Setting The LCLS-II Phase Shifters

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Abstract

This note describes a technique for setting the LCLS-II phase shifters. Each phase shifter must make corrections for both its upstream and downstream undulators. The method of combining these corrections is discussed. The phase shifters must also make jumps in order to stay within their phase integral range. An algorithm to make the jumps is discussed.

1 Introduction[1]

LCLS-II requires phase shifters between the undulators in order for the phase relation between the electron bunch and the light wave to be maintained as the undulator gaps are changed. In order to analyze how to set the phase shifters, we conceptually divide a phase shifter into two phase shifters, one phase shifter to correct the upstream undulator and another to correct the downstream undulator. The layout of the two phase shifters is shown in figure 1. The technique of conceptually

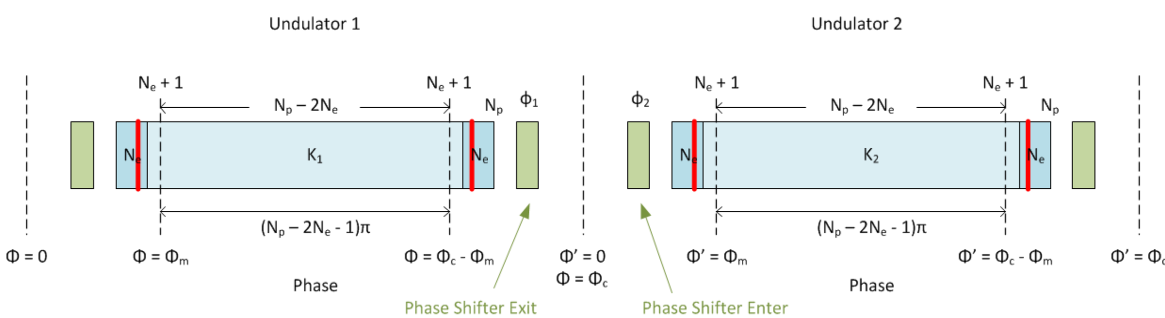

Figure 1: The phase shifter between undulators is conceptually broken into a phase shifter which sets the phase matching error at the cell boundary to a multiple of 2π, and a phase shifter which sets the phase matching error in the central core of the downstream undulator to a multiple of 2π.

dividing the phase shifter in two is described in detail in another technical note.[2] In this note we discuss how to combine the two conceptual phase shifters into the single actual phase shifter, and how we set its phase integral so that it is within the range of the phase shifter.

Two adjacent undulators are typically used with different K values. This is done, for instance, to keep the resonant wavelength constant even though the average electron energy is lower in the downstream undulator. In this note we analyze how to set the phase shifters for two cases. In the

[1] Work supported in part by the DOE Contract DE-AC02-76SF00515. This work was performed in support of the LCLS project at SLAC.

[2] Z. Wolf, "Phase Matching The LCLS-II Undulators", LCLS-TN-16-3, July, 2016.

first case, the undulators have a constant K value along their length and adjacent undulators are set to different K values. This is a step taper and was used for LCLS-I. The LCLS-II undulators also allow the gap to be tapered linearly along its length. We also consider setting the phase shifters with a linear taper.

2 Step Taper

The production measurements of the undulators are typically made with a constant gap. The undulator parameters are calculated assuming that γ, the electron Lorentz factor, is constant in the undulator. In this section we assume that γ is constant in each undulator, but is different in different undulators. The K values are different in order to keep the undulators resonant to a fixed wavelength of light. The situation is shown in figure 2. The step change in γ is an approximation

