Fits To The Delta Undulator K Measurements

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Abstract

Magnetic measurements of the Delta undulator have been performed at different quadrant row settings. The settings include linear, circular, and elliptical modes and numerous K values. In this note fits to the data are made in an effort to find a single function that gives the K value at any quadrant row setting. Such a function will aid in making the Delta undulator resonant with the microbunched beam from LCLS. The form of the fit may also give information about errors in the undulator.

1 Introduction And Overview[1]

The K value and polarization mode of the Delta undulator are set by longitudinally moving the four quadrants[2]. Magnetic measurements were made with many different quadrant position settings, which put the undulator in different polarization modes with different K values. Typically, setting the polarization mode does not require high quadrant position accuracy. However, setting the K value to be resonant with the LCLS undulators does require high accuracy. For operating the undulator, we wish to find a single function which gives the K value as a function of quadrant positions. This function is determined by fitting the measurements. In order for the fit to be accurate, the form of the fitting function must be correct. In this note we explore such fits to the measurements.

The note starts by stating the requirements of the fit. It then defines the terminology with a discussion of how the K value for an ideal undulator is calculated. The explicit method of performing the fits is given. To do a fit, a model of the undulator is made and an expression for the K value using that model is determined. The fit determines values for the parameters of the model. Residuals of the fit are calculated and the smaller the residuals, the more confidence we have that the model is correct.

The first model considered assumes an ideal undulator measured off the magnetic center. We will see that the fit is fairly good, but the fit coefficients are too large to be described by the model. We then assume the undulator has quadrant strength and position errors. The fit is again fairly good at large K, but one of the fit coefficients is too large to be explained by the model. We then consider quadrant longitudinal position errors. The new terms in the fit improve the fit slightly, but not significantly. Finally, we consider possible effects from the magnet sorting. Again, the fit is slightly improved, but not significantly. By the end of the note, we have a fit that meets the requirements for setting the Delta undulator, but the fits do not provide the anticipated information about the inner workings of the undulator, at least not at low K.

[1] Work supported in part by the DOE Contract DE-AC02-76SF00515. This work was performed in support of the LCLS project at SLAC.

[2] Z. Wolf and H.-D. Nuhn, "Setting the K Value And Polarization Mode Of The Delta Undulator", LCLS-TN-14-2, September, 2014.

2 Requirements, Notation, And Fit Methodology

2.1 Requirements Of The Fit

The half width of the radiation spectrum $\Delta\omega/\omega = \Delta\lambda/\lambda$ of an undulator is equal to $1/N_p$, where N_p is the number of periods[3]. For the Delta undulator $N_p = 96$. The resonant wavelength is given by

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\sqrt{1 + \frac{1}{2}K^2} \tag{1}$$

If our fit gives an error ΔK when setting K, the resonant wavelength will change by

$$\frac{\Delta\lambda_r}{\lambda_r} = \frac{\frac{1}{2}K\Delta K}{1 + \frac{1}{2}K^2} \simeq \frac{\Delta K}{K}$$

The approximation is valid for the Delta undulator when $K \sim 3$ which we will consider below. In order for the Delta undulator to be resonant with the LCLS, we need

$$\frac{\Delta\lambda_r}{\lambda_r} < \frac{1}{N_p}$$

so we need

$$\frac{\Delta K}{K} < \frac{1}{N_p} \tag{2}$$

or $\Delta K/K < 0.01$. This means our fits are required to have residuals smaller than $\Delta K/K < 0.01$.

2.2 K Value Of An Ideal Undulator

An expression for the K value of the Delta undulator was given in a previous note[4] and is summarized briefly here. This summary gives the notation used in the rest of the note. When undulator errors are considered later in the note, they are treated as perturbations to the ideal undulator considered here.

2.2.1 Magnetic Field

The scalar potential for a quadrant is assumed to have the form

$$\phi = \phi_0 \cos(k_s s) \exp\left(-k_r r\right) \cos\left(k_u \left(z - z_0\right)\right) \tag{3}$$

where ϕ_0 is a constant, $k_u = 2\pi/\lambda_u$ where λ_u is the undulator period, z is the coordinate down the undulator, and z_0 gives the quadrant position along z. k_r and k_s determine the behavior of the potential in the transverse directions. The potential decreases as one moves radially away from the magnet array and this is expressed by $\exp\left(-k_r r\right)$. The potential also decreases as one moves to the side and this is given by the $\cos(k_s s)$ dependence over a limited range of s. The coordinate system is shown in figure 1. Since the Laplacian of the potential is zero, we have the constraint

$$-k_s^2 + k_r^2 - k_u^2 = 0 \tag{4}$$

We use the form of the potential given in equation 3 to calculate the magnetic scalar potential in the undulator by rotating the four quadrants and summing their rotated scalar potentials. The Delta undulator is oriented as shown on the left side of figure 2 and we call this the laboratory frame. In the laboratory frame, z is along the beam direction, y_L is up, and x_L makes a right

[3] A. Hofmann, "The Physics Of Synchrotron Radiation", Cambridge University Press, Cambridge, 2007, p.137.

[4] Z. Wolf, "Position Dependence Of The K Parameter In The Delta Undulator", LCLS-TN-16-1, January, 2016.

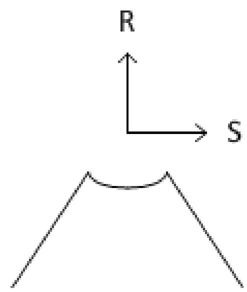

Figure 1: Coordinate system for the scalar potential of a single quadrant.

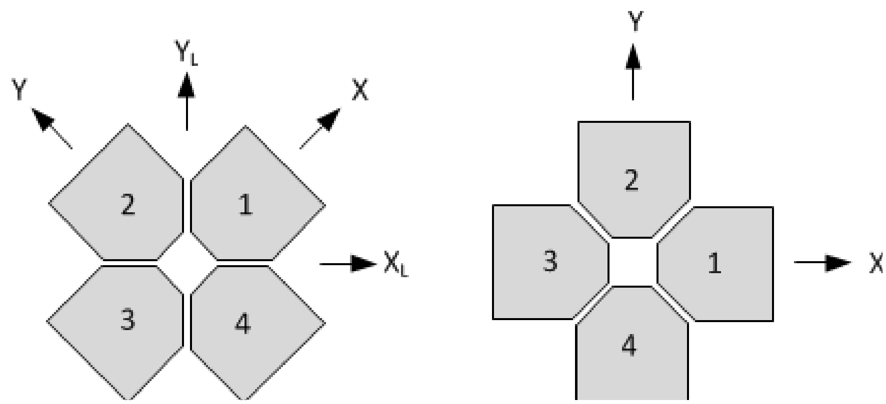

Figure 2: The left side of the figure shows the Delta undulator in its configuration in the tunnel where y_L is up, z is in the beam direction, and x_L makes a right handed system. For our calculations, we use the rotated coordinate system on the right, where x is along the line pointing from quadrant 3 to quadrant 1, y is along the line pointing from quadrant 4 to quadrant 2, and z makes a right handed system.

handed system. For our calculations, it is more convenient to use the rotated system x, y, z, where x is in the direction from quadrant 3 to quadrant 1, y is in the direction from quadrant 4 to quadrant 2, and z is in the beam direction. This is shown on the right side of figure 2. Using equation 3, the scalar potential for each of the quadrants in the x, y, z system is

$$\phi_1(x,y,z) = \phi_{0Q}\cos(k_s y)\exp\left(k_r x\right)\cos\left(k_u\left(z-z_{01}\right)\right) \tag{5}$$
$$\phi_2(x,y,z) = \phi_{0Q}\cos(k_s x)\exp\left(k_r y\right)\cos\left(k_u\left(z-z_{02}\right)\right) \tag{6}$$
$$\phi_3(x,y,z) = -\phi_{0Q}\cos(k_s y)\exp\left(-k_r x\right)\cos\left(k_u\left(z-z_{03}\right)\right) \tag{7}$$
$$\phi_4(x,y,z) = -\phi_{0Q}\cos(k_s x)\exp\left(-k_r y\right)\cos\left(k_u\left(z-z_{04}\right)\right) \tag{8}$$

where z_{0i} is the longitudinal shift of quadrant i, and ϕ_{0Q} is the amplitude of the scalar potential of all the identical quadrants on the axis of the undulator where $x = 0$ and $y = 0$. Quadrants 3 and 4 are loaded with opposite polarity magnets as quadrants 1 and 2 in order to make a vertical field planar undulator in the laboratory frame when all the rows are aligned. This accounts for the minus signs, $-\phi_{0Q}$, in the potentials for quadrants 3 and 4.

