) \quad (6)$$

where $\alpha = 1/4$ or $3/4$ for the one quarter and three quarter strength poles, and $\alpha = 1$ for the full strength poles. The slippage past a pole is given by

$$\Delta S_p = \frac{1}{2} \frac{1}{\gamma^2} (1 + \frac{1}{2} \alpha^2 K^2) \Delta z_p \quad (7)$$

Using equations 2, 3, and 7 with $\Delta z_p = \lambda_u/2$, the phase advance past a pole is

$$\phi_\alpha = \pi \frac{(1 + \frac{1}{2} \alpha^2 K^2)}{(1 + \frac{1}{2} K^2)} \quad (8)$$

When we add the drift phase, the phase past the quarter and three quarter strength poles, and $\pi$ per pole for the full strength poles to the first core pole, we get for the phase advance to reach the first core pole

$$\phi_{\text{enter}} = \phi_d + \phi_{1/4} + \phi_{3/4} + (N_e - 2)\pi + \pi \quad (9)$$

where $N_e$ is the number of end poles.

In figure 3 we plot $\phi_{\text{enter}}$ as a function of $K$. Also shown in the figure is the measured phase in SXU-001 at the first core pole as a function of $K$. The model agrees well with the measurements. The phase advance into the undulator is a smooth function of $K$.

In practice we average the phase over the core poles rather than use the phase of the first core pole by itself. For pole $n$ in the core with measured phase $\phi_n$, we calculate $\phi_{\text{enter}}$ as

$$\phi_{\text{enter}} = \frac{1}{N_p - 2N_e} \sum_{n=N_e+1}^{N_p-N_e} [\phi_n - \pi (n - N_e - 1)] \quad (10)$$

where $N_p$ is the number of undulator poles. There are $N_p - 2N_e$ core poles. The phase advances by $\pi$ in the spaces between the core poles by the resonance condition.

### 3.2 Phase Advance Into the Undulator Required For Synchronization

In order for the electron beam to be synchronized with a light wave, only certain values of $\phi_{\text{enter}}$ are allowed. Consider the six pole undulator with one end pole at each end shown in figure 4. The phase advance from one core pole to the next is $\pi$ by the resonance condition. The phase advance from the entrance cell boundary to the exit cell boundary must be a multiple of $2\pi$ for the cells to be independent of each other. In the figure the cell phase is chosen to be $6\pi$. Since there are 4 core poles, there are 3 phase advances of $\pi$ between poles.
Since the total phase of the cell is $6\pi$, the total phase of the ends is $6\pi - 3\pi = 3\pi$. The end phase advances are equal by symmetry, so the phase advance from the cell boundary to the first core pole is $3\pi/2$, and the phase advance from the last core pole to the exit cell boundary is $3\pi/2$. Note that this condition, or any multiple of $2\pi$ added to this condition, guarantees that the phase advance from one undulator pole to the corresponding pole in the next undulator will be a multiple of $2\pi$. In this way, the phase at any point in an undulator will be the same, modulo $2\pi$, for all undulators.

For a general undulator with $N_p$ poles and $N_e$ end poles, there are $N_p - 2N_e$ core poles with $N_p - 2N_e - 1$ spaces between core poles where the phase advances by $\pi$ because of the resonance condition. The total phase advance through the core is

$$\Phi_{core} = \pi(N_p - 2N_e - 1) \tag{11}$$

The phase advance of the cell must be a multiple of $2\pi$ in order for the cells to be independent.

$$\Phi_{cell} = 2\pi M \tag{12}$$

where $M$ is a positive integer greater than the number of undulator periods.

Let the phase advance from the cell boundary to the first core pole be $\Phi_{pm}$. By symmetry, the phase advance from the last core pole to the exit cell boundary is also $\Phi_{pm}$. 

Figure 3: The measured phase from the cell boundary to the first core pole agrees well with the model given in the text.
The phase advance in the cell is

$$\Phi_{\text{cell}} = 2\Phi_{\text{pm}} + \Phi_{\text{core}}$$  \hspace{1cm} (13)

The phase matching phase $\Phi_{\text{pm}}$ is then given by

$$\Phi_{\text{pm}} = \frac{\Phi_{\text{cell}} - \Phi_{\text{core}}}{2}$$  \hspace{1cm} (14)

$$= \frac{2\pi M - \pi(N_p - 2N_e - 1)}{2}$$  \hspace{1cm} (15)

As we have seen above, the phase advance from the cell boundary to the undulator core depends on the $K$ value of the undulator in a smooth way. As we have just seen, however, only certain values of the phase advance are allowed for synchronization of the electron beam to a light wave. It is the job of the phase shifter to add phase to achieve $\Phi_{\text{pm}}$, or $\Phi_{\text{pm}}$ with any additional multiple of $2\pi$, for all $K$ values of the undulator.