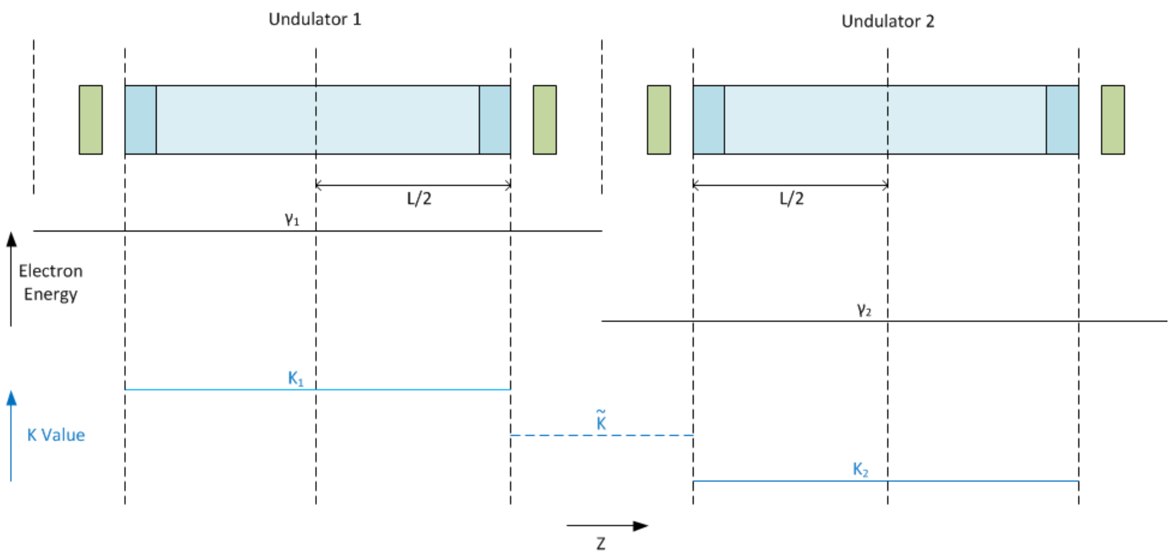

Figure 2: In the step taper model, the undulators are set to fixed K values along their length. The K values change between undulators. The electron Lorentz factor is assumed constant in the analysis of each undulator.

to the way γ decreases along the undulator line. If we keep the resonant wavelength the same in each undulator, we have a relation between K and γ.

$$\lambda_r = \frac{\lambda_u}{2\gamma_1^2}\left(1 + \frac{1}{2}K_1^2\right) = \frac{\lambda_u}{2\gamma_2^2}\left(1 + \frac{1}{2}K_2^2\right) \tag{1}$$

Consider phase matching in this model. Let ψ_1 be the phase matching error at the exit of the upstream undulator. This is the difference between the phase advance from the central core of the undulator to the cell boundary and the ideal phase advance from the central core of the undulator to the cell boundary. The phase matching error is corrected to a multiple of 2π by adding a phase shift ϕ_1, as illustrated in figure 3. The phase shift ϕ_1 as a function of the undulator parameter K_1 is illustrated in figure 4. The phase shift ϕ_1 is added by a phase shifter which adds phase by setting its magnetic field to have a phase integral PI_1. The phase integral PI_1 is calculated from the required phase ϕ_1 using the undulator parameter K_1. The phase integral as a function of K_1 is illustrated in figure 5.

Similarly, let ψ_2 be the phase matching error at the entrance of the downstream undulator. This is the difference between the phase advance from the cell boundary to the central core of the undulator and the ideal phase advance from the cell boundary to the central core of the undulator. This is corrected to a multiple of 2π by a phase shifter which adds phase ϕ_2. The phase shifter adds the phase by setting its magnetic field to have a phase integral PI_2. The phase integral PI_2 is calculated from the required phase ϕ_2 using the undulator parameter K_2. The phase matching error, added phase for correction, and required phase integral for undulator 2 are shown in the figures mentioned above.

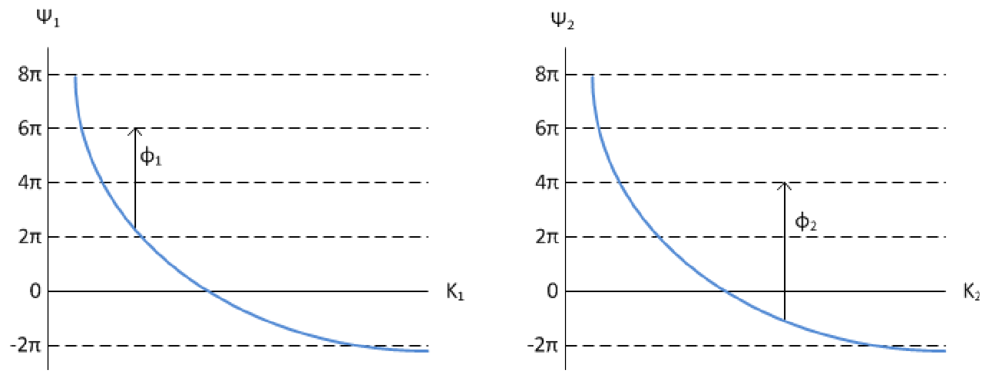

Figure 3: A phase shifter adds phase to the phase matching error to bring it to a multiple of 2π.