The scalar potential for the undulator is the sum of the scalar potentials for the quadrants.

Performing the sum, we find after some calculation

$$\begin{aligned}\phi &= 2\phi_{0Q}\cos(k_s y)\sinh(k_r x)\cos\left(k_u\frac{\Delta_{13}}{2}\right)\cos(k_u(z-Z_{13}))\\ &+2\phi_{0Q}\cos(k_s y)\cosh(k_r x)\sin\left(k_u\frac{\Delta_{13}}{2}\right)\sin(k_u(z-Z_{13}))\\ &+2\phi_{0Q}\cos(k_s x)\sinh(k_r y)\cos\left(k_u\frac{\Delta_{24}}{2}\right)\cos(k_u(z-Z_{24}))\\ &+2\phi_{0Q}\cos(k_s x)\cosh(k_r y)\sin\left(k_u\frac{\Delta_{24}}{2}\right)\sin(k_u(z-Z_{24}))\end{aligned} \tag{9}$$

where

$$Z_{13} = \frac{z_{01}+z_{03}}{2} \tag{10}$$

$$\Delta_{13} = z_{01}-z_{03} \tag{11}$$

$$Z_{24} = \frac{z_{02}+z_{04}}{2} \tag{12}$$

$$\Delta_{24} = z_{02}-z_{04} \tag{13}$$

By putting in the various values for Z_{13}, Z_{24}, Δ_{13}, and Δ_{24}, we get the scalar potential for the various undulator modes at different K values.

The magnetic field in the undulator is given by $B = \nabla\phi$. In order to simplify the formulas further, let

$$B_0 = 2\phi_{0Q}k_u \tag{14}$$

$$B_{xc} = B_0\cos\left(k_u\frac{\Delta_{13}}{2}\right) \tag{15}$$

$$B_{xs} = B_0\sin\left(k_u\frac{\Delta_{13}}{2}\right) \tag{16}$$

$$B_{yc} = B_0\cos\left(k_u\frac{\Delta_{24}}{2}\right) \tag{17}$$

$$B_{ys} = B_0\sin\left(k_u\frac{\Delta_{24}}{2}\right) \tag{18}$$

If we let

$$\delta = k_u(Z_{13}-Z_{24}) \tag{19}$$

we write the fields as a function of position as

$$\begin{aligned}B_x(x,y,z) &= B_{xc}\frac{k_r}{k_u}\cos(k_s y)\cosh(k_r x)\cos(k_u z)\\ &+B_{xs}\frac{k_r}{k_u}\cos(k_s y)\sinh(k_r x)\sin(k_u z)\\ &-B_{yc}\frac{k_s}{k_u}\sin(k_s x)\sinh(k_r y)\cos(k_u z+\delta)\\ &-B_{ys}\frac{k_s}{k_u}\sin(k_s x)\cosh(k_r y)\sin(k_u z+\delta)\end{aligned} \tag{20}$$

$$
\begin{aligned}
B_y(x,y,z) &= -B_{xc}\frac{k_s}{k_u}\sin(k_s y)\sinh(k_r x)\cos(k_u z) \\
&\quad -B_{xs}\frac{k_s}{k_u}\sin(k_s y)\cosh(k_r x)\sin(k_u z) \\
&\quad +B_{yc}\frac{k_r}{k_u}\cos(k_s x)\cosh(k_r y)\cos(k_u z+\delta) \\
&\quad +B_{ys}\frac{k_r}{k_u}\cos(k_s x)\sinh(k_r y)\sin(k_u z+\delta) \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
B_z(x,y,z) &= -B_{xc}\cos(k_s y)\sinh(k_r x)\sin(k_u z) \\
&\quad +B_{xs}\cos(k_s y)\cosh(k_r x)\cos(k_u z) \\
&\quad -B_{yc}\cos(k_s x)\sinh(k_r y)\sin(k_u z+\delta) \\
&\quad +B_{ys}\cos(k_s x)\cosh(k_r y)\cos(k_u z+\delta) \qquad (22)
\end{aligned}
$$

2.2.2 K Value

The K value is determined by how the slippage, the distance between a light wave and the electron beam, changes with position down the undulator. We start with the equation[5]

$$K^2 = 2\frac{\gamma^2}{c^2}\left(\langle v_x^2\rangle + \langle v_y^2\rangle\right) \qquad (23)$$

where the average is taken along z. The transverse velocities are calculated from the transverse accelerations, which to first order are given by the Lorentz force law:

$$\frac{dv_x}{dz} = -\frac{q}{\gamma m}B_y \qquad (24)$$

$$\frac{dv_y}{dz} = \frac{q}{\gamma m}B_x \qquad (25)$$

Let

$$I_x(z) = \int_0^z B_x(z')dz' \qquad (26)$$

$$I_y(z) = \int_0^z B_y(z')dz' \qquad (27)$$

The transverse velocities are

$$v_x = -\frac{q}{\gamma m}I_y \qquad (28)$$

$$v_y = \frac{q}{\gamma m}I_x \qquad (29)$$

Inserting the expressions for the velocities into K^2, we find

$$K^2 = 2\left(\frac{q}{mc}\right)^2\left(\langle I_x^2\rangle + \langle I_y^2\rangle\right) \qquad (30)$$

where the average is taken along z. Performing the squares and averages using the magnetic field given above, we can write an expression for K^2.

$$
\begin{aligned}
K^2 &= \left(a_1^2+a_2^2+a_3^2+a_4^2\right)+\left(b_1^2+b_2^2+b_3^2+b_4^2\right) \\
&\quad +2(a_1a_3+a_2a_4+b_1b_3+b_2b_4)\cos(\delta) \\
&\quad +2(-a_1a_4+c_2c_3-b_1b_4+b_2b_3)\sin(\delta) \qquad (31)
\end{aligned}
$$

[5] Z. Wolf, "Position Dependence Of The K Parameter In The Delta Undulator", LCLS-TN-16-1, January, 2016.

where the a_i and b_i in this expression have values

$$a_1 = K_0 \frac{k_s}{k_u} \cos\left(k_u \frac{\Delta_{13}}{2}\right) \sin(k_s y_0) \sinh(k_r x_0) \tag{32}$$

$$a_2 = -K_0 \frac{k_s}{k_u} \sin\left(k_u \frac{\Delta_{13}}{2}\right) \sin(k_s y_0) \cosh(k_r x_0) \tag{33}$$

$$a_3 = -K_0 \frac{k_r}{k_u} \cos\left(k_u \frac{\Delta_{24}}{2}\right) \cos(k_s x_0) \cosh(k_r y_0) \tag{34}$$

$$a_4 = K_0 \frac{k_r}{k_u} \sin\left(k_u \frac{\Delta_{24}}{2}\right) \cos(k_s x_0) \sinh(k_r y_0) \tag{35}$$

$$b_1 = K_0 \frac{k_r}{k_u} \cos\left(k_u \frac{\Delta_{13}}{2}\right) \cos(k_s y_0) \cosh(k_r x_0) \tag{36}$$

$$b_2 = -K_0 \frac{k_r}{k_u} \sin\left(k_u \frac{\Delta_{13}}{2}\right) \cos(k_s y_0) \sinh(k_r x_0) \tag{37}$$

$$b_3 = -K_0 \frac{k_s}{k_u} \cos\left(k_u \frac{\Delta_{24}}{2}\right) \sin(k_s x_0) \sinh(k_r y_0) \tag{38}$$

$$b_4 = K_0 \frac{k_s}{k_u} \sin\left(k_u \frac{\Delta_{24}}{2}\right) \sin(k_s x_0) \cosh(k_r y_0) \tag{39}$$

where

$$K_0 = \frac{qB_0}{mk_u c} \tag{40}$$

and (x_0, y_0) is the transverse position where the beam enters the undulator. With these relations, we know K as a function of row settings for all polarization modes of the undulator.