### 3.3 Entrance Phase Shifter Correction

We add the phase shifter phase $\phi_{\text{ps, enter}}$ to the undulator entrance phase $\phi_{\text{enter}}$ in such a way that the sum is equal to the phase matching phase $\Phi_{\text{pm}}$ plus arbitrary multiples of $2\pi$.

$$\phi_{\text{ps, enter}} + \phi_{\text{enter}} = \Phi_{\text{pm}} + 2\pi k$$  \hspace{1cm} (16)

for integer $k$. The phase shifter is required to add

$$\phi_{\text{ps, enter}} = 2\pi k - (\phi_{\text{enter}} - \Phi_{\text{pm}})$$  \hspace{1cm} (17)

$$= 2\pi k - \phi_{\text{pm, enter}}$$  \hspace{1cm} (18)

where we defined

$$\phi_{\text{pm, enter}} = \phi_{\text{enter}} - \Phi_{\text{pm}}$$  \hspace{1cm} (19)

This is illustrated by the arrows in figure 5.
3.4 Exit Phase Shifter Correction

A phase shifter is required at the exit end of the undulator in order to make the phase at the exit end cell boundary a multiple of $2\pi$. The phase advance from the last core pole to the exit end cell boundary is given by $\phi_{exit}$. The exit phase shifter adds phase $\phi_{ps\_exit}$ to $\phi_{exit}$ in order to make the sum equal to the phase matching phase $\Phi_{pm}$ plus arbitrary multiples of $2\pi$.

\[
\phi_{exit} + \phi_{ps\_exit} = \Phi_{pm} + 2\pi m 
\]  \hspace{1cm} (20)

for integer $m$. The exit phase shifter is required to add

\[
\phi_{ps\_exit} = 2\pi m - (\phi_{exit} - \Phi_{pm}) 
\]  \hspace{1cm} (21)

\[
= 2\pi m - \phi_{pme\_exit} 
\]  \hspace{1cm} (22)

where we defined

\[
\phi_{pme\_exit} = \phi_{exit} - \Phi_{pm} 
\]  \hspace{1cm} (23)

Since we are always correcting to the required phase in the undulator for phase matching, the exit phase shifter also guarantees that the phase will be a multiple of $2\pi$ at the exit cell boundary.

As for the entrance, in practice we average the phase over the core poles rather than use the phase of the last core pole by itself. For pole $n$ in the core with measured phase $\phi_n$, we calculate $\phi_{exit}$ as

\[
\phi_{exit} = \phi_{cell} - \frac{1}{N_p - 2N_e} \sum_{n=N_p-N_e}^{N_e+1} [\phi_n + \pi (N_p - N_e - n)] 
\]  \hspace{1cm} (24)

$\phi_{cell}$ is the measured phase advance through the cell. The sum is made over decreasing pole number from the last core pole to the first core pole with $\pi$ being added when going from one pole to the next upstream pole.
The expression for \( \phi_{exit} \) can be written in terms of \( \phi_{enter} \):

\[
\phi_{exit} = \phi_{cell} - \frac{1}{N_p - 2N_e} \sum_{n=N_p-N_e}^{N_e+1} \left[ \phi_n - \pi (n - N_e - 1) + \pi (N_p - N_e - n) + \pi (n - N_e - 1) \right] 
\]

\[
= \phi_{cell} - \phi_{enter} - \frac{1}{N_p - 2N_e} \sum_{n=N_p-N_e}^{N_e+1} \pi (N_p - 2N_e - 1) 
\]

\[
= \phi_{cell} - \phi_{enter} - \pi (N_p - 2N_e - 1) 
\]

(25)

(26)

(27)

The last term in the last expression is just \( \Phi_{core} \).

\[
\phi_{exit} = \phi_{cell} - \phi_{enter} - \Phi_{core} 
\]

(28)

Since

\[
\Phi_{core} = \Phi_{cell} - 2\Phi_{pm} 
\]

(29)

we have

\[
\phi_{exit} = \phi_{cell} - \phi_{enter} - \Phi_{cell} + 2\Phi_{pm} 
\]

(30)

or

\[
(\phi_{exit} - \Phi_{pm}) = (\phi_{cell} - \Phi_{cell}) - (\phi_{enter} - \Phi_{pm}) 
\]

(31)

With the definitions

\[
\phi_{pme\_exit} = \phi_{exit} - \Phi_{pm} 
\]

(32)

\[
\phi_{pme\_cell} = \phi_{cell} - \Phi_{cell} 
\]

(33)

\[
\phi_{pme\_enter} = \phi_{enter} - \Phi_{pm} 
\]

(34)

we have

\[
\phi_{pme\_exit} = \phi_{pme\_cell} - \phi_{pme\_enter} 
\]

(35)

### 3.5 Phase Integral

We can calculate from the measurements \( \phi_{pme\_enter} \) and \( \phi_{pme\_exit} \) for a given undulator \( K \) value and given nominal cell phase. This in turn gives us, up to a multiple of \( 2\pi \), the phase that the phase shifters need to add: \( \phi_{ps\_enter} \) and \( \phi_{ps\_exit} \). The added phase of the phase shifter depends on the strength of its magnetic field and on the radiation wavelength. The added slippage through the phase shifter from the magnetic field is given by

\[
S'(z) = \int_{z_0}^{z} \frac{1}{2} x'^2 \, dz_1 
\]