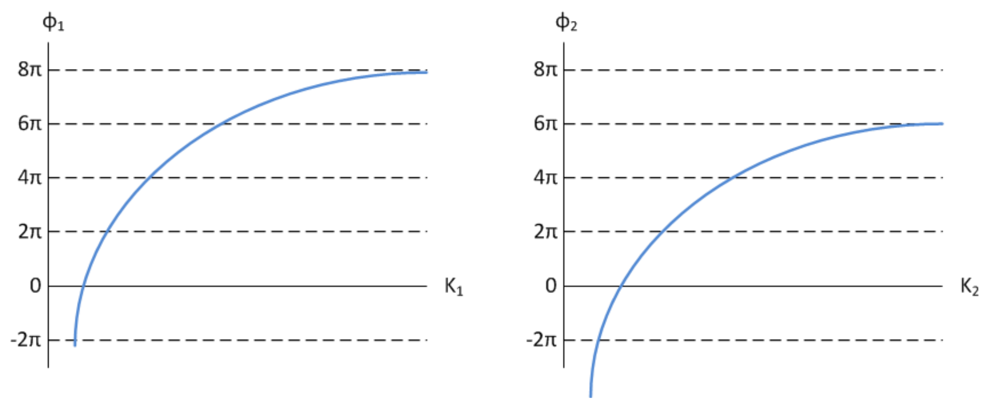

Figure 4: Required phase corrections as a function of K.

The phase integrals required for the given phase corrections are given by[3]

$$PI_1 = \phi_1 \frac{\lambda_u}{2\pi} \left(1 + \frac{1}{2} K_1^2\right) \left(\frac{mc}{e}\right)^2 \tag{2}$$

$$PI_2 = \phi_2 \frac{\lambda_u}{2\pi} \left(1 + \frac{1}{2} K_2^2\right) \left(\frac{mc}{e}\right)^2 \tag{3}$$

Note that γ has dropped out of these relations.

[3] Z. Wolf, "A PPM Phase Shifter Design", LCLS-TN-11-2, July, 2011.

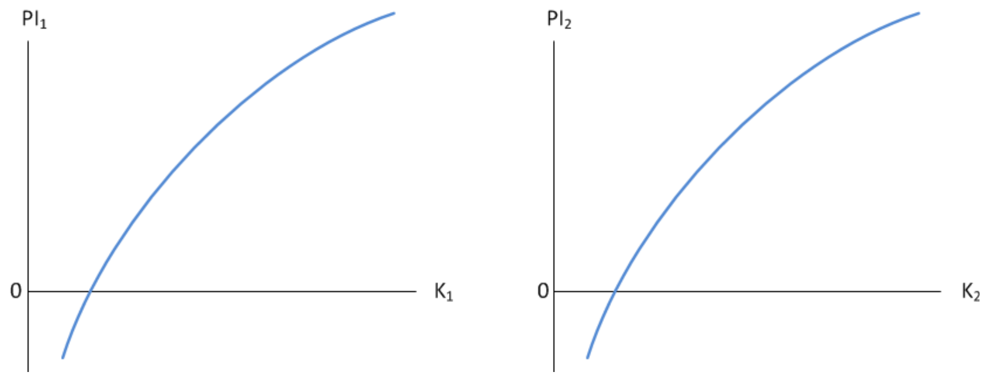

Figure 5: Phase integral from the phase shifter required to add the appropriate phase as a function of K.