2.2.3 K Value Near The Magnetic Center

Typically the undulator is aligned so that the beam is close to the magnetic center. In this case we expand the a and b parameters of equations 32 to 39 to second order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$. We get the following results.

$$a_1 = K_0 \frac{k_s^2 k_r}{k_u} x_0 y_0 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \tag{41}$$

$$a_2 = -K_0 \frac{k_s^2}{k_u} y_0 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \tag{42}$$

$$a_3 = -K_0 \frac{k_r}{k_u} \left(1 - \frac{1}{2} k_s^2 x_0^2 + \frac{1}{2} k_r^2 y_0^2\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \tag{43}$$

$$a_4 = K_0 \frac{k_r^2}{k_u} y_0 \sin\left(k_u \frac{\Delta_{24}}{2}\right) \tag{44}$$

$$b_1 = K_0 \frac{k_r}{k_u} \left(1 - \frac{1}{2} k_s^2 y_0^2 + \frac{1}{2} k_r^2 x_0^2\right) \cos\left(k_u \frac{\Delta_{13}}{2}\right) \tag{45}$$

$$b_2 = -K_0 \frac{k_r^2}{k_u} x_0 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \tag{46}$$

$$b_3 = -K_0 \frac{k_s^2 k_r}{k_u} x_0 y_0 \cos\left(k_u \frac{\Delta_{24}}{2}\right) \tag{47}$$

$$b_4 = K_0 \frac{k_s^2}{k_u} x_0 \sin\left(k_u \frac{\Delta_{24}}{2}\right) \tag{48}$$

We can now find K^2 to second order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$. Equation 31 gives

$$\begin{aligned} K^2 = \frac{K_0^2}{k_u^2} \{ & \\ & k_r^2 \left[\cos^2\left(k_u \frac{\Delta_{13}}{2}\right) + \cos^2\left(k_u \frac{\Delta_{24}}{2}\right)\right] \\ & + 2 k_s^2 k_r \left[\begin{array}{c} y_0 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \\ -x_0 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \end{array} \right] \sin(\delta) \\ & + \left(k_s^4 y_0^2 + k_r^4 x_0^2\right) \sin^2\left(k_u \frac{\Delta_{13}}{2}\right) + k_r^2 \left(k_r^2 x_0^2 + k_s^2 y_0^2\right) \cos^2\left(k_u \frac{\Delta_{13}}{2}\right) \\ & + \left(k_s^4 x_0^2 + k_r^4 y_0^2\right) \sin^2\left(k_u \frac{\Delta_{24}}{2}\right) + k_r^2 \left(k_s^2 x_0^2 + k_r^2 y_0^2\right) \cos^2\left(k_u \frac{\Delta_{24}}{2}\right) \\ & - 2 k_s^2 k_r^2 \left[\begin{array}{c} 2 x_0 y_0 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \\ + \left(x_0^2 + y_0^2\right) \sin\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \end{array} \right] \cos(\delta) \} \end{aligned} \tag{49}$$

This result gives the form of the function used to fit K^2 measured off the magnetic axis. It is the first fit to the data that we perform.

2.3 Fits To The Measured K Values

The Delta undulator was extensively measured from September to October, 2015. In this data set, 260 measurements were made in various modes and at different K values; 88 measurements were made in the primary linear and circular modes at different K values uniformly spaced, and in modes where one pair of quadrants was "turned off" and the other pair was set to different K values; and 80 measurements were made in the four primary linear and circular modes at a dense set of K values near the desired operating point. The 428 measurements listed above are initially included in the fits. Afterward, some cuts are made to fit only the parameter range where the undulator is used.

The fits are done as follows. Let the point p_i represent the parameters used to set the Delta for measurement number i.

$$p_i = (\Delta_{13}, \Delta_{24}, \delta)_i \tag{50}$$

The fit is of the form

$$K_i^2 = c_1 f_1(p_i) + c_2 f_2(p_i) + \ldots + c_n f_n(p_i) \tag{51}$$

where the fit parameters c_j are chosen to minimize the squares of the residuals. There is one equation for each measurement so i ranges from 1 to 428 before the cuts are applied. A matrix equation for the measurements is formed as follows:

$$\begin{bmatrix} f_1(p_1) & f_2(p_1) & \ldots & f_n(p_1) \\ f_1(p_2) & f_2(p_2) & \ldots & f_n(p_2) \\ \ldots & \ldots & \ldots & \ldots \\ f_1(p_m) & f_2(p_m) & \ldots & f_n(p_m) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \ldots \\ c_n \end{bmatrix} = \begin{bmatrix} K_1^2 \\ K_2^2 \\ \ldots \\ K_m^2 \end{bmatrix} \tag{52}$$

This can be written in shorthand as

$$F * C = M \tag{53}$$

where F is the matrix of fit function values, C is an array of fit coefficients, and M is an array of measured values. This equation is solved for the fit coefficients in Matlab in a least squares sense using the "\" operator.

$$C = F \backslash M \tag{54}$$

Once the fit coefficients are know, the fitted values for each measurement are given by

$$M_{fit} = F * C \tag{55}$$

The residuals are given by

$$r' = M - M_{fit} \tag{56}$$

Since we are fitting for K^2 and we want the residuals for K, we have

$$r = \sqrt{M} - \sqrt{M_{fit}} \tag{57}$$

This procedure will now be used to fit the measured K values.

3 Fit For An Ideal Undulator Measured Off The Magnetic Axis

Formula 49 for K^2 has the form

$$\begin{aligned} K^2 \quad = \quad & c_1 \cos^2\left(k_u \frac{\Delta_{13}}{2}\right) + c_2 \cos^2\left(k_u \frac{\Delta_{24}}{2}\right) \\ & + d_1 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \sin(\delta) + d_2 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \sin(\delta) \\ & + e_1' \sin^2\left(k_u \frac{\Delta_{13}}{2}\right) + e_2' \sin^2\left(k_u \frac{\Delta_{24}}{2}\right) \\ & + e_3 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \cos(\delta) + e_4 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \cos(\delta) \end{aligned} \tag{58}$$

Note, however, that $\sin^2\left(k_u \frac{\Delta_{13}}{2}\right)$ is equal to a constant plus $\cos^2\left(k_u \frac{\Delta_{13}}{2}\right)$. Similarly for the $\sin^2\left(k_u \frac{\Delta_{24}}{2}\right)$ term. In order for the fit to be well defined, we replace these terms with a constant in the fitting function. The $\cos^2\left(k_u \frac{\Delta_{13}}{2}\right)$ and $\cos^2\left(k_u \frac{\Delta_{24}}{2}\right)$ terms are already included.

$$\begin{aligned} K^2 \quad = \quad & c_1 \cos^2\left(k_u \frac{\Delta_{13}}{2}\right) + c_2 \cos^2\left(k_u \frac{\Delta_{24}}{2}\right) \\ & d_1 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \sin(\delta) + d_2 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \sin(\delta) \\ & + e_1 \\ & + e_2 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \cos(\delta) + e_3 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \cos(\delta) \end{aligned} \tag{59}$$

We know the row settings for the quadrants, so we know Δ_{13}, Δ_{24}, δ, and we also know k_u from the undulator periodicity. We measure the K value so we know K^2. We can thus apply the fit procedure given in the previous section in order to determine the fit coefficients.

The coefficients c_1 and c_2 are zeroth order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$. We will start with a fit using only these largest terms. The coefficients d_1 and d_2 are first order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and

$k_s y_0$. We will next do a fit including these smaller terms. The coefficients e_1, e_2, and e_3 are second order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$. We will then do a fit including these small terms.

In order to estimate the size of the terms in the various order of fits, assume k_r, k_s, and k_u are all on the order of 200 1/m, and that x_0 and y_0 are on the order of 200 μm. In this case, the d_i coefficients are roughly $(200\ 1/\mathrm{m})(2\times10^{-4}\ \mathrm{m}) = 0.04$ times smaller than the c_i coefficients, and the e_i coefficients are roughly $(0.04)^2$ times smaller than the c_i coefficients. We expect the c_i coefficients to dominate the fit to K^2 at large K, however, they go to zero when the undulator is "turned off", that is when $k_u \frac{\Delta_{13}}{2} = \frac{\pi}{2}$ or $k_u \frac{\Delta_{24}}{2} = \frac{\pi}{2}$. For the small K region of the fit, we need to keep the second order terms which do not go to zero. The d_i coefficients will make $0.04 * K^2_{\max}$ changes to K^2, which corresponds to $0.02 * K_{\max}$ changes to K, or contributions of approximately 0.06 to K. The e_i coefficients will make approximately $0.0016 * K^2_{\max}$ changes to K^2, or $.0008 * K_{\max}$ changes to K, or contributions of approximately 0.0024 to K.