(36)

where the slippage term due to free space drift through the phase shifter has not been included since it is accounted for in the drift distance at the end of the undulator and is not added slippage. The small \( y' \) term has been ignored for this discussion, but is included in practice. The trajectory slope is given by

\[
x'(z) = -\frac{q}{\gamma m v_z} \int_{z_0}^{z} B_y(z_1) \, dz_1 
\]

(37)
So the added slippage from the phase shifter is

\[ S' = \frac{1}{2\gamma^2} \left( \frac{q}{mv_z} \right)^2 \int_{-L_p/2}^{L_p/2} \left( \int_{-L_p/2}^{z} B_y(z_1) \, dz_1 \right)^2 \, dz \]  

(38)

We define the phase integral of the phase shifter as

\[ PI = \int_{-L_p/2}^{L_p/2} \left( \int_{-L_p/2}^{z} B_y(z_1) \, dz_1 \right)^2 \, dz \]  

(39)

Using this expression for the phase integral, the added slippage of the phase shifter is

\[ S' = \frac{1}{2\gamma^2} \left( \frac{q}{mv_z} \right)^2 PI \]  

(40)

The phase added by the phase shifter is given by

\[ \phi_{ps} = 2\pi \frac{S'}{\lambda_r} \]  

(41)

Inserting the expression for the radiation wavelength, we have

\[ \phi_{ps} = 2\pi \left( \frac{q}{mv_z} \right)^2 \frac{PI}{\lambda_u \left(1 + \frac{1}{2}K^2\right)} \]  

(42)

The phase integral depends on the magnetic field of the phase shifter and is calibrated as a function phase shifter gap. For given \( \phi_{ps} \), the phase integral required by the phase shifter is given by

\[ PI = \frac{\phi_{ps}}{2\pi} \lambda_u \left(1 + \frac{1}{2}K^2\right) \left(\frac{mv_z}{q}\right)^2 \]  

(43)

We have calculated the phases \( \phi_{ps\_enter} \) and \( \phi_{ps\_exit} \) to do the phase matching. This expression tells us the required phase integrals of the entrance and exit phase shifters.

3.6 Additional Phase

The phase shifter can add phase in addition to the phase matching. If the operator desires to add \( \phi_{add} \), the added phase integral is

\[ PI = \frac{\phi_{add}}{2\pi} \lambda_u \left(1 + \frac{1}{2}K^2\right) \left(\frac{mv_z}{q}\right)^2 \]  

(44)

3.7 Phase Shifter Jumps

We will see that the phase shifters are not strong enough to smoothly cover the required range of \( \phi_{ps} \). Furthermore, in the beam line a phase shifter corrects more than one end of one undulator. A phase shifter corrects the phase matching of the exit end of the upstream undulator, the phase advance in any drift sections or open undulators before the
next undulator, the phase matching of the entrance end of the next undulator, and any desired additional phase from the operator. When these four terms are added, the total phase is much larger than the phase that a phase shifter can provide. The solution is to change the phase by multiples of $2\pi$ in order to stay within the range of the phase shifter. When multiples of $2\pi$ are added to the phase, the setting of the phase shifter has a jump. We wish to control the jumps in order to have as few jumps as possible and to make the locations of the jumps as fixed as possible. In order to achieve this, we make each of the four terms vary smoothly through positive values. After the four terms are added, the jumps are calculated. In this section we describe the procedure.

As we have seen, the phase shifter correction for an undulator entrance or exit adds phase to $\phi_{pme}$ in order to get to a multiple of $2\pi$.

$$\phi_{ps} + \phi_{pme} = 2\pi k$$  

(45)

giving

$$\phi_{ps} = 2\pi k - \phi_{pme}$$  

(46)

We need $\phi_{ps}$ to be positive because a phase shifter can only add phase. We also need $\phi_{ps}$ to vary smoothly because jumps will be calculated later after all terms are added and we don’t want additional discontinuities at this point. In order for $\phi_{ps}$ to be positive and vary smoothly, we must choose the integer $k$ such that $2\pi k$ is larger than the largest value of $\phi_{pme}$. This is illustrated in figure 6. As an example, the phase matching error at the

![Figure 6: The arrows indicate the phase that the phase shifter must add in order to be positive and vary smoothly over the entire K range.](image)

entrance of SXU-001 is shown in figure 7. The correction that the phase shifter must add (the arrows in figure 6) is shown in figure 8. The phase integral that corresponds to this correction is shown in figure 9. The maximum phase integral required$^{3}$ of the phase shifter

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Figure 7: Phase matching error at the entrance of SXU-001.

is 3814 T²mm³. One can see from the figure that the phase shifter can not smoothly cover the range of phase integrals required to phase match the undulator entrance. Jumps are required.

