

Initial Tests Of Tapering An LCLS-II Undulator

Zachary Wolf, Yuri Levashov, Heinz-Dieter Nuhn
SLAC

March 6, 2018

Abstract

The LCLS-II undulators allow the gap to be tapered along the undulator length. This allows the K value to be tapered. New possibilities arise for keeping an undulator resonant along its length as the electron beam energy decreases due to radiation and other losses. In this note we study the performance of a tapered undulator including its phase errors and phase matching errors.

1 Introduction¹

The LCLS-II undulators have flexures which allow the gap to be tapered. This opens the possibility to keep an undulator resonant along its length as the electron beam loses energy due to radiation and other losses. In this test we performed initial measurements of a tapered undulator. We determined whether the phase errors remain within tolerance with both an energy taper and a matching K taper. We also determined how the phase matching errors changed from the untapered case. The production measurements of the undulators are done without taper. We determined whether the constant gap measurements could be used for operating the tapered undulators.

The measurements for this test were made on undulator SXU-006. The calculation to find the proper taper for a given energy loss taper is described. The equations to calculate phase errors with energy taper are presented. With the energy loss taper and K taper, we calculate the phase errors and phase matching errors from the measurements. We compare these quantities to the untapered case. Our conclusion is that if the K taper matches the energy taper, the phase errors are only slightly different than the untapered case, and the phase matching is essentially the same as the untapered case using the K value at each end of the undulator.

2 Matching A Linear Gap Taper To An Energy Taper

The electron energy decreases along each undulator due to radiation and other losses. The energy loss in each undulator is assumed small and we take it as a linear decrease with distance. The electron Lorentz factor γ as a function of position is shown decreasing in figure 1. The resonant wavelength depends on γ and at any position is given by

$$\lambda_r = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{1}{2}K^2 \right) \quad (1)$$

We wish to keep λ_r constant. Assume that γ , or equivalently the electron energy, changes along the undulator as

$$\gamma = \gamma_0 \left(1 - \alpha_\gamma \frac{z}{L} \right) \quad (2)$$

¹Work supported in part by the DOE Contract DE-AC02-76SF00515. This work was performed in support of the LCLS project at SLAC.

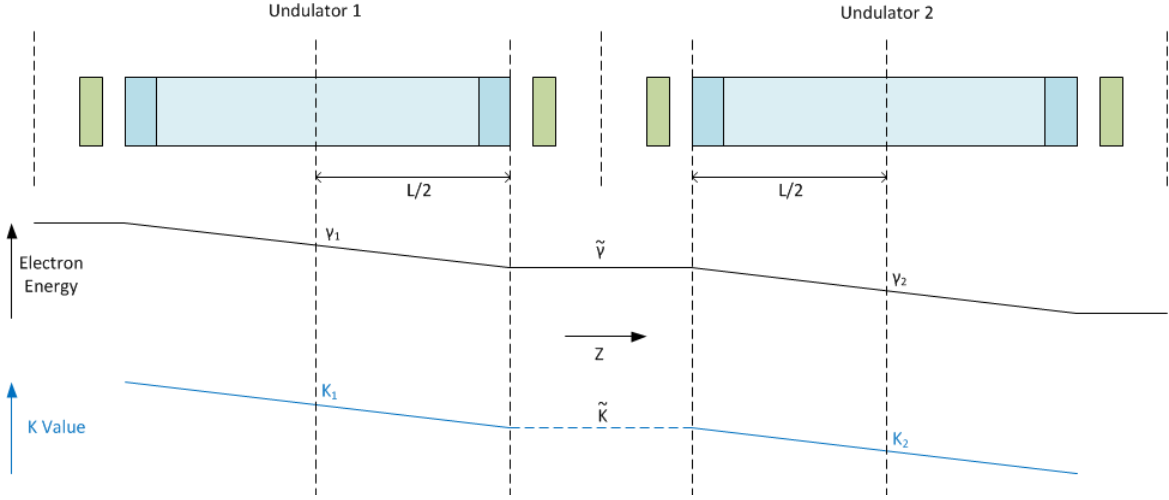


Figure 1: Linear taper model. Assume the electron energy decreases linearly in each undulator due to radiation. The wavelength is kept constant by putting a linear taper on K .

where z is the position along the undulator with $z = 0$ at the entrance of the undulator, γ_0 is the Lorentz factor at the entrance of the undulator, L is the undulator length, and α_γ is the parameter describing the slope of the energy decrease. In order to keep λ_r constant, we put a linear gap taper into the undulator. The linear gap taper makes an approximate linear K taper for small tapers. In this case, we parametrize the position dependence of K as

$$K = K_0 \left(1 - \alpha_K \frac{z}{L}\right) \quad (3)$$

where K_0 is the undulator K value at the entrance. At any position along the undulator,

$$\lambda_r = \frac{\lambda_u}{2} \frac{1 + \frac{1}{2}K_0^2 \left(1 - \alpha_K \frac{z}{L}\right)^2}{\gamma_0^2 \left(1 - \alpha_\gamma \frac{z}{L}\right)^2} \quad (4)$$