3.1 Zeroth Order Fit

We start by assuming a fitting function using the expression for K^2 to zeroth order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$. The form of the fitting function is

$$K^2 = c_1 \cos^2\left(k_u \frac{\Delta_{13}}{2}\right) + c_2 \cos^2\left(k_u \frac{\Delta_{24}}{2}\right) \tag{60}$$

The residuals of the fit, calculated as

$$r = K_{meas} - \sqrt{K^2_{fit}} \tag{61}$$

are shown in figure 3 plotted against the measured K value. The fit coefficients are

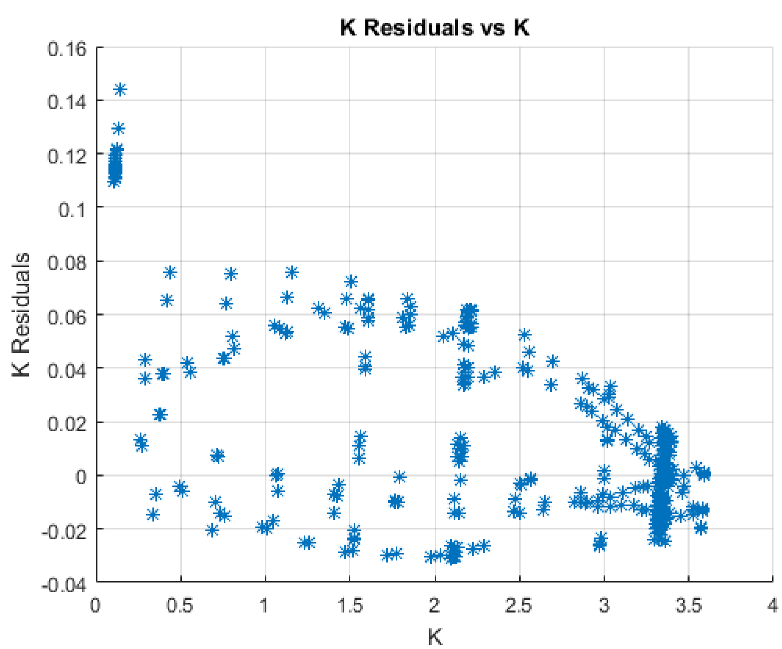

Figure 3: K residuals of the zero'th order fit to K^2.

$$c_1 = 6.270 \quad (62)$$
$$c_2 = 6.614 \quad (63)$$

The sub-undulator made of quadrants 2 and 4 is stronger than the sub-undulator made of quadrants 1 and 3 since $c_2 > c_1$.

When we look at the residuals as a function of Δ_{13}, we get the plot shown in figure 4. The

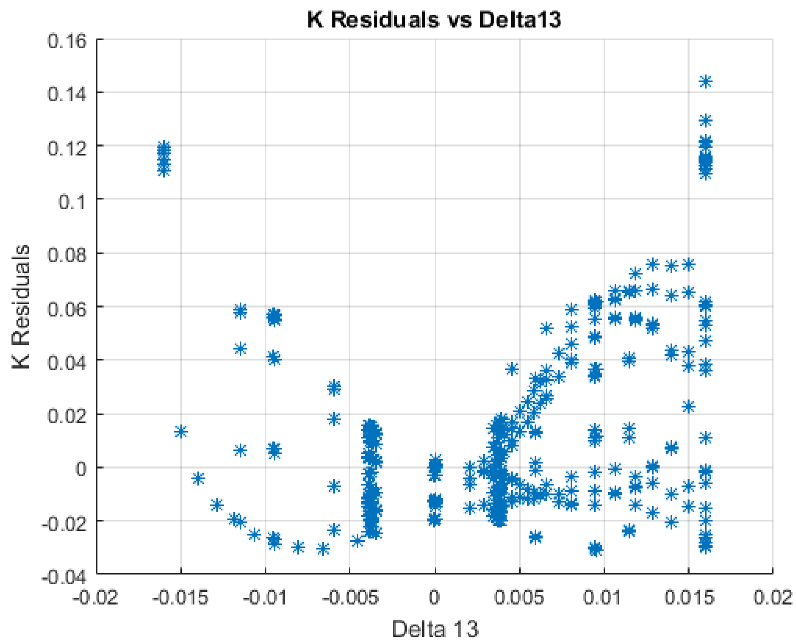

Figure 4: K residuals as a function of Δ_{13}.

plot of the residuals as a function of Δ_{24} looks similar. When the undulator is "turned off", when $\Delta_{13} = \pm.016$ m or $\Delta_{24} = \pm.016$ m, the residuals get very large. The K value does not go to zero as expected. This behavior is not understood at present. However, since the K value does not need to be accurately set to these low values in operation, we will exclude points with $|\Delta_{13}| > .010$ m or $|\Delta_{24}| > .010$ m from our fits. Since the Delta undulator is not used in this regime, we do not lose any functionality with this cut. The new fit with these points excluded is shown in figure 5.

In this plot we have introduced a quantity R which is meant to help quantify improvements to the fit when we add more terms. We define R as

$$R = \sqrt{\frac{1}{N}\sum_{i=1}^{N} r_i^2} \quad (64)$$

N is the number of measurements used in the fit. R is the square root of the mean of the square of the residuals. The average values of the residuals is not subtracted from each residual.

With the cut, the fit gives

$$c_1 = 6.765 \quad (65)$$
$$c_2 = 6.110 \quad (66)$$

Note that now $c_2 < c_1$. The cut has changed the relative magnitudes of c_1 and c_2, so we lose our interpretation that the 2-4 undulator is stronger than the 1-3 undulator. The residuals are too

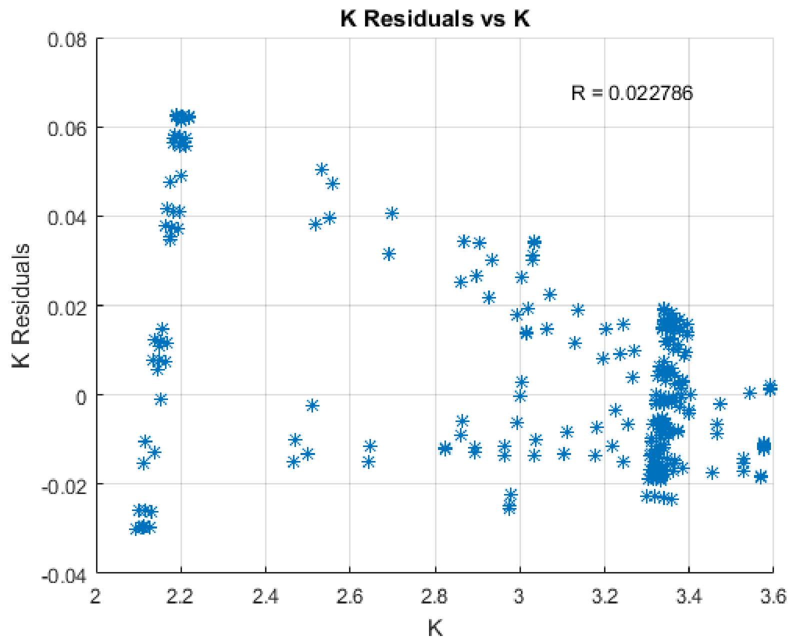

Figure 5: K residuals for the zero'th order fit with the points with $|\Delta_{13}| > .01$ m or $|\Delta_{24}| > .01$ m excluded.

large to set the undulator to be resonant with the rest of the LCLS undulators, so further work is required.

3.2 First Order Fit

We now use equation 59, including the first order terms, as the form of the fitting function. The fitting function is

$$\begin{aligned} K^2 &= c_1 \cos^2\left(k_u \frac{\Delta_{13}}{2}\right) + c_2 \cos^2\left(k_u \frac{\Delta_{24}}{2}\right) \\ &+ d_1 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \sin(\delta) + d_2 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \sin(\delta) \end{aligned} \tag{67}$$

The coefficients c_1 and c_2 are zeroth order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$ and the coefficients d_1 and d_2 are first order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$. When we use this form of the fitting function and exclude points with $|\Delta_{13}| > .01$ m or $|\Delta_{24}| > .01$ m, we get the residuals shown in figure 6. The fit coefficients are

$$c_1 = 6.765 \tag{68}$$
$$c_2 = 6.110 \tag{69}$$
$$d_1 = -0.194 \tag{70}$$
$$d_2 = -0.073 \tag{71}$$

Surprisingly, the first order terms have a small effect on the residuals of the fit.