In order to discuss the procedure for calculating jumps, we continue with the example of the entrance end of SXU-001, although as noted above, other corrections are summed before the jumps are calculated in practice. The phase integral of a phase shifter is related to the phase value by equation 43:

$$PI = \frac{\phi_{ps}}{2\pi} \lambda_u \left( 1 + \frac{1}{2} K^2 \right) \left( \frac{mv_z}{q} \right)^2$$  \hspace{1cm} (47)

We preserve the phase relation when multiples of $2\pi$ are added to $\phi_{ps}$.

$$\phi_{ps\_jump} = \phi_{ps} + 2\pi n$$  \hspace{1cm} (48)

where $n$ is an integer. We vary $n$ and make jumps in order to keep $PI$ within the range of the phase shifter. The phase integral with jumps is given by

$$PI_{jump} = \frac{\phi_{ps\_jump}}{2\pi} \lambda_u \left( 1 + \frac{1}{2} K^2 \right) \left( \frac{mv_z}{q} \right)^2$$  \hspace{1cm} (49)

$$= PI_{orig} + n\lambda_u \left( 1 + \frac{1}{2} K^2 \right) \left( \frac{mv_z}{q} \right)^2$$  \hspace{1cm} (50)

where $PI_{orig}$ is the original phase integral with $n = 0$. Figure 10 shows the phase integral required to correct the entrance end of SXU-001 with additional curves for values of $n$ from
7 to $-9$ changing by 1 from curve to curve. The phase integral range of the phase shifter is also shown. The values of $n$ shown are required to keep $PI_{jump}$ within the range of the phase shifter over the range of $K$.

Figure 11 is an expanded view of figure 10 around the phase integral range of the phase shifter. At small $K$, a value of $n = 7$ is chosen to keep the phase integral above $PI_{min}$. As $K$ increases, we keep $n = 7$ until $PI_{max}$ is reached. At this $K$ value, we change to $n = -3$ in order to make $PI_{jump}$ as small as possible within the range of the phase shifter so that the number of jumps is minimized. We keep $n = -3$ as $K$ increases until we reach $PI_{max}$ again. At this point we set $n = -7$. We keep $n = -7$ as $K$ increases until $PI_{max}$ is reached for the final time. At this point we set $n = -9$ and use this value until the maximum value of $K$ is reached. For any undulator $K$ value, we use this curve of $PI_{jump}$ vs $K$ to set the phase shifter.

This same procedure is used for an actual phase shifter correcting an undulator exit, drift, undulator entrance, and making additional phase. The sum of the required phase integrals is calculated for a dense set of $K$ values. The jumps are determined. The $K$ value corresponding to the phase shifter location is determined, $K_{val}$. The phase integral $PI_{jump}$ at $K_{val}$ is used to set the phase shifter.
Figure 9: Phase integral required by the phase shifter to correct the phase matching error at the entrance of SXU-001.

Figure 10: Phase integral values for various values of added phase $2\pi n$. The phase integral limits of the phase shifter are shown in red.
Figure 11: Phase integral with jumps in order to stay within the range of the phase shifter.
3.8 Relation Of This Test To The SXR Line

In the test described in this note, the phase shifters only corrected the entrance and exit phase matching errors of one undulator. They also added a user specified phase. In practice the situation is as shown in figure 12. The LCLS-II phase shifters are downstream of the undulators. Each cell contains an undulator and phase shifter, except undulators can be open or missing, and the last undulator does not need a phase shifter.

![Diagram of undulators and phase shifters](image)

Figure 12: Schematic of the undulators and phase shifters in an undulator line.

Consider phase shifter PS2. It corrects the exit end of undulator U2, the drift through cell 3, the entrance phase matching of U4, and any desired additional phase. An array of $K$ values is generated. The phase integral of each of the four terms is calculated for all $K$ values in the array. The four phase integral arrays are added. Jumps are calculated. The phase integral is chosen for the array $K$ value that most closely matches the undulator $K$ value. The undulator $K$ value is further discussed for the cases of jump tapers and linear undulator line tapers in another note.\(^6\)

3.9 Summary Of The Phase Matching Calculations

As a summary to this section of the note, the steps to set PS2 in the previous example are listed below. The equations given above are referenced when they are used. Some corrections, such as for temperature differences, are not discussed here but must be made in practice.

3.9.1 Undulator Calibration

In preparation to set PS2, the following undulator measurements and calculations must be made:

1. Measure the fields in each undulator at many undulator gap values. For each undulator, make the following calculations.

2. Calculate the $K$ value at each gap. Compile the gaps and $K$ values in a spline file in the Controls Folder, e.g. "V:\MET\MagServe\MagData\LCLS-II\Undulator\SXU_001\DATASET0007\Final Results\Controls Data\sxu_001_k_vs_gap spline.dat".

3. Calculate the phase advance from the cell boundary to each peak field position in the core of the undulator.

4. Calculate $\phi_{\text{enter}}$ using equation 10.

5. Calculate $\phi_{\text{exit}}$ using equation 24.

6. Choose $\Phi_{\text{cell}}$ using equation 12.

7. Calculate the phase matching phase $\Phi_{\text{pm}}$ using equation 15.

8. Calculate the phase matching error $\phi_{\text{pm enter}}$ at the undulator entrance using equation 19.

9. Calculate the phase that the phase shifter must add to correct the phase matching error $\phi_{\text{ps enter}}$ using equation 18. Choose integer $k$ large enough so that $\phi_{\text{ps enter}}$ is a smooth positive function of $K$ as shown in figure 6.