We assume α_γ and α_K are small. We expand the squared terms and find the relation between α_γ and α_K which makes the first order terms cancel. Performing the expansion, we get

$$\lambda_r = \frac{\lambda_u}{2} \frac{1 + \frac{1}{2}K_0^2 \left(1 - 2\alpha_K \frac{z}{L} + \alpha_K^2 \left(\frac{z}{L}\right)^2\right)}{\gamma_0^2 \left(1 - 2\alpha_\gamma \frac{z}{L} + \alpha_\gamma^2 \left(\frac{z}{L}\right)^2\right)} \quad (5)$$

For small α_γ we find

$$\lambda_r = \frac{\lambda_u}{2} \frac{1 + \frac{1}{2}K_0^2 \left(1 - 2\alpha_K \frac{z}{L} + \alpha_K^2 \left(\frac{z}{L}\right)^2\right)}{\gamma_0^2} \left(1 + 2\alpha_\gamma \frac{z}{L} + 3\alpha_\gamma^2 \left(\frac{z}{L}\right)^2 + O(\alpha_\gamma^3)\right) \quad (6)$$

At this point we neglect the quadratic terms in α_γ and α_K , and expand to first order.

$$\lambda_r = \frac{\lambda_u}{2\gamma_0^2} \left[1 + \frac{1}{2}K_0^2 - K_0^2\alpha_K \frac{z}{L} + \left(1 + \frac{1}{2}K_0^2\right) 2\alpha_\gamma \frac{z}{L}\right] \quad (7)$$

In order to eliminate the z dependence, we set

$$-K_0^2 \alpha_K \frac{z}{L} + \left(1 + \frac{1}{2} K_0^2\right) 2\alpha_\gamma \frac{z}{L} = 0 \quad (8)$$

or

$$\alpha_K = \alpha_\gamma \frac{1 + \frac{1}{2} K_0^2}{\frac{1}{2} K_0^2} \quad (9)$$

This expression tells us how to set the K taper for a given energy taper. In our test it tells us how to set the γ taper for a measured K taper. We use this tapered γ in our calculations of phase errors and phase matching errors.

When we taper the undulator gap, we know the gap at each end of the undulator. From the production measurements without taper, we know the K value that corresponds to the gap at each end of the undulator. We also know the phase matching error correction for the K value at each end of the undulator in the untapered case. We wish to see if the phase matching error with taper agrees with the phase matching error without taper but at the K value at the undulator ends. We also wish to see if the phase errors grow with the combined energy taper and K taper.

3 Calculations With z -Dependent γ

In order to test the ideas presented above on making the K taper compensate the γ taper, we must perform calculations that include the z dependence of γ . The equations for constant γ were derived in a previous note². We revisit those equations, but now make γ a function of z .

A relativistic electron in a magnetic field obeys the Lorentz force equation

$$\frac{d}{dt} (\gamma m \vec{v}) = q \vec{v} \times \vec{B} \quad (10)$$

It is convenient to change variables from time to the z position of the electron bunch. With this change of variables $dz = v_z dt$, and we have

$$v_z \frac{d}{dz} (\gamma m \vec{v}) = q \vec{v} \times \vec{B} \quad (11)$$

Equation 11 can be written in component form, ignoring small terms in the cross product, as

$$v_z \frac{d}{dz} (\gamma v_x) = -\frac{q}{m} v_z B_y \quad (12)$$

$$v_z \frac{d}{dz} (\gamma v_y) = \frac{q}{m} v_z B_x \quad (13)$$

Cancelling v_z and integrating, we find

$$\gamma v_x = -\frac{q}{m} \int_{z_0}^z B_y(z_1) dz_1 \quad (14)$$

$$\gamma v_y = \frac{q}{m} \int_{z_0}^z B_x(z_1) dz_1 \quad (15)$$

Changing variables from time to z -position in the velocities, we have $v_x = \frac{d}{dt} x = v_z x'$ and $v_y = \frac{d}{dt} y = v_z y'$, where the prime indicates differentiation with respect to z . Using this notation, we find

²Z. Wolf, "Introduction To LCLS Undulator Tuning", LCLS-TN-04-7, June, 2004.