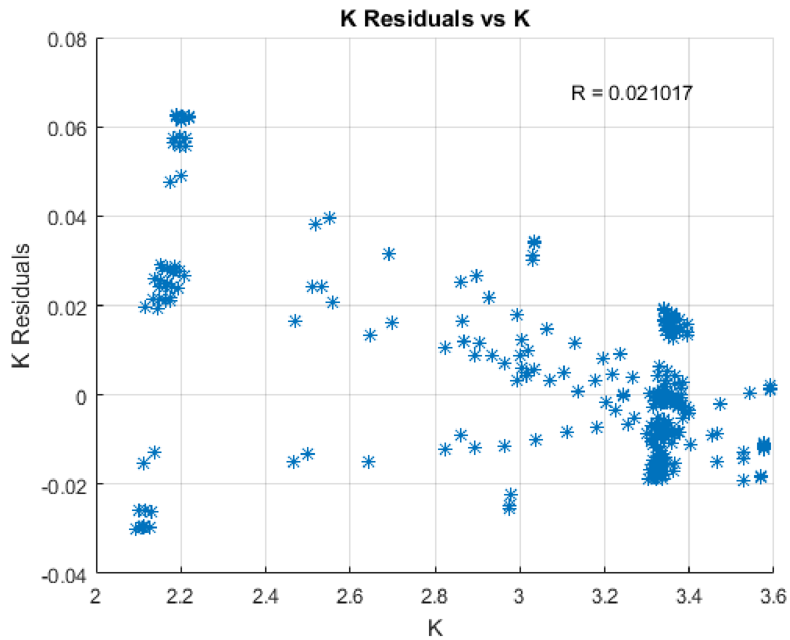

Figure 6: Residuals of the first order fit.

3.3 Second Order Fit

We now use equation 59 including the second order terms as the form of the fitting function. The fitting function is

$$\begin{aligned} K^2 &= c_1 \cos^2\left(k_u \frac{\Delta_{13}}{2}\right) + c_2 \cos^2\left(k_u \frac{\Delta_{24}}{2}\right) \\ & d_1 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \sin(\delta) + d_2 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \sin(\delta) \\ & +e_1 \\ & +e_2 \cos\left(k_u \frac{\Delta_{13}}{2}\right) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \cos(\delta) + e_3 \sin\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \cos(\delta) \end{aligned} \tag{72}$$

The coefficients c_1 and c_2 are zeroth order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$. The coefficients d_1 and d_2 are first order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$. The coefficients e_1, e_2, and e_3 are second order in $k_r x_0$, $k_s x_0$, $k_r y_0$ and $k_s y_0$. When we use this form of the fitting function and exclude points with $|\Delta_{13}| > .010$ m or $|\Delta_{24}| > .010$ m, we get the residuals shown in figure 7. The fit coefficients are

$$c_1 = 6.503 \tag{73}$$
$$c_2 = 6.159 \tag{74}$$
$$d_1 = -0.194 \tag{75}$$
$$d_2 = -0.073 \tag{76}$$
$$e_1 = 0.169 \tag{77}$$
$$e_2 = -0.056 \tag{78}$$
$$e_3 = 0.298 \tag{79}$$

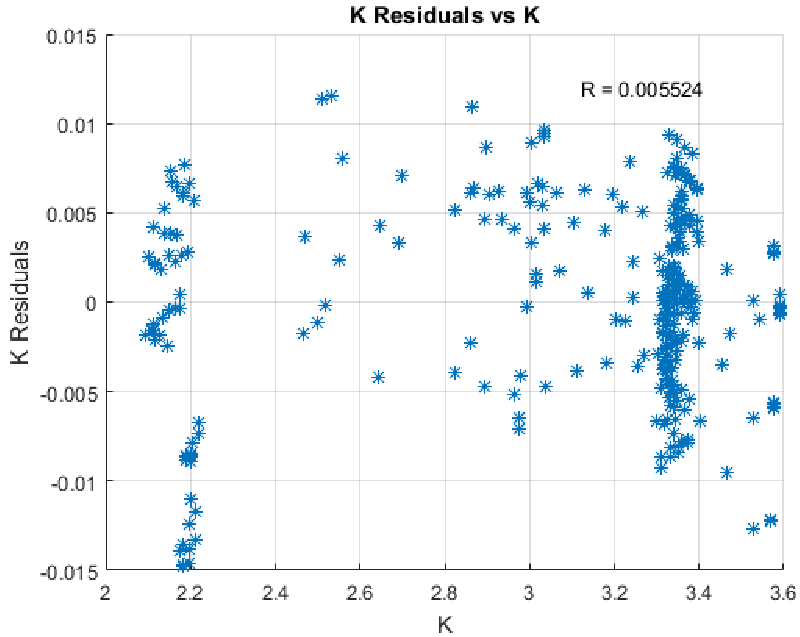

Figure 7: Residuals from the second order fit to K^2.

The fit is greatly improved. It is adequate for setting the K value of the Delta undulator.

If the Delta undulator is used over a smaller range in K, the fit is improved. When we use this form of the fitting function and exclude points with $|\Delta_{13}| > .008$ m or $|\Delta_{24}| > .008$ m, we get the residuals shown in figure 8. The fit coefficients are

$$c_1 = 6.180 \quad (80)$$
$$c_2 = 6.365 \quad (81)$$
$$d_1 = -0.189 \quad (82)$$
$$d_2 = -0.073 \quad (83)$$
$$e_1 = 0.272 \quad (84)$$
$$e_2 = -0.056 \quad (85)$$
$$e_3 = 0.441 \quad (86)$$

This fit should work well for $K > 2.6$ with rms errors less than $\Delta K < .004$, or $\frac{\Delta K}{K} < .002$.

3.4 Discussion

The ideal undulator measured off the magnetic axis provides a form of the fitting function that performs fairly well. It meets the requirements for setting the Delta undulator. The interpretation of the fitting coefficients is problematic, however. The added terms in the fitting function are known to be small, and so they should have a small effect on the fit. The $\sin\left(k_u\frac{\Delta_{13}}{2}\right)\sin\left(k_u\frac{\Delta_{24}}{2}\right)\cos(\delta)$ is very important to the fit, yet its coefficient is expected to be second order and small. Also the relative strengths of the 1-3 and 2-4 undulators change depending on the cuts applied to the data. We expect that other error terms contribute to the fit with functional forms similar to those corresponding to the e_i terms. We continue to look for a fitting function whose form tells us about the inner workings of the Delta undulator.

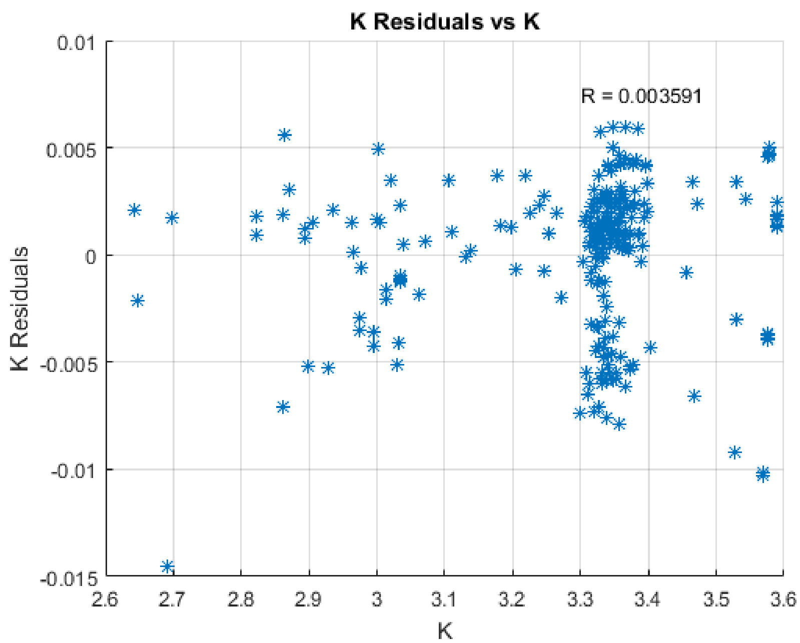

Figure 8: Residuals from the second order fit limited to a smaller range in K.

4 Fit For An Undulator With Errors

4.1 Quadrant Strength And Position Errors

In this section we consider an undulator where each quadrant has a different strength and each quadrant has a position error. Radial position errors give strength errors and are included in the strength errors we consider. Angular position errors of the quadrants give new terms different than the strength error terms. We begin by taking the quadrant strengths to be arbitrary, and we assume the angular position errors are small. We work to first order in the angle errors. We consider only fields on the undulator axis, since the previous section showed that being off the undulator axis does not have a large first order effect. The position errors are illustrated in figure 9.