As noted above, the phase shifters can add any multiple of 2π to the phase and still preserve the proper phase relationship between the light wave and the electron beam. Adding multiples of 2π to the phase gives allowed phase integrals

$$PI_1' = (\phi_1 + 2\pi n_1)\frac{\lambda_u}{2\pi}\left(1 + \frac{1}{2}K_1^2\right)\left(\frac{mc}{e}\right)^2 \tag{4}$$

$$PI_2' = (\phi_2 + 2\pi n_2)\frac{\lambda_u}{2\pi}\left(1 + \frac{1}{2}K_2^2\right)\left(\frac{mc}{e}\right)^2 \tag{5}$$

In practice the two phase shifters are combined into a single phase shifter with phase integral

$$PI = PI_1' + PI_2' \tag{6}$$

The phase integral of the single phase shifter is given by

$$PI = PI_1(K_1) + PI_2(K_2) + n_1\lambda_u\left(1 + \frac{1}{2}K_1^2\right)\left(\frac{mc}{e}\right)^2 + n_2\lambda_u\left(1 + \frac{1}{2}K_2^2\right)\left(\frac{mc}{e}\right)^2 \tag{7}$$

The phase shifters for LCLS-II do not have a large enough phase integral range to continuously supply the phase matching correction required by the undulators. Since only a phase correction up to a multiple of 2π is required, the values of n_1 and n_2 can be chosen to bring the phase integral within the range of the phase shifter. Jumps in the phase shifter phase integral can be made by changing n_1 and n_2.

Making jumps as a function of two parameters K_1 and K_2 is difficult. We simplify the problem of bringing the required phase integral within the range of the phase shifter by considering the case of a single jump parameter n and a single K value $\widetilde{K}$ as shown in figure 6. The phase shifter has maximum phase integral $PI_{\max}$ and minimum phase integral $PI_{\min}$. We set operational limits $PI_{up\,\mathrm{lim}}$ and $PI_{lo\,\mathrm{lim}}$ within the range of $PI_{\max}$ and $PI_{\min}$. As one approaches the upper or lower phase integral limit, one can jump to a new curve by changing the value of n. Also note that the value of n can be chosen to minimize the number of jumps. The red line shows how the phase integral would be set for different values of $\widetilde{K}$. Having a single value of n as a function of a single parameter $\widetilde{K}$ makes for a simple algorithm for determining where the jumps take place.

We now look for a way to use the simplicity of a single value of n with a single parameter $\widetilde{K}$ in equation 7. In the alternative, if n_1 is a function of K_1 and n_2 is a function of K_2, then if K_1 and K_2 have values near each other, we can have a jump when K_1 reaches a certain value, and then

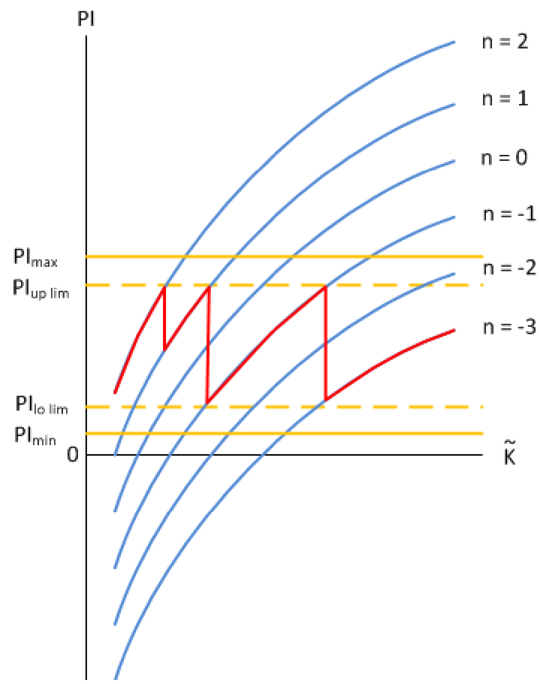

Figure 6: Jumps in the phase integral of the phase shifter must be made in order to keep the phase integral within the range of the phase shifter.

another jump soon afterward when K_2 reaches a nearby value, which complicates the operation and use of the machine. This problem can be eliminated if the jumps occur at certain values of a single variable. Let

$$\widetilde{K} = \widetilde{K}(K_1, K_2) \tag{8}$$

be a function of K_1 and K_2. For example, we can have $\widetilde{K}$ be the mean of K_1 and K_2. Since we have many measurements giving us $PI_1(K_1)$, we can fit this data to get $PI_1(\widetilde{K})$. The same is true for $PI_2(\widetilde{K})$. From this we find the phase shifter phase integral at $K_1 = K_2 = \widetilde{K}$.