the trajectory slopes

$$x' = -\frac{q}{mv_z \gamma} \int_{z_0}^z B_y(z_1) dz_1 \quad (16)$$

$$y' = \frac{q}{mv_z \gamma} \int_{z_0}^z B_x(z_1) dz_1 \quad (17)$$

For large γ , we make the approximation $v_z \simeq c$ with negligible error of the trajectory slopes.

$$x' = -\frac{q}{mc \gamma} \int_{z_0}^z B_y(z_1) dz_1 \quad (18)$$

$$y' = \frac{q}{mc \gamma} \int_{z_0}^z B_x(z_1) dz_1 \quad (19)$$

The trajectory slopes are used to find the slippage. The radiation in the undulator moves with the speed of light c in the z direction and the electron moves with speed v_z . In time dt , the slippage changes by

$$dS = (c - v_z)dt \quad (20)$$

Using $dz = v_z dt$, we have

$$dS = \left(\frac{c}{v_z} - 1\right)dz \quad (21)$$

If the electron has velocity v with components v_x , v_y and v_z , $v^2 = v_z^2 + v_x^2 + v_y^2$. We can write this as

$$v^2 = v_z^2 \left(1 + \frac{v_x^2}{v_z^2} + \frac{v_y^2}{v_z^2}\right) = v_z^2 (1 + x'^2 + y'^2) \quad (22)$$

The velocity can be expressed in terms of γ , $v^2 = c^2 \left(1 - \frac{1}{\gamma^2}\right)$. Combining these formulas, we have $c^2 \left(1 - \frac{1}{\gamma^2}\right) = v_z^2 (1 + x'^2 + y'^2)$. This lets us solve for $\frac{c}{v_z}$

$$\frac{c}{v_z} = \sqrt{\frac{(1 + x'^2 + y'^2)}{\left(1 - \frac{1}{\gamma^2}\right)}} \quad (23)$$

It is a very good assumption for relativistic electrons that $\frac{1}{\gamma^2} \ll 1$, $x'^2 \ll 1$, and $y'^2 \ll 1$. Using these approximations and keeping only first order terms in small quantities, we find

$$\frac{c}{v_z} \simeq 1 + \frac{1}{2\gamma^2} + \frac{1}{2}(x'^2 + y'^2) \quad (24)$$

The slippage differential can now be written as

$$dS = \left(\frac{1}{2\gamma^2} + \frac{1}{2}(x'^2 + y'^2)\right) dz \quad (25)$$

Integrating from initial position z_0 to z , we find the change in slippage

$$\Delta S = \int_{z_0}^z \left(\frac{1}{2\gamma^2} + \frac{1}{2}(x'^2 + y'^2)\right) dz \quad (26)$$

If we define the initial position to have zero slippage, $S(z_0) = 0$, we can define the slippage as

$$S(z) = \int_{z_0}^z \left(\frac{1}{2\gamma^2} + \frac{1}{2}(x'^2 + y'^2)\right) dz \quad (27)$$

We now put in the expressions for x' and y' in order to illustrate how the z dependence of γ must be taken into account.

$$S(z) = \int_{z_0}^z \left[\frac{1}{2\gamma(z_2)^2} + \frac{1}{2} \left(\frac{q}{mc} \right)^2 \left(\frac{1}{\gamma(z_2)} \int_{z_0}^{z_2} B_y(z_1) dz_1 \right)^2 + \frac{1}{2} \left(\frac{q}{mc} \right)^2 \left(\frac{1}{\gamma(z_2)} \int_{z_0}^{z_2} B_x(z_1) dz_1 \right)^2 \right] dz_2 \quad (28)$$

The resonance condition states that the average slippage change from pole to pole is one half the radiation wavelength λ_r . We can calculate $S(z)$ and evaluate it at the field peaks. Let ΔS_p be the difference in the slippage between two adjacent field peaks. Multiplying ΔS_p by 2, and averaging over all adjacent peaks gives λ_r .

$$2 \langle \Delta S_p \rangle = \lambda_r \quad (29)$$

We define phase by dividing the slippage by λ_r and multiplying by 2π . Let P be the phase,

$$P(z) = \frac{2\pi S(z)}{\lambda_r} \quad (30)$$

The phase is zero at the initial point where the slippage was defined to be zero. If we let ΔP_p be the phase change between two adjacent field peaks, we have

$$\Delta P_p = \frac{2\pi}{\lambda_r} \Delta S_p \quad (31)$$

Averaging over all pairs of field peaks, we have

$$\langle \Delta P_p \rangle = \pi \quad (32)$$

since $\langle \Delta S_p \rangle = \lambda_r/2$. Let P_k be the phase at field peak k minus the phase at the first peak, numbered 0. If the phase advance between adjacent field peaks all had the average value, then $P_k = k\pi$. The difference between this nominal phase increase and the actual phase increase is called the phase error at pole k , ϵ_k .

$$\epsilon_k = P_k - k\pi \quad (33)$$

This gives us the phase errors. Once we know the phase, the phase matching errors can also be found. They are calculated the same way as in the untapered case³.