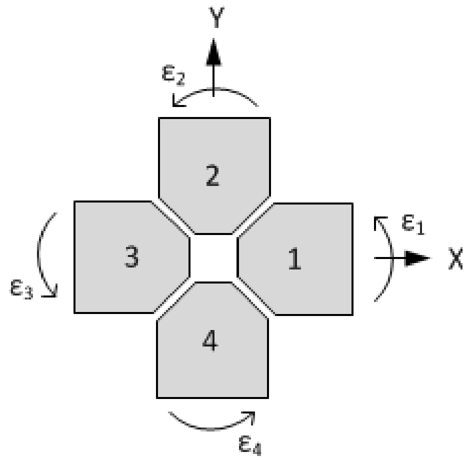

Figure 9: Undulator with different magnetic strengths for each quadrant and different angular position errors for each quadrant.

The fields on the undulator axis from each quadrant are given by

$$\overrightarrow{B_1} = \widehat{x}B_{01}\cos(k_u(z-z_{01})) + \widehat{y}\epsilon_1 B_{01}\cos(k_u(z-z_{01})) \quad (87)$$

$$\overrightarrow{B_3} = \widehat{x}B_{03}\cos(k_u(z-z_{03})) + \widehat{y}\epsilon_3 B_{03}\cos(k_u(z-z_{03})) \quad (88)$$

$$\overrightarrow{B_2} = \widehat{y}B_{02}\cos(k_u(z-z_{02})) - \widehat{x}\epsilon_2 B_{02}\cos(k_u(z-z_{02})) \quad (89)$$

$$\overrightarrow{B_4} = \widehat{y}B_{04}\cos(k_u(z-z_{04})) - \widehat{x}\epsilon_4 B_{04}\cos(k_u(z-z_{04})) \quad (90)$$

The horizontal and vertical fields are given by the sum of the fields from the quadrants.

$$\begin{aligned} B_x &= B_{01}\cos(k_u(z-z_{01})) + B_{03}\cos(k_u(z-z_{03})) \\ &\quad -\epsilon_2 B_{02}\cos(k_u(z-z_{02})) - \epsilon_4 B_{04}\cos(k_u(z-z_{04})) \quad (91) \end{aligned}$$

$$\begin{aligned} B_y &= B_{02}\cos(k_u(z-z_{02})) + B_{04}\cos(k_u(z-z_{04})) \\ &\quad +\epsilon_1 B_{01}\cos(k_u(z-z_{01})) + \epsilon_3 B_{03}\cos(k_u(z-z_{03})) \quad (92) \end{aligned}$$

The integrals of the fields are given by

$$\begin{aligned} I_x &= B_{01}\frac{1}{k_u}\sin(k_u(z-z_{01})) + B_{03}\frac{1}{k_u}\sin(k_u(z-z_{03})) \\ &\quad -\epsilon_2 B_{02}\frac{1}{k_u}\sin(k_u(z-z_{02})) - \epsilon_4 B_{04}\frac{1}{k_u}\sin(k_u(z-z_{04})) \end{aligned} \tag{93}$$

$$\begin{aligned} I_y &= B_{02}\frac{1}{k_u}\sin(k_u(z-z_{02})) + B_{04}\frac{1}{k_u}\sin(k_u(z-z_{04})) \\ &\quad +\epsilon_1 B_{01}\frac{1}{k_u}\sin(k_u(z-z_{01})) + \epsilon_3 B_{03}\frac{1}{k_u}\sin(k_u(z-z_{03})) \end{aligned} \tag{94}$$

The K value is proportional to

$$K^2 \sim \langle I_x^2 \rangle + \langle I_y^2 \rangle \tag{95}$$

where the average is taken along z. Performing the squares and averages, we have

$$\begin{aligned} \langle I_x^2 \rangle &= \frac{1}{k_u^2}\{\frac{1}{2}B_{01}^2 + \frac{1}{2}B_{03}^2 + B_{01}B_{03}\cos(k_u(z_{01}-z_{03})) \\ &\quad -\epsilon_2 B_{01}B_{02}\cos(k_u(z_{01}-z_{02})) - \epsilon_4 B_{01}B_{04}\cos(k_u(z_{01}-z_{04})) \\ &\quad -\epsilon_2 B_{03}B_{02}\cos(k_u(z_{03}-z_{02})) - \epsilon_4 B_{03}B_{04}\cos(k_u(z_{03}-z_{04})) \\ &\quad +\frac{1}{2}\epsilon_2^2 B_{02}^2 + \frac{1}{2}\epsilon_4^2 B_{04}^2 + \epsilon_2\epsilon_4 B_{02}B_{04}\cos(k_u(z_{02}-z_{04}))\} \end{aligned} \tag{96}$$

$$\begin{aligned} \langle I_y^2 \rangle &= \frac{1}{k_u^2}\{\frac{1}{2}B_{02}^2 + \frac{1}{2}B_{04}^2 + B_{02}B_{04}\cos(k_u(z_{02}-z_{04})) \\ &\quad +\epsilon_1 B_{02}B_{01}\cos(k_u(z_{02}-z_{01})) + \epsilon_3 B_{02}B_{03}\cos(k_u(z_{02}-z_{03})) \\ &\quad +\epsilon_1 B_{04}B_{01}\cos(k_u(z_{04}-z_{01})) + \epsilon_3 B_{04}B_{03}\cos(k_u(z_{04}-z_{03})) \\ &\quad +\frac{1}{2}\epsilon_1^2 B_{01}^2 + \frac{1}{2}\epsilon_3^2 B_{03}^2 + \epsilon_1\epsilon_3 B_{01}B_{03}\cos(k_u(z_{01}-z_{03}))\} \end{aligned} \tag{97}$$

Inserting these expressions in relation 95, we get

$$\begin{aligned} K^2 &\sim \frac{1}{k_u^2}\{\frac{1}{2}B_{01}^2 + \frac{1}{2}B_{03}^2 + B_{01}B_{03}\cos(k_u(z_{01}-z_{03})) \\ &\quad +\frac{1}{2}B_{02}^2 + \frac{1}{2}B_{04}^2 + B_{02}B_{04}\cos(k_u(z_{02}-z_{04})) \\ &\quad -\epsilon_2 B_{01}B_{02}\cos(k_u(z_{01}-z_{02})) - \epsilon_4 B_{01}B_{04}\cos(k_u(z_{01}-z_{04})) \\ &\quad -\epsilon_2 B_{03}B_{02}\cos(k_u(z_{03}-z_{02})) - \epsilon_4 B_{03}B_{04}\cos(k_u(z_{03}-z_{04})) \\ &\quad +\epsilon_1 B_{02}B_{01}\cos(k_u(z_{02}-z_{01})) + \epsilon_3 B_{02}B_{03}\cos(k_u(z_{02}-z_{03})) \\ &\quad +\epsilon_1 B_{04}B_{01}\cos(k_u(z_{04}-z_{01})) + \epsilon_3 B_{04}B_{03}\cos(k_u(z_{04}-z_{03})) \\ &\quad +\frac{1}{2}\epsilon_2^2 B_{02}^2 + \frac{1}{2}\epsilon_4^2 B_{04}^2 + \epsilon_2\epsilon_4 B_{02}B_{04}\cos(k_u(z_{02}-z_{04})) \\ &\quad +\frac{1}{2}\epsilon_1^2 B_{01}^2 + \frac{1}{2}\epsilon_3^2 B_{03}^2 + \epsilon_1\epsilon_3 B_{01}B_{03}\cos(k_u(z_{01}-z_{03}))\} \end{aligned} \tag{98}$$

Simplifying, we have

$$\begin{aligned} K^2 &\sim \frac{1}{k_u^2}\{\frac{1}{2}B_{01}^2 + \frac{1}{2}B_{03}^2 + \frac{1}{2}B_{02}^2 + \frac{1}{2}B_{04}^2 \\ &\quad +\frac{1}{2}\epsilon_1^2 B_{01}^2 + \frac{1}{2}\epsilon_3^2 B_{03}^2 + \frac{1}{2}\epsilon_2^2 B_{02}^2 + \frac{1}{2}\epsilon_4^2 B_{04}^2 \\ &\quad +(1+\epsilon_1\epsilon_3)B_{01}B_{03}\cos(k_u(z_{01}-z_{03})) + (1+\epsilon_2\epsilon_4)B_{02}B_{04}\cos(k_u(z_{02}-z_{04})) \\ &\quad +(\epsilon_1-\epsilon_2)B_{01}B_{02}\cos(k_u(z_{01}-z_{02})) + (\epsilon_1-\epsilon_4)B_{01}B_{04}\cos(k_u(z_{01}-z_{04})) \\ &\quad +(\epsilon_3-\epsilon_2)B_{03}B_{02}\cos(k_u(z_{03}-z_{02})) + (\epsilon_3-\epsilon_4)B_{03}B_{04}\cos(k_u(z_{03}-z_{04}))\} \end{aligned} \tag{99}$$