$$PI(\widetilde{K}) = PI_1(\widetilde{K}) + PI_2(\widetilde{K}) + n_1(\widetilde{K})\lambda_u \left(1 + \frac{1}{2}\widetilde{K}^2\right)\left(\frac{mc}{e}\right)^2 + n_2(\widetilde{K})\lambda_u \left(1 + \frac{1}{2}\widetilde{K}^2\right)\left(\frac{mc}{e}\right)^2 \tag{9}$$

or

$$PI(\widetilde{K}) = PI_1(\widetilde{K}) + PI_2(\widetilde{K}) + \left(n_1(\widetilde{K}) + n_2(\widetilde{K})\right)\lambda_u \left(1 + \frac{1}{2}\widetilde{K}^2\right)\left(\frac{mc}{e}\right)^2 \tag{10}$$

Let $n = n_1 + n_2$. Then

$$PI(\widetilde{K}) = PI_1(\widetilde{K}) + PI_2(\widetilde{K}) + n(\widetilde{K})\lambda_u \left(1 + \frac{1}{2}\widetilde{K}^2\right)\left(\frac{mc}{e}\right)^2 \tag{11}$$

We use this formula to find the jumps in PI. As one reaches the upper limit phase integral, the value of n is reduced to another integer as small as possible such that the phase integral is still larger than the lower limit of the phase shifter. This procedure gives us a function $n(\widetilde{K})$ consisting of jumps between integers at discrete $\widetilde{K}$ values. One possibility is that the integers n_1 and n_2 are found as follows.

$$n_1 = floor(n/2) \tag{12}$$
$$n_2 = n - n1 \tag{13}$$

where $floor$ represents the integer part of $n/2$. This procedure keeps n_1 and n_2 close to each other, and it makes them jump at the same value of $\widetilde{K}$.

We now return to the original problem where we want the phase shifter phase integral when the adjacent undulators have K values of K_1 and K_2. In this case we form the function $\widetilde{K} = \widetilde{K}(K_1, K_2)$ and use the jump functions that we just found. The formula is

$$PI(K_1, K_2) = PI_1(K_1) + PI_2(K_2) + n_1(\widetilde{K})\lambda_u \left(1 + \frac{1}{2}K_1^2\right)\left(\frac{mc}{e}\right)^2 + n_2(\widetilde{K})\lambda_u \left(1 + \frac{1}{2}K_2^2\right)\left(\frac{mc}{e}\right)^2 \tag{14}$$

This formula is valid for any K_1 and K_2 with integers n_1 and n_2. The jump functions $n_1(\widetilde{K})$ and $n_2(\widetilde{K})$ are shown in figure 7. The jumps always occur at the same values of $\widetilde{K}$. The integer values

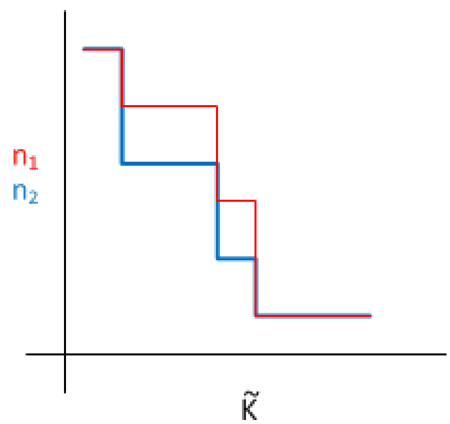

Figure 7: Jumps in n_1 and n_2 occur at the same values of $\widetilde{K}$. Jumps in n_1 and n_2 can occur between different integers.

of n_1 and n_2 can be different. Note that setting $K_1 = K_2 = \widetilde{K}$ is only an aid to establish the jump functions. Once the jump functions are established, the phase integral is a well defined function of K_1 and K_2.

We made PI_{uplim} less than $PI_{\max}$ and PI_{lolim} larger than $PI_{\min}$ so that there is a buffer region between the limits and the maximum and minimum values. Since $PI_1(K_1) \neq PI_1(\widetilde{K})$ and $PI_2(K_2) \neq PI_2(\widetilde{K})$, we use the buffer region to account for the small difference and still stay within the range of the phase shifter. The jumps only have to be between integers and give phase integrals within the range of the phase shifter.