10. Calculate the phase integral $PI_{\text{enter}}$ required by a phase shifter to correct the phase matching error using equation 43 with $\phi_{\text{ps}} = \phi_{\text{ps enter}}$.

11. Compile the undulator gaps and $PI_{\text{enter}}$ values in a spline file in the Controls Folder, e.g. "V:\MET\MagServe\MagData\LCLS-II\Undulator\SXU_001 \DATASET0007\Final Results\Controls Data \sxu_001_phase_match_enter_vs_gap_spline.dat".

12. Calculate the phase matching error $\phi_{\text{pm exit}}$ at the undulator exit using equation 23.

13. Calculate the phase that the phase shifter must add to correct the phase matching error $\phi_{\text{ps exit}}$ using equation 22. Choose integer $m$ large enough so that $\phi_{\text{ps exit}}$ is a smooth positive function of $K$ as shown in figure 6.

14. Calculate the phase integral $PI_{\text{exit}}$ required by a phase shifter to correct the phase matching error using equation 43 with $\phi_{\text{ps}} = \phi_{\text{ps exit}}$.

15. Compile the undulator gaps and $PI_{\text{exit}}$ values in a spline file in the Controls Folder, e.g. "V:\MET\MagServe\MagData\LCLS-II\Undulator\SXU_001 \DATASET0007\Final Results\Controls Data \sxu_001_phase_match_exit_vs_gap_spline.dat".

3.9.2 Phase Shifter Calibration

In preparation to set a phase shifter, the following phase shifter measurements and calculations must be made:

1. Measure the fields in each phase shifter at many phase shifter gaps. For each phase shifter, make the following calculations.
2. For each gap, calculate the phase integral $PI$ using equation 39, but including the small $B_x$ terms.

3. Compile the gaps and $PI$ values in a spline file in the Controls Folder, e.g. "V:\MET\MagServe\MagData\LCLS-II\Phase Shifter\SXPS_16349\DATASET0002\Final Results\Controls Data\sxps_16349_pivsgap_spline.dat".

### 3.9.3 Calculate The Phase Shifter Settings

To set PS2, perform the following steps:

1. Make a dense array of $K$ values, $K_{array}$.

2. For each $K$ value in $K_{array}$, use the $K$ vs gap spline data for undulator U2 to determine the undulator gap. Use the $PI_{exit}$ vs gap spline data to determine $PI_{exit}$ for each $K$ value in $K_{array}$.

3. For each $K$ value in $K_{array}$, use equation 4 to calculate the phase advance $\phi_{drift}$ in the drift length of cell 3. If cell 3 were filled, the drift length would be zero. The steps to correct the phase advance in the drift length are very similar to the steps to correct the phase advance when entering an undulator. Calculate the correction to $\phi_{drift}$ required to make a multiple of $2\pi$ over the length of cell 3. This is given by

   $$\phi_{ps\_drift} = 2\pi r - \phi_{drift}$$

   for integer $r$ such that the correction $\phi_{ps\_drift}$ is a smooth positive function over all values in $K_{array}$. Calculate the phase integral $PI_{drift}$ that the phase shifter must make to add $\phi_{ps\_drift}$. This is done for all values in $K_{array}$ using equation 43 with $\phi_{ps} = \phi_{ps\_drift}$.

4. For each $K$ value in $K_{array}$, use the $K$ vs gap spline data for undulator U4 to determine the undulator gap. Use the $PI_{enter}$ vs gap spline data to determine $PI_{enter}$ for each $K$ value in $K_{array}$.

5. For any additional phase $\phi_{add}$ that the operator wishes to add, use equation 44 to calculate the phase integral $PI_{add}$ for each value in $K_{array}$.

6. Add the phase integrals to get the total for each value of $K_{array}$.

   $$PI_{total} = PI_{exit} + PI_{drift} + PI_{enter} + PI_{add}$$  \hspace{1cm} (51)

   The total phase integral should be a smooth positive function over $K_{array}$.

7. Calculate $2\pi$ phase jumps in order to keep the phase shifter within its range. Use the algorithm described above. After this procedure, there is a value of $PI_{jump}$ including jumps for all values of $K$ in $K_{array}$.
8. Choose a value $K_{\text{int}}$ corresponding to the interspace region. For a linear taper, use the exit $K$ value of U2, which is equal to the entrance $K$ value of U4. For step tapes, use the mean of the $K$ values of U2 and U4, which is an approximation to the $K$ value of a linear taper.

9. Find the value in $K_{\text{array}}$ closest to $K_{\text{int}}$. Find the corresponding phase integral value $PI_{\text{jump}}$.

10. Use the spline data to find the phase shifter gap and set the gap.

The test described below is a partial test of this procedure. No drift section was included. Only entrance and exit undulator phase matching and adding phase were tested.

4 Measurement Results

Undulator SXU-001 and entrance phase shifter SXPS-16349 and exit phase shifter SXPS-16345 were set up at the measurement bench as described above. A computer program had as input either the $K$ value or the gap of the undulator. If the $K$ value was entered, the undulator gap was calculated using the spline fit from the Controls Data. The gap of the undulator was set.