4 Measurements

The measurements for this test were done with undulator SXU-006. Production measurements were completed before this test so those measurements were available and were used for determining quantities in the untapered case. Fits were made to the untapered data to determine the gap as a function of K , K as a function of gap, and the phase matching errors as a function of K .

Tapered measurements for this test were made with the upstream end gap set to 8.000 mm and the downstream end gap set to 8.100 mm, 8.300 mm, and 7.900 mm. We define taper to be the difference between the downstream gap and the upstream gap. So measurements for the test were made with an upstream gap of 8.000 mm and tapers of +0.100, +0.300, and -0.100 mm. Measurements were also made with an upstream gap of 20.000 mm and tapers of +0.100, +0.300, and -0.100 mm.

In order to illustrate the measurement results, we present the case where the upstream gap was 8.000 mm and the taper was +0.300 mm. Other cases showed similar results. They are shown in tables at the end of this section.

³Z. Wolf, "Phase Matching The LCLS-II Undulators", LCLS-TN-16-3, July, 2016.

For the test, a gap taper was put into the undulator. The gap was measured at the ends of the undulator. A fit was made to the untapered measurements to determine the K value at each end of the undulator that corresponded to the gap. A linear K taper was determined. From the K taper, a linear energy taper was calculated. The linear energy taper was used with the measured magnetic fields to calculate the slippage, wavelength, phase, phase errors, and phase matching errors. Comparisons were made to the untapered case.

4.1 K Taper

For measurements with 8 mm gap and 0.3 mm taper, the measured gap at the upstream end of the undulator was 7.999 mm, and the measured gap at the downstream end was 8.299 mm. From fits to the untapered production measurements, the K value at the upstream end was 5.31018, and the K value at the downstream end was 5.13017. The encoders at the upstream end downstream ends were 3.400 m apart. Using the formula

$$K = K_0 \left(1 - \alpha_K \frac{z}{L}\right) \quad (34)$$

we determine the K taper parameters

$$K_0 = 5.31018 \quad (35)$$

$$\alpha_K = 0.03386 \quad (36)$$

$$L = 3.400 \text{ m} \quad (37)$$

The K taper function is shown in figure 2.

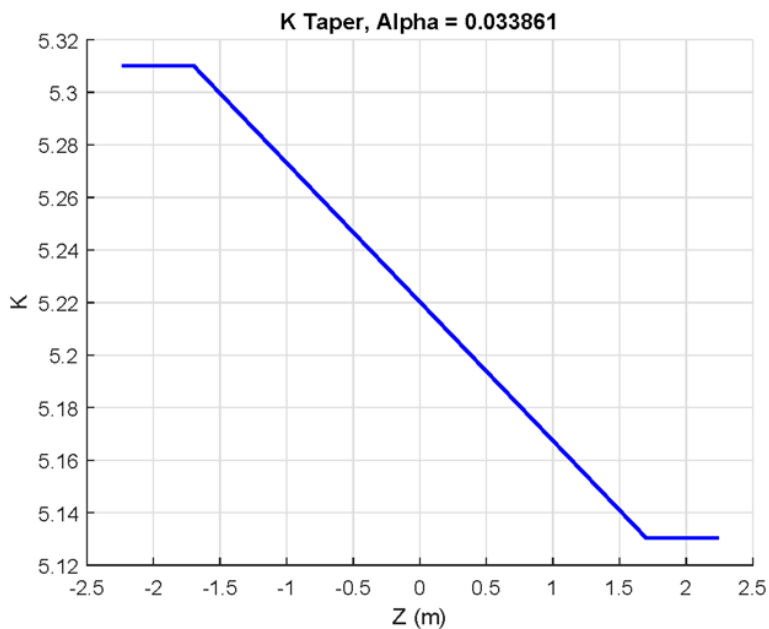


Figure 2: Illustration of the K taper function. The function only exists inside the undulator.

The gap taper reduces the peak fields progressing down the undulator. The absolute value of the peak field at each pole is shown in figure 3. The linear gap taper produces an approximately linear field strength taper.