Keeping only the largest term for each function involving Δ_{13}, Δ_{24}, or δ, we get

$$\begin{aligned} K^2 \sim\ & \frac{1}{k_u^2}\{\frac{1}{2}B_{01}^2 + \frac{1}{2}B_{03}^2 + \frac{1}{2}B_{02}^2 + \frac{1}{2}B_{04}^2 \\ & + B_{01}B_{03}\cos(k_u(z_{01}-z_{03})) + B_{02}B_{04}\cos(k_u(z_{02}-z_{04})) \\ & + (\epsilon_1-\epsilon_2)B_{01}B_{02}\cos(k_u(z_{01}-z_{02})) + (\epsilon_1-\epsilon_4)B_{01}B_{04}\cos(k_u(z_{01}-z_{04})) \\ & + (\epsilon_3-\epsilon_2)B_{03}B_{02}\cos(k_u(z_{03}-z_{02})) + (\epsilon_3-\epsilon_4)B_{03}B_{04}\cos(k_u(z_{03}-z_{04}))\} \end{aligned} \tag{100}$$

The functional form of K^2 is

$$\begin{aligned} K^2 =\ & c_1 + c_2\cos(k_u(z_{01}-z_{03})) + c_3\cos(k_u(z_{02}-z_{04})) \\ & + d_1\cos(k_u(z_{01}-z_{02})) + d_2\cos(k_u(z_{01}-z_{04})) \\ & + d_3\cos(k_u(z_{03}-z_{02})) + d_4\cos(k_u(z_{03}-z_{04})) \end{aligned} \tag{101}$$

The difference between c_2 and c_3 gives information about the strength differences of the quadrants. The d_i give information about the angle errors of the quadrants. Up to a common factor of $2\left(\frac{q}{mk_uc}\right)^2$, the resulting fit coefficients are

$$c_1 = \frac{1}{2}B_{01}^2 + \frac{1}{2}B_{03}^2 + \frac{1}{2}B_{02}^2 + \frac{1}{2}B_{04}^2 \tag{102}$$
$$c_2 = B_{01}B_{03} \tag{103}$$
$$c_3 = B_{02}B_{04} \tag{104}$$
$$d_1 = (\epsilon_1-\epsilon_2)B_{01}B_{02} \tag{105}$$
$$d_2 = (\epsilon_1-\epsilon_4)B_{01}B_{04} \tag{106}$$
$$d_3 = (\epsilon_3-\epsilon_2)B_{03}B_{02} \tag{107}$$
$$d_4 = (\epsilon_3-\epsilon_4)B_{03}B_{04} \tag{108}$$

With this functional form, the residuals of the fit are as shown in figure 10. The fit coefficients are

$$c_1 = 6.500 \tag{109}$$
$$c_2 = 3.252 \tag{110}$$
$$c_3 = 3.080 \tag{111}$$
$$d_1 = 0.091 \tag{112}$$
$$d_2 = -0.022 \tag{113}$$
$$d_3 = -0.155 \tag{114}$$
$$d_4 = 0.030 \tag{115}$$

We can get information about the undulator errors from the fit coefficients. We make the following approximations, again ignoring the factor of $2\left(\frac{q}{mk_uc}\right)^2$ common to all the coefficients.

$$c_1 = 2B_0^2 \tag{116}$$
$$c_2 = B_0^2 + B_0(\delta B_{01} + \delta B_{03}) \tag{117}$$
$$c_3 = B_0^2 + B_0(\delta B_{02} + \delta B_{04}) \tag{118}$$
$$d_1 = (\epsilon_1-\epsilon_2)B_0^2 \tag{119}$$
$$d_2 = (\epsilon_1-\epsilon_4)B_0^2 \tag{120}$$
$$d_3 = (\epsilon_3-\epsilon_2)B_0^2 \tag{121}$$
$$d_4 = (\epsilon_3-\epsilon_4)B_0^2 \tag{122}$$

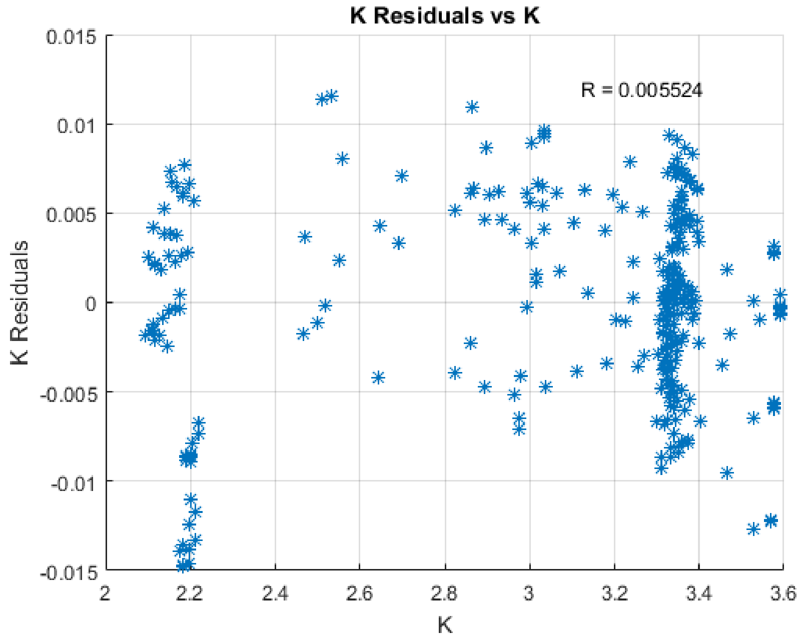

Figure 10: Residuals with a functional form including both strength errors and angular positioning errors of the quadrants.

where B_0 is an average quadrant strength, and δB_{0i} is the difference in strength between quadrant i and the average: $B_{0i} = B_0 + \delta B_{0i}$.

The relative strength of the 1-3 undulator compared to the 2-4 undulator is

$$\frac{(B_{01}+B_{03})-(B_{02}+B_{04})}{B_0} = \frac{(\delta B_{01}+\delta B_{03})-(\delta B_{02}+\delta B_{04})}{B_0} \quad (123)$$

$$= \frac{c_2-c_3}{c_1}2 \quad (124)$$

The angle errors of each quadrant relative to quadrant 1 are

$$\epsilon_2-\epsilon_1 \simeq \frac{-d_1}{c_1}2 \quad (125)$$

$$\epsilon_3-\epsilon_1 \simeq \frac{d_3-d_1}{c_1}2 \quad (126)$$

$$\epsilon_4-\epsilon_1 \simeq \frac{-d_2}{c_1}2 \quad (127)$$

We can calculate the errors from the fit coefficients.

$$\frac{(B_{01}+B_{03})-(B_{02}+B_{04})}{B_0} = 0.053 \quad (128)$$

$$\epsilon_2-\epsilon_1 = -0.028 \quad (129)$$

$$\epsilon_3-\epsilon_1 = -0.076 \quad (130)$$

$$\epsilon_4-\epsilon_1 = 0.007 \quad (131)$$

The fit coefficients say that the 1-3 undulator is 5.3% stronger than the 2-4 undulator, although we know that this depends on the applied cuts. The difference in the quadrant angles is up to 0.076

radians. This corresponds to 300 μm quadrant position errors at a radius of 4 mm. These angle errors seem reasonable.

The rms of the residuals is the same for this fit with undulator errors as for the case of an ideal undulator measured off the magnetic axis. In the appendix we show that both the functional form of the undulator with errors fit and the functional form of the ideal undulator fit are equivalent.