Spline fits are also in the Controls Data for the phase integral required to correct the phase matching error. Given the undulator gap, the entrance and exit phase integrals were determined from the spline fits. Additional phase integrals from any additional phase that the operator entered in the program was added. The calculation of the phase shifter jumps was then made and the phase shifter phase integrals were determined. Spline fits from the Controls Data were used to determine the phase shifter gaps. The gaps of the phase shifters were set.

The measurements for this test are in the SXU-001, Dataset 7, Tuning folder, runs 47 to 60. Runs 47 to 57 were done in sequence increasing the gap from run to run. The calibration data were taken this way so that no hysteresis effects would be present. The gap was decreased for run 58 setting the $K$ value to 4.000. Adding additional phase was studied. Some hysteresis effects may be present for this run. Afterward, the gap was decreased and then increased setting the $K$ value back to 4.000. Runs 59 and 60 should again have no hysteresis artifacts.

4.1 Undulator $K$ Value

In runs 47 to 57, the desired undulator $K$ value was input to the program. The undulator gap and phase shifter gaps were set. A scan of the magnetic fields was made using the measurement bench. A typical scan (Run 47) is shown in figure 13. The fields from the phase shifters at each end of the undulator are clearly visible.

In order to determine the undulator $K$ value, the segments of the scan through the phase shifters were replaced by background field values near the phase shifter. The standard undulator analysis program then calculated the $K$ value.

The temperature of the undulator during calibration was 20.09 deg C. The temperature during these tests was 20.02 deg C. No significant temperature effects should be present.
Figure I3: Typical measurement scan showing the upstream phase shifter, undulator, and downstream phase shifter.

The measured $K$ value is compared to the desired $K$ value in the following table.

<table>
<thead>
<tr>
<th>Run</th>
<th>Des K</th>
<th>Meas K</th>
<th>$dK/K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>5.7300</td>
<td>5.729938</td>
<td>$-1.08 \times 10^{-5}$</td>
</tr>
<tr>
<td>48</td>
<td>5.5000</td>
<td>5.499627</td>
<td>$-6.78 \times 10^{-5}$</td>
</tr>
<tr>
<td>49</td>
<td>5.0000</td>
<td>4.999452</td>
<td>$-1.09 \times 10^{-4}$</td>
</tr>
<tr>
<td>50</td>
<td>4.5000</td>
<td>4.499495</td>
<td>$-1.12 \times 10^{-4}$</td>
</tr>
<tr>
<td>51</td>
<td>4.0000</td>
<td>3.999507</td>
<td>$-1.23 \times 10^{-4}$</td>
</tr>
<tr>
<td>52</td>
<td>3.5000</td>
<td>3.499593</td>
<td>$-1.16 \times 10^{-4}$</td>
</tr>
<tr>
<td>53</td>
<td>3.0000</td>
<td>2.999552</td>
<td>$-1.49 \times 10^{-4}$</td>
</tr>
<tr>
<td>54</td>
<td>2.5000</td>
<td>2.499853</td>
<td>$-5.88 \times 10^{-5}$</td>
</tr>
<tr>
<td>55</td>
<td>2.0000</td>
<td>1.999879</td>
<td>$-6.05 \times 10^{-5}$</td>
</tr>
<tr>
<td>56</td>
<td>1.5000</td>
<td>1.499886</td>
<td>$-7.60 \times 10^{-5}$</td>
</tr>
<tr>
<td>57</td>
<td>1.0000</td>
<td>0.999843</td>
<td>$-1.57 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

The relative difference is given by

$$\frac{dK}{K} = \frac{K_{\text{meas}} - K_{\text{des}}}{K_{\text{des}}}$$

(52)

For all scans, the relative difference in $K$ is less than the tolerance of $\pm 3 \times 10^{-4}$ for SXR
undulators\textsuperscript{7}. The operating range of the undulator includes gaps from 7.2 mm to 22.0 mm. The corresponding \( K \) range is 5.74 to 1.38. Only run 57 is out of the operating range of the undulator.

### 4.2 Phase Shifter Phase Integral Values

For each \( K \) value in Runs 47 to 57, the phase shifters were set to perform the upstream and downstream phase matching. In this section we compare the desired phase shifter phase integrals to the measured values. As an illustration, the phase integral for the upstream phase shifter for each of the runs was set according to figure 14. The plot for the downstream phase shifter is very similar.

![Phase Shifter PI vs Interspace K](image)

**Figure 14**: Phase integral settings for upstream phase shifter SXPS-16349 during this test.

In order to determine the phase integral from each phase shifter, the part of the scan through the phase shifter was isolated. The standard phase shifter analysis program was then used to determine the phase integral. The desired phase integral and the measured phase integral for the upstream phase shifter are given in the following table.

The relative difference is given by

\[ \frac{dPI}{PI} = \frac{PI_{\text{meas}} - PI_{\text{des}}}{PI_{\text{des}}} \]  

(53)

The ambient temperature of the Dover bench room was 19.65 deg C during the calibration. The ambient temperature of the Kugler bench room during these tests was 20.02 deg C. The temperature dependence of the phase integral has not been measured at the time of this writing, but the higher temperature during these tests would make the phase shifter weaker as seen.