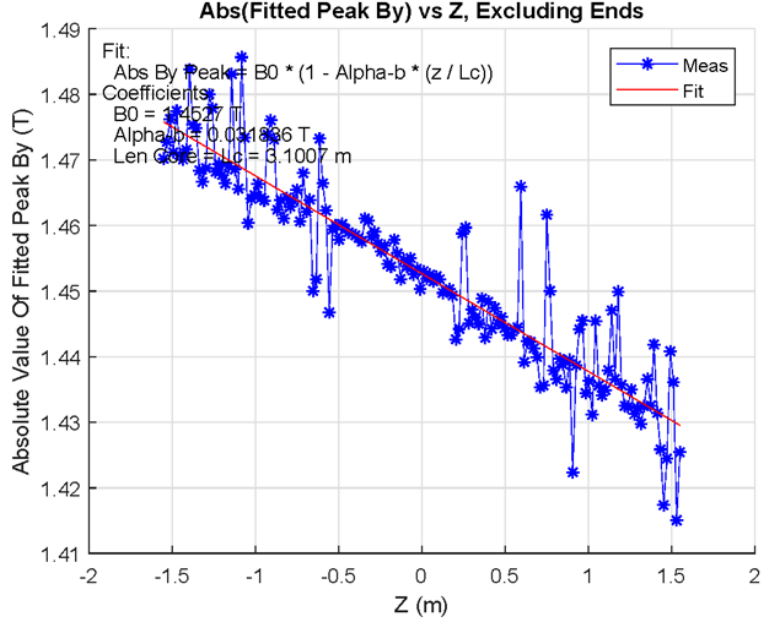


Figure 3: Absolute value of the B_y peak field strengths.

4.2 Energy Taper

The energy taper is given by

$$\gamma = \gamma_0 \left(1 - \alpha_\gamma \frac{z}{L}\right) \quad (38)$$

Knowing the K taper parameters, we can calculate the corresponding energy taper parameters. The slope of the energy taper is calculated from the slope of the K taper using the formula

$$\alpha_\gamma = \alpha_K \frac{\frac{1}{2}K_0^2}{1 + \frac{1}{2}K_0^2} \quad (39)$$

We take the Lorentz factor at the undulator entrance to be $\gamma_0 = 7827.8$. The energy taper parameters are

$$\gamma_0 = 7827.8 \quad (40)$$

$$\alpha_\gamma = 0.031618 \quad (41)$$

$$L = 3.400 \text{ m} \quad (42)$$

A plot of the energy taper is shown in figure 4.

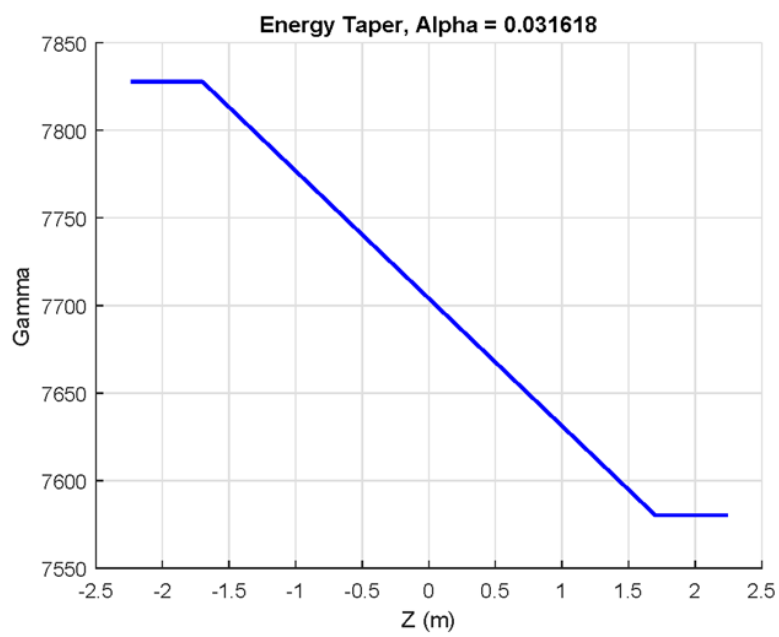


Figure 4: Energy taper which corresponds to the gap taper in the undulator.

4.3 Slippage

The slippage is calculated from equation 28. We have determined the energy taper $\gamma(z)$ and we have measured the fields $B_x(z)$ and $B_y(z)$ in the tapered undulator. The slippage evaluated at the peak field positions is shown in figure 5. Also shown is a linear fit to the slippage at the field peaks. The