Another advantage of the buffer region is that we can turn off jumps when doing small energy scans where a jump is undesirable. We turn off jumps and go into the buffer region by a small enough amount that we still stay within the range of the phase shifter.

A problem with this approach is that if we had chosen n_1 and n_2 a different way, even if $n_1 + n_2 = n$, we would get a different phase shifter setting. The effect is small if K_1 and K_2 are close to each other. This problem of ambiguity is inherent in the unphysical way γ changes between undulators. We next consider a more realistic linear taper model.

3 Linear Taper

Consider a model in which the electron energy decreases along each undulator. The energy loss is due to radiation and other losses. The energy loss in each undulator is assumed small and we take it as a linear decrease with distance. The electron Lorentz factor as a function of position is shown in figure 8. The resonant wavelength at any position is given by

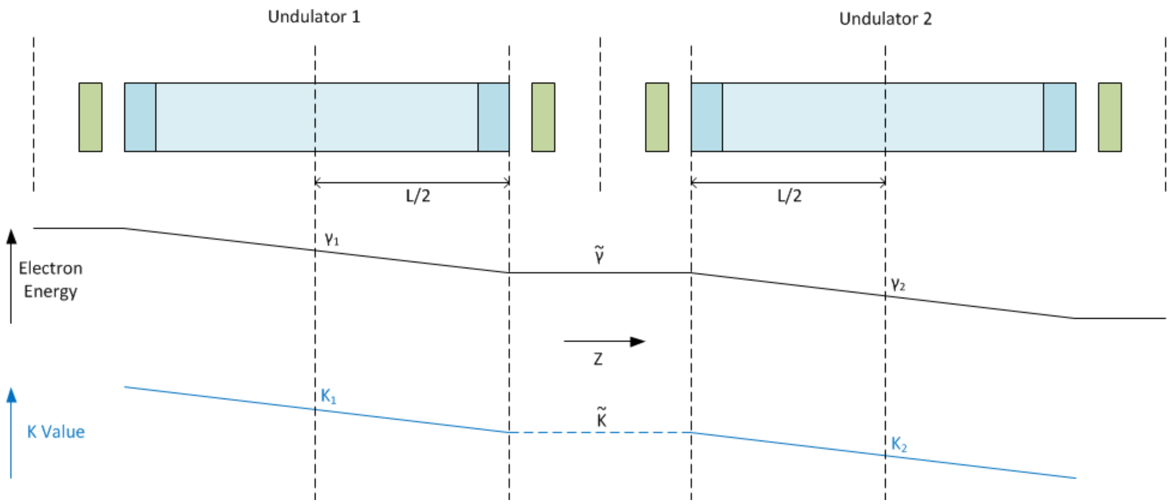

Figure 8: Linear taper model. Assume the electron energy decreases linearly in each undulator due to radiation. The wavelength is kept constant by putting a linear taper on K.

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{1}{2}K^2\right) \tag{15}$$

We wish to keep λ_r constant. Assume that γ, or equivalently the electron energy, changes along the undulator as

$$\gamma = \gamma_0\left(1 - \alpha\frac{z}{L/2}\right) \tag{16}$$

where z is the position along the undulator with $z = 0$ in the center of the undulator, γ_0 is the Lorentz factor in the center of the undulator, L is the undulator length, and α is the parameter describing the slope of the energy decrease. In order to keep λ_r constant, we put a linear gap taper into the undulator. The linear gap taper makes an approximate linear K taper for small tapers. In this case, we parametrize the position dependence of K as

$$K = K_0\left(1 - \beta\frac{z}{L/2}\right) \tag{17}$$

At any position along the undulator,

$$\lambda_r = \frac{\lambda_u}{2}\frac{1 + \frac{1}{2}K_0^2\left(1 - \beta\frac{z}{L/2}\right)^2}{\gamma_0^2\left(1 - \alpha\frac{z}{L/2}\right)^2} \tag{18}$$

We assume α and β are small. We expand the squared terms and find the relation between α and β which makes the first order terms cancel. Performing the expansion, we get

$$\lambda_r = \frac{\lambda_u}{2} \frac{1 + \frac{1}{2}K_0^2 \left(1 - 2\beta \frac{z}{L/2} + \beta^2 \left(\frac{z}{L/2}\right)^2\right)}{\gamma_0^2 \left(1 - 2\alpha \frac{z}{L/2} + \alpha^2 \left(\frac{z}{L/2}\right)^2\right)} \tag{19}$$