The desired phase integral and the measured phase integral for the downstream phase shifter are given in the following table.

<table>
<thead>
<tr>
<th>Run</th>
<th>Des K</th>
<th>Des PI (T²mm³)</th>
<th>Meas PI (T²mm³)</th>
<th>dPI/PI</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>5.7300</td>
<td>3431.530</td>
<td>3428.583</td>
<td>-8.59 × 10⁻⁴</td>
</tr>
<tr>
<td>48</td>
<td>5.5000</td>
<td>3061.352</td>
<td>3059.254</td>
<td>-6.85 × 10⁻⁴</td>
</tr>
<tr>
<td>49</td>
<td>5.0000</td>
<td>2298.189</td>
<td>2296.290</td>
<td>-8.26 × 10⁻⁴</td>
</tr>
<tr>
<td>50</td>
<td>4.5000</td>
<td>1596.185</td>
<td>1594.986</td>
<td>-7.51 × 10⁻⁴</td>
</tr>
<tr>
<td>51</td>
<td>4.0000</td>
<td>3011.092</td>
<td>3008.401</td>
<td>-8.93 × 10⁻⁴</td>
</tr>
<tr>
<td>52</td>
<td>3.5000</td>
<td>2038.204</td>
<td>2036.459</td>
<td>-8.56 × 10⁻⁴</td>
</tr>
<tr>
<td>53</td>
<td>3.0000</td>
<td>3692.051</td>
<td>3688.837</td>
<td>-8.70 × 10⁻⁴</td>
</tr>
<tr>
<td>54</td>
<td>2.5000</td>
<td>2361.788</td>
<td>2360.028</td>
<td>-7.45 × 10⁻⁴</td>
</tr>
<tr>
<td>55</td>
<td>2.0000</td>
<td>1276.945</td>
<td>1275.978</td>
<td>-7.57 × 10⁻⁴</td>
</tr>
<tr>
<td>56</td>
<td>1.5000</td>
<td>2845.052</td>
<td>2842.220</td>
<td>-9.95 × 10⁻⁴</td>
</tr>
<tr>
<td>57</td>
<td>1.0000</td>
<td>1541.908</td>
<td>1540.888</td>
<td>-6.61 × 10⁻⁴</td>
</tr>
</tbody>
</table>

The ambient temperature of the Dover bench room during calibration was 19.81 deg C. The ambient temperature of the phase shifter during these tests was 20.02 deg C. The calibration temperature is closer to the test temperature and the agreement between the desired phase integral and the measured phase integral is better than for the upstream phase shifter.
4.3 Phase Matching Errors

The measured phase matching errors at the undulator entrance are given in the table below. The values modulo 360 degrees are also given. All values include the added phase of the phase shifter. In general, the errors in setting the phase shifter are very small.

<table>
<thead>
<tr>
<th>Run</th>
<th>Des K</th>
<th>PME Enter (deg)</th>
<th>Mod PME Enter (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>5.7300</td>
<td>719.02</td>
<td>−0.99</td>
</tr>
<tr>
<td>48</td>
<td>5.5000</td>
<td>719.33</td>
<td>−0.67</td>
</tr>
<tr>
<td>49</td>
<td>5.0000</td>
<td>719.75</td>
<td>−0.26</td>
</tr>
<tr>
<td>50</td>
<td>4.5000</td>
<td>719.67</td>
<td>−0.33</td>
</tr>
<tr>
<td>51</td>
<td>4.0000</td>
<td>1438.88</td>
<td>−1.12</td>
</tr>
<tr>
<td>52</td>
<td>3.5000</td>
<td>1439.47</td>
<td>−0.53</td>
</tr>
<tr>
<td>53</td>
<td>3.0000</td>
<td>2879.16</td>
<td>−0.84</td>
</tr>
<tr>
<td>54</td>
<td>2.5000</td>
<td>2879.20</td>
<td>−0.80</td>
</tr>
<tr>
<td>55</td>
<td>2.0000</td>
<td>2879.45</td>
<td>−0.55</td>
</tr>
<tr>
<td>56</td>
<td>1.5000</td>
<td>6476.30</td>
<td>−3.70</td>
</tr>
<tr>
<td>57</td>
<td>1.0000</td>
<td>6477.58</td>
<td>−2.42</td>
</tr>
</tbody>
</table>

The measured phase matching errors at the undulator exit are given in the table below. The values modulo 360 degrees are also given. The errors in setting the phase shifter are again small.