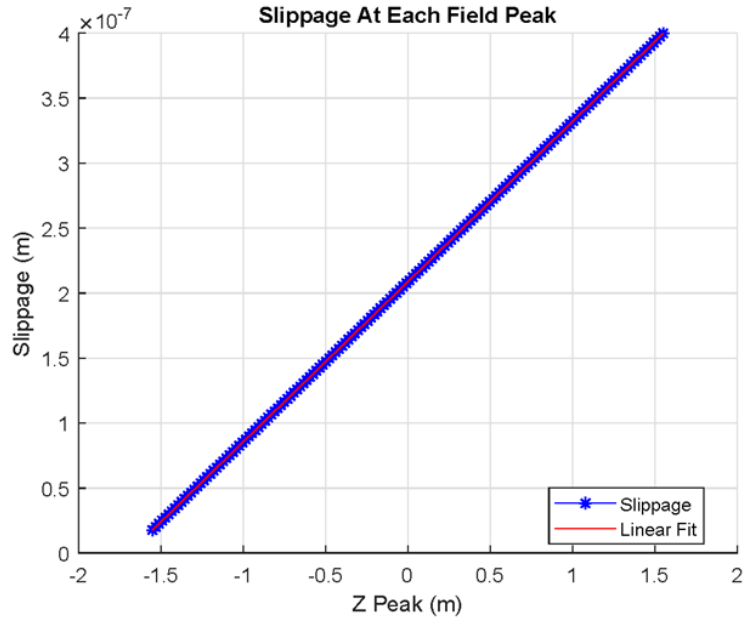


Figure 5: Slippage evaluated at the peak field positions.

residuals to the linear fit are shown in figure 6. The residuals are small showing that the slippage increases to good approximation by half a radiation wavelength per half undulator period.

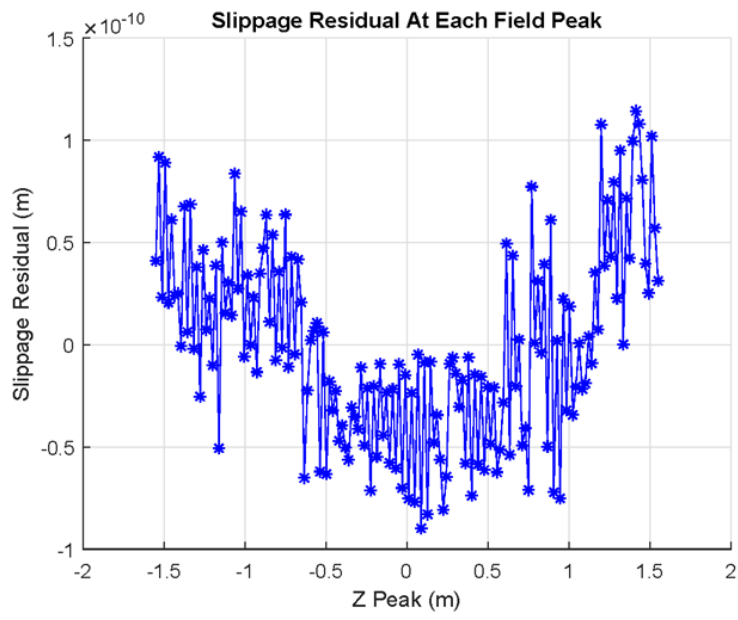


Figure 6: Residuals to a linear fit of the slippage at the field peaks.

4.4 Wavelength

Figure 7 is a plot of twice the slippage change between field peaks, which is the local radiation wavelength from the resonance condition. The local radiation wavelength is fairly constant along the undulator. The average value is the radiation wavelength for the undulator, λ_r .

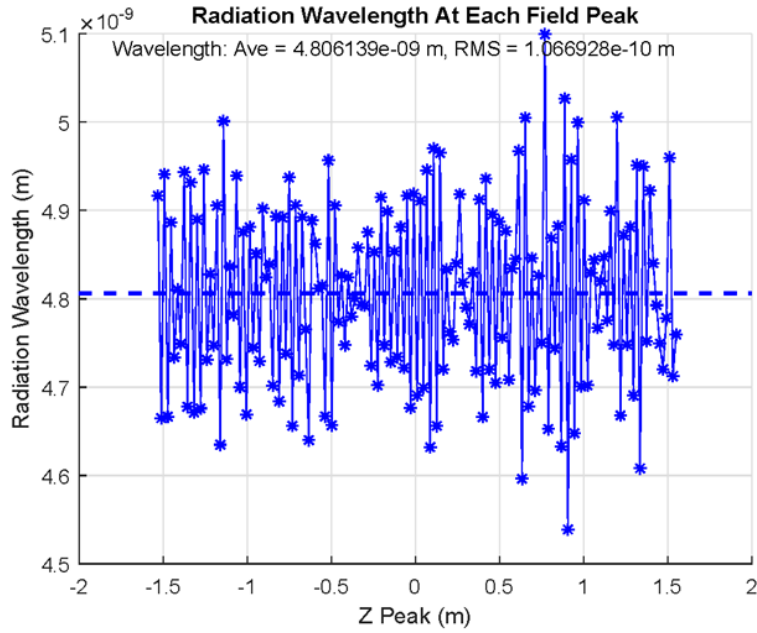


Figure 7: This figure shows twice the slippage change between field peaks. It gives the local radiation wavelength.