For small α we find

$$\lambda_r = \frac{\lambda_u}{2} \frac{1 + \frac{1}{2}K_0^2 \left(1 - 2\beta \frac{z}{L/2} + \beta^2 \left(\frac{z}{L/2}\right)^2\right)}{\gamma_0^2} \left(1 + 2\alpha \frac{z}{L/2} + 3\alpha^2 \left(\frac{z}{L/2}\right)^2 + O(\alpha^3)\right) \tag{20}$$

At this point we neglect the quadratic terms in α and β, and expand to first order.

$$\lambda_r = \frac{\lambda_u}{2\gamma_0^2} \left[1 + \frac{1}{2}K_0^2 - K_0^2 \beta \frac{z}{L/2} + \left(1 + \frac{1}{2}K_0^2\right) 2\alpha \frac{z}{L/2}\right] \tag{21}$$

In order to eliminate the z dependence, we set

$$-K_0^2 \beta \frac{z}{L/2} + \left(1 + \frac{1}{2}K_0^2\right) 2\alpha \frac{z}{L/2} = 0 \tag{22}$$

or

$$\beta = \alpha \frac{1 + \frac{1}{2}K_0^2}{\frac{1}{2}K_0^2} \tag{23}$$

This expression tells us how to set the K taper for a given energy taper.

In order to set the phase shifters, we assume that the production undulator measurements that are made without taper can be applied to the tapered undulator at a position with a given K value. In particular, assume the exit end of undulator 1 in figure 8 has a K value of $\widetilde{K}$. We assume that we can use the non-tapered measurements to find the fitted value of the exit phase matching error at $\widetilde{K}$. Similarly, the entrance of undulator 2 is set to a K value of $\widetilde{K}$. We use the non-tapered measurements of undulator 2 to find the phase matching error at $\widetilde{K}$.

The value of $\widetilde{K}$ is found by noting that the center of undulator 1 is set to K_1 and the center of undulator 2 is set to K_2. We assume the energy taper is the same in both undulators, so that the K taper is the same also. In this case,

$$\frac{K_1 - \widetilde{K}}{L/2} = \frac{\widetilde{K} - K_2}{L/2} \tag{24}$$

Solving for $\widetilde{K}$, we find that

$$\widetilde{K} = \frac{K_1 + K_2}{2} \tag{25}$$

The phase integral for the combined phase shifter for the exit of undulator 1 and the entrance of undulator 2 is

$$PI(\widetilde{K}) = PI_1(\widetilde{K}) + PI_2(\widetilde{K}) + n_1(\widetilde{K})\lambda_u \left(1 + \frac{1}{2}\widetilde{K}^2\right) \left(\frac{mc}{e}\right)^2 + n_2(\widetilde{K})\lambda_u \left(1 + \frac{1}{2}\widetilde{K}^2\right) \left(\frac{mc}{e}\right)^2 \tag{26}$$

which we have seen above in the step taper discussion. Simplifying, we get

$$PI(\widetilde{K}) = PI_1(\widetilde{K}) + PI_2(\widetilde{K}) + n(\widetilde{K})\lambda_u \left(1 + \frac{1}{2}\widetilde{K}^2\right) \left(\frac{mc}{e}\right)^2 \tag{27}$$

This expression is calculated using the non-tapered measurements evaluated at $\widetilde{K}$. Similarly, the jumps are calculated as a function of a single parameter, $\widetilde{K}$. This model of a linear taper is much more physical than a step taper model. It leads to a unique algorithm for setting the phase shifters.

4 Conclusion

An algorithm was proposed to determine jump functions $n_1(\widetilde{K})$ and $n_2(\widetilde{K})$ for a step taper,and $n(\widetilde{K})$ for a linear taper, which are functions of a single parameter $\widetilde{K}$. With these jump functions, we have a deterministic way to calculate $PI(K_1, K_2)$ in order to set the phase shifters within their operating range. Formulas for the phase shifter phase integral for both the step taper and the linear taper were given.

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