<table>
<thead>
<tr>
<th>Run</th>
<th>Des K</th>
<th>PME Exit (deg)</th>
<th>Mod PME Exit (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>5.7300</td>
<td>720.64</td>
<td>0.64</td>
</tr>
<tr>
<td>48</td>
<td>5.5000</td>
<td>720.74</td>
<td>0.71</td>
</tr>
<tr>
<td>49</td>
<td>5.0000</td>
<td>720.82</td>
<td>0.82</td>
</tr>
<tr>
<td>50</td>
<td>4.5000</td>
<td>721.02</td>
<td>1.02</td>
</tr>
<tr>
<td>51</td>
<td>4.0000</td>
<td>1441.03</td>
<td>1.03</td>
</tr>
<tr>
<td>52</td>
<td>3.5000</td>
<td>1441.02</td>
<td>1.02</td>
</tr>
<tr>
<td>53</td>
<td>3.0000</td>
<td>2880.44</td>
<td>0.44</td>
</tr>
<tr>
<td>54</td>
<td>2.5000</td>
<td>2880.78</td>
<td>0.78</td>
</tr>
<tr>
<td>55</td>
<td>2.0000</td>
<td>2882.13</td>
<td>2.13</td>
</tr>
<tr>
<td>56</td>
<td>1.5000</td>
<td>6479.46</td>
<td>−0.54</td>
</tr>
<tr>
<td>57</td>
<td>1.0000</td>
<td>6480.80</td>
<td>0.80</td>
</tr>
</tbody>
</table>

The overall phase advance in the cell is given in the table below along with the values modulo 360 degrees. The errors are small.
<table>
<thead>
<tr>
<th>Run</th>
<th>Des K</th>
<th>Cell Phase (deg)</th>
<th>Mod Cell Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>5.7300</td>
<td>32399.66</td>
<td>−0.34</td>
</tr>
<tr>
<td>48</td>
<td>5.5000</td>
<td>32400.06</td>
<td>0.06</td>
</tr>
<tr>
<td>49</td>
<td>5.0000</td>
<td>32400.57</td>
<td>0.57</td>
</tr>
<tr>
<td>50</td>
<td>4.5000</td>
<td>32400.69</td>
<td>0.69</td>
</tr>
<tr>
<td>51</td>
<td>4.0000</td>
<td>33839.91</td>
<td>−0.09</td>
</tr>
<tr>
<td>52</td>
<td>3.5000</td>
<td>33840.49</td>
<td>0.49</td>
</tr>
<tr>
<td>53</td>
<td>3.0000</td>
<td>36719.60</td>
<td>−0.40</td>
</tr>
<tr>
<td>54</td>
<td>2.5000</td>
<td>36719.98</td>
<td>−0.02</td>
</tr>
<tr>
<td>55</td>
<td>2.0000</td>
<td>36721.59</td>
<td>1.59</td>
</tr>
<tr>
<td>56</td>
<td>1.5000</td>
<td>43915.76</td>
<td>−4.24</td>
</tr>
<tr>
<td>57</td>
<td>1.0000</td>
<td>43918.38</td>
<td>−1.62</td>
</tr>
</tbody>
</table>

### 4.4 Added Phase

Runs 58, 59, and 60 had an additional phase requested of the phase shifter. Run 58 had 20 degrees added at the upstream end. Run 59 had 20 degrees added at the downstream end. Run 60 had 20 degrees added at the upstream end and −20 degrees added at the downstream end.

The phase matching errors at the upstream end of the undulator are shown in the following table.

<table>
<thead>
<tr>
<th>Run</th>
<th>Des K</th>
<th>PME Enter (deg)</th>
<th>Mod PME Enter (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>58</td>
<td>4.0000</td>
<td>1458.39</td>
<td>18.39</td>
</tr>
<tr>
<td>59</td>
<td>4.0000</td>
<td>1439.05</td>
<td>−0.95</td>
</tr>
<tr>
<td>60</td>
<td>4.0000</td>
<td>1459.02</td>
<td>19.02</td>
</tr>
</tbody>
</table>

The phase matching errors at the downstream end of the undulator are shown in the following table.

<table>
<thead>
<tr>
<th>Run</th>
<th>Des K</th>
<th>PME Exit (deg)</th>
<th>Mod PME Exit (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>58</td>
<td>4.0000</td>
<td>1440.76</td>
<td>0.76</td>
</tr>
<tr>
<td>59</td>
<td>4.0000</td>
<td>1460.83</td>
<td>20.83</td>
</tr>
<tr>
<td>60</td>
<td>4.0000</td>
<td>1420.74</td>
<td>−19.26</td>
</tr>
</tbody>
</table>

The overall phase advance in the cell is given in the following table.

<table>
<thead>
<tr>
<th>Run</th>
<th>Des K</th>
<th>Cell Phase (deg)</th>
<th>Mod Cell Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>58</td>
<td>4.0000</td>
<td>33859.15</td>
<td>19.15</td>
</tr>
<tr>
<td>59</td>
<td>4.0000</td>
<td>33859.88</td>
<td>19.88</td>
</tr>
<tr>
<td>60</td>
<td>4.0000</td>
<td>33839.76</td>
<td>−0.24</td>
</tr>
</tbody>
</table>

In each case the measured added phase has the expected value within error.
5 Summary

This note summarized a test of phase matching the LCLS-II undulators by performing measurements of an undulator with a phase shifter at each end. In general, the phase shifters are cross calibrated with the undulators and the phase matching errors are small.

Acknowledgements
We are grateful to Heinz-Dieter Nuhn for many discussions about this work.