4.5 Phase

Knowing the slippage and the radiation wavelength, we can calculate the phase.

$$P(z) = \frac{2\pi S(z)}{\lambda_r} \quad (43)$$

The phase can be evaluated at the peak field positions. Ideally, the phase increases by π between the field peaks. The difference between the phase at the field peaks and a linear function that increases as π per pole is the phase error. The phase errors are plotted in figure 8. The rms phase error is 3.54° . For the untapered undulator with 8 mm gap, the rms phase error was 2.04° . Tapering caused only a small increase in the phase errors. Note that the energy taper had to be correct in order to have small phase errors. If the energy had been assumed to be constant in the undulator, the rms phase error would have been 61° . In order to test the sensitivity, the energy taper was increased by 10% and the rms phase error increased to 9.24° .

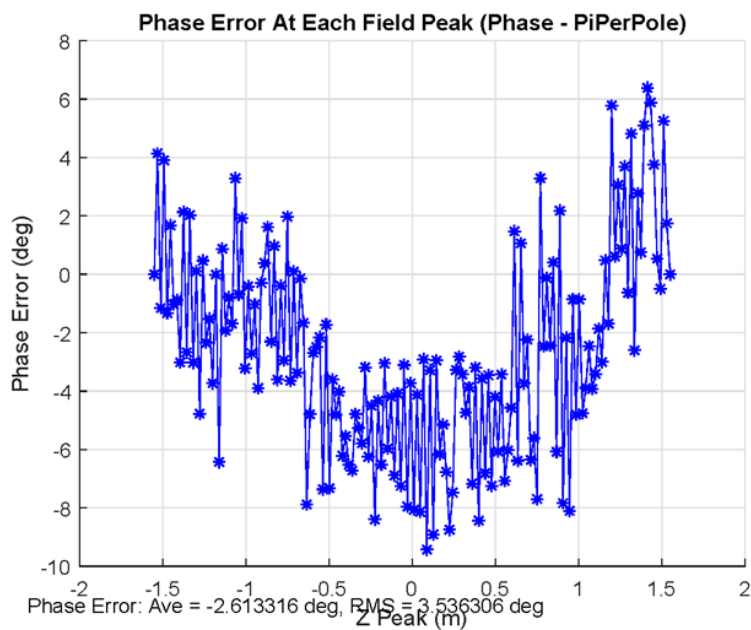


Figure 8: Phase errors in the tapered undulator with a linear energy taper.

4.6 Phase Matching Error

We need to know how to set the phase shifters for tapered undulators. We know the phase matching error at the entrance and exit gap widths of an untapered undulator from the production measurements. We compare the phase matching error of the tapered undulator to the value of the untapered undulator at the corresponding gap.

At the tapered undulator entrance, the gap was 7.999 mm. The phase matching error at the entrance is shown in figure 9. The average phase matching error is 130.1° . For the untapered

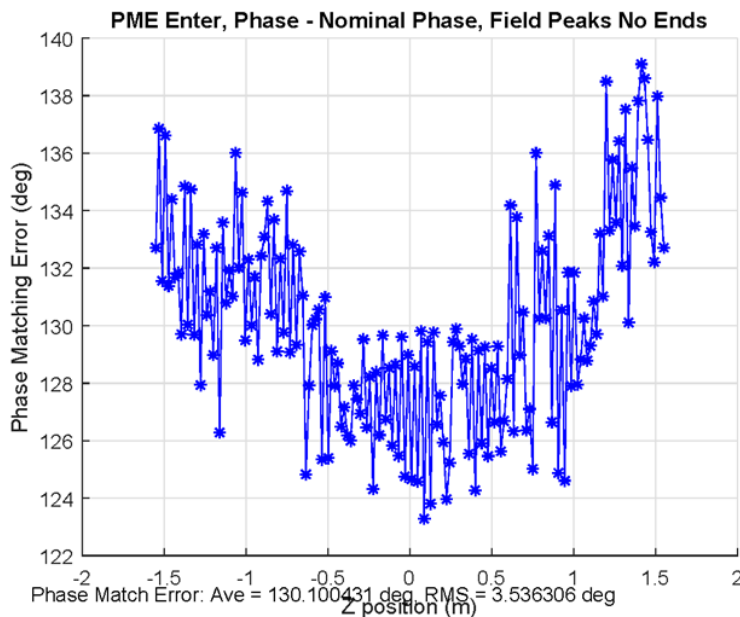


Figure 9: Phase matching error at the undulator entrance.

undulator at 7.999 mm gap, the phase matching error at the entrance was 135.0° .

At the tapered undulator exit, the gap was 8.299 mm. The phase matching error at the exit is shown in figure 10. The average phase matching error is 158.4° . For the untapered undulator at 8.299 mm gap, the phase matching error at the exit was 152.8° .

The phase matching errors are different than the untapered case by about the tolerance for setting the phase shifters. We conclude that the untapered measurements can be used at each end of the tapered undulator, but the errors will increase.

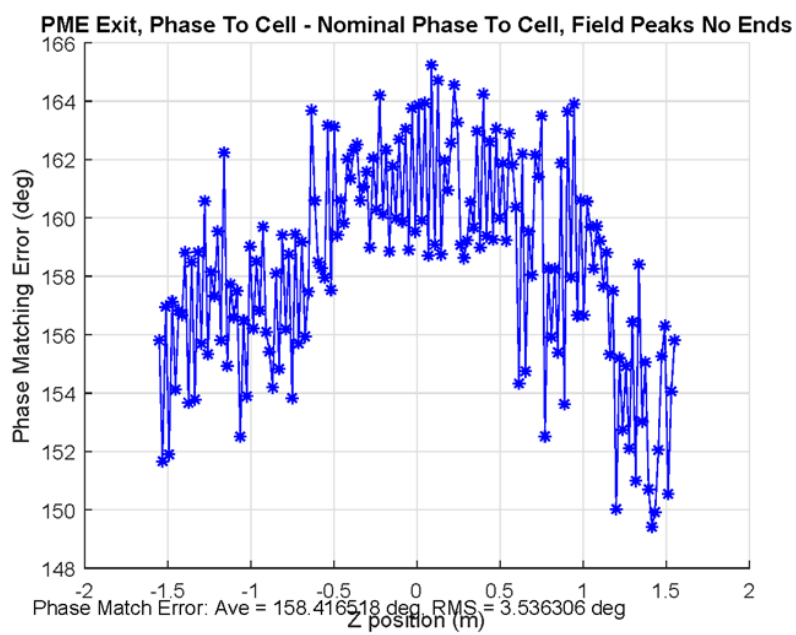


Figure 10: Phase matching error at the undulator exit.

4.7 Measurement Summary

Results from measurements with a gap of 8 mm and a taper of 0.3 mm were presented above. The other measurements in this test are summarized below.

The gaps and tapers, along with the K values at the ends of the undulator and the α_K and α_γ parameters are given in the table below.

Gap (mm)	Taper (mm)	K_{enter}	K_{exit}	α_K	α_γ
8.000	0.100	5.3103	5.2504	0.01127	0.01053
8.000	0.300	5.3102	5.1304	0.03386	0.03162
8.000	-0.100	5.3101	5.3694	-0.01117	-0.01043
20.000	0.100	1.6466	1.6325	0.008597	0.004937
20.000	0.300	1.6466	1.6047	0.02549	0.01467
20.000	-0.100	1.6466	1.6608	-0.008644	-0.004974

The phase errors in the tapered undulator are given in the table below. Also shown are the phase errors at the given gap with no taper. "NT" indicates no taper.

Gap (mm)	Taper (mm)	Phase Err (deg)	NT Phase Err (deg)
8.000	0.100	2.50	2.04
8.000	0.300	3.17	2.04
8.000	-0.100	2.12	2.04
20.000	0.100	2.84	2.73
20.000	0.300	2.99	2.73
20.000	-0.100	2.76	2.73

The phase matching errors for the tapered and untapered cases are given in the table below.

Gap (mm) (mm)	Taper (mm) (mm)	PME Enter (deg)	NT PME Enter (deg)	PME Exit (deg)	NT PME Exit (deg)
8.000	0.100	132.8	135.0	140.6	137.5
8.000	0.300	130.6	135.0	157.9	152.8
8.000	-0.100	137.6	135.0	123.1	124.1
20.000	0.100	1986.8	1986.6	2018.7	2012.1
20.000	0.300	1985.5	1986.5	2063.3	2054.9
20.000	-0.100	1987.5	1986.5	1974.9	1969.4

5 Conclusion

This study shows that the LCLS-II SXR undulators can be tapered without significantly increasing the phase errors or the phase matching errors. The gap taper must be carefully chosen to match the energy taper. The untapered measurements at the corresponding gap at each end of the undulator can be used to get the K values at each end and determine the K taper. The K values at the undulator ends can be used to determine the entrance and exit phase matching errors, and thereby set the phase shifters.