4.2 Longitudinal Row Position Errors

From equation 101, the fit function for K^2 with no transverse quadrant position errors is

$$K^2 = c_1 + c_2 \cos(k_u(z_{01} - z_{03})) + c_3 \cos(k_u(z_{02} - z_{04})) \tag{132}$$

or

$$K^2 = c_1 + c_2 \cos(k_u \Delta_{13}) + c_3 \cos(k_u \Delta_{24})$$

If the row positions have errors, then

$$\Delta_{13} \rightarrow \Delta_{13} + \epsilon_{13} \tag{133}$$
$$\Delta_{24} \rightarrow \Delta_{24} + \epsilon_{24} \tag{134}$$

where ϵ_{13} is the error in the row shift between quadrants 1 and 3, and ϵ_{24} is the error in the row shift between quadrants 2 and 4. We insert these errors in the fit function for K^2. The cosine functions change as follows.

$$\begin{aligned} \cos\left(k_u\left(\Delta_{13}+\epsilon_{13}\right)\right) &= \cos\left(k_u\Delta_{13}\right)\cos\left(k_u\epsilon_{13}\right) - \sin\left(k_u\Delta_{13}\right)\sin\left(k_u\epsilon_{13}\right) \\ &\simeq \cos\left(k_u\Delta_{13}\right) - \sin\left(k_u\Delta_{13}\right) k_u\epsilon_{13} \end{aligned} \tag{135}$$

Similarly

$$\begin{aligned} \cos\left(k_u\left(\Delta_{24}+\epsilon_{24}\right)\right) &= \cos\left(k_u\Delta_{24}\right)\cos\left(k_u\epsilon_{24}\right) - \sin\left(k_u\Delta_{24}\right)\sin\left(k_u\epsilon_{24}\right) \\ &\simeq \cos\left(k_u\Delta_{24}\right) - \sin\left(k_u\Delta_{24}\right) k_u\epsilon_{24} \end{aligned} \tag{136}$$

The form of the fitting function including this error to first order is

$$\begin{aligned} K^2 &= c_1 + c_2\cos(k_u\Delta_{13}) + c_3\cos(k_u\Delta_{24}) \\ &\quad + d_5\sin\left(k_u\Delta_{13}\right) + d_6\sin\left(k_u\Delta_{24}\right) \end{aligned} \tag{137}$$

When we add these error terms to equation 101, the fit function for K^2 including quadrant strength and transverse position errors, we get

$$\begin{aligned} K^2 &= c_1 + c_2\cos(k_u(z_{01}-z_{03})) + c_3\cos(k_u(z_{02}-z_{04})) \\ &\quad + d_1\cos(k_u(z_{01}-z_{02})) + d_2\cos(k_u(z_{01}-z_{04})) \\ &\quad + d_3\cos(k_u(z_{03}-z_{02})) + d_4\cos(k_u(z_{03}-z_{04})) \\ &\quad + d_5\sin\left(k_u(z_{01}-z_{03})\right) + d_6\sin\left(k_u(z_{02}-z_{04})\right) \end{aligned} \tag{138}$$

The residuals to the fit are shown in figure 11. The residuals are slightly improved, but not significantly.

4.3 Errors From The Magnet Sort

The quadrants have error fields which may have a periodic pattern from the method of magnet sorting. The magnets were sorted in an effort to make each quadrant produce straight trajectories

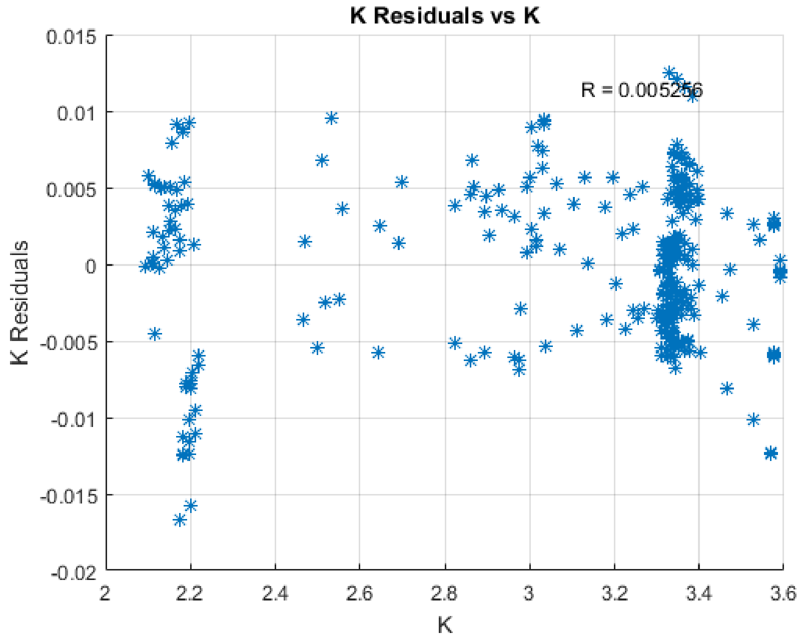

Figure 11: Residuals with quadrant strength, transverse position, and longitudinal position errors.

with no phase errors[6]. Each vertically magnetized magnet was paired in a period with an equal strength vertically magnetized magnet of the opposite polarity, so the periods did not steer the beam. The horizontal error fields were also paired so that there was no net steering. This means that the horizontal error fields could have a periodic pattern. The method of pairing magnets makes such a systematic effect unlikely, but we assume worst case and assume the error fields had a systematic periodic pattern. Additionally, each strong pair of magnets in a period was placed next to a weak pair in a period in an effort to minimize phase errors. As the rows are shifted by half a period, this could potentially produce a residual field. This effect is also explored.

4.3.1 Trajectory Error Sort

Let the main field from each quadrant be denoted by B and the smaller transverse error field by b. Errors in the main field have already been studied. The error fields come from alternating the transverse error fields from the magnet blocks so that the error fields cancel in each period. We assume the error fields have the same periodicity. The error fields b can have either sign, but we assume they are in phase with the main field as the worst case. The fields on the undulator axis from each quadrant are given by

$$\overrightarrow{B_1} = \widehat{x} B_{01} \cos(k_u(z - z_{01})) + \widehat{y} b_{01} \cos(k_u(z - z_{01})) \tag{139}$$

$$\overrightarrow{B_2} = \widehat{y} B_{02} \cos(k_u(z - z_{02})) + \widehat{x} b_{02} \cos(k_u(z - z_{02})) \tag{140}$$

$$\overrightarrow{B_3} = \widehat{x} B_{03} \cos(k_u(z - z_{03})) + \widehat{y} b_{03} \cos(k_u(z - z_{03})) \tag{141}$$

$$\overrightarrow{B_4} = \widehat{y} B_{04} \cos(k_u(z - z_{04})) + \widehat{x} b_{04} \cos(k_u(z - z_{04})) \tag{142}$$

[6] Z. Wolf, "Delta Undulator Magnet Block Sorting Algorithm", LCLS-TN-13-1, January, 2013.

The horizontal and vertical fields are given by the sum of the fields from the quadrants.

$$\begin{aligned} B_x &= B_{01}\cos(k_u(z-z_{01})) + B_{03}\cos(k_u(z-z_{03})) \\ &\quad + b_{02}\cos(k_u(z-z_{02})) + b_{04}\cos(k_u(z-z_{04})) \end{aligned} \tag{143}$$

$$\begin{aligned} B_y &= B_{02}\cos(k_u(z-z_{02})) + B_{04}\cos(k_u(z-z_{04})) \\ &\quad + b_{01}\cos(k_u(z-z_{01})) + b_{03}\cos(k_u(z-z_{03})) \end{aligned} \tag{144}$$

The integrals of the fields are

$$\begin{aligned} I_x &= B_{01}\frac{1}{k_u}\sin(k_u(z-z_{01})) + B_{03}\frac{1}{k_u}\sin(k_u(z-z_{03})) \\ &\quad + b_{02}\frac{1}{k_u}\sin(k_u(z-z_{02})) + b_{04}\frac{1}{k_u}\sin(k_u(z-z_{04})) \end{aligned} \tag{145}$$

$$\begin{aligned} I_y &= B_{02}\frac{1}{k_u}\sin(k_u(z-z_{02})) + B_{04}\frac{1}{k_u}\sin(k_u(z-z_{04})) \\ &\quad + b_{01}\frac{1}{k_u}\sin(k_u(z-z_{01})) + b_{03}\frac{1}{k_u}\sin(k_u(z-z_{03})) \end{aligned} \tag{146}$$

The K value is proportional to

$$K^2 \sim \langle I_x^2 \rangle + \langle I_y^2 \rangle \tag{147}$$

where the average is taken along z. Performing the squares and averages, we have

$$\begin{aligned} \langle I_x^2 \rangle &= \frac{1}{k_u^2}\{\frac{1}{2}B_{01}^2 + \frac{1}{2}B_{03}^2 + B_{01}B_{03}\cos(k_u(z_{01}-z_{03})) \\ &\quad + B_{01}b_{02}\cos(k_u(z_{01}-z_{02})) + B_{01}b_{04}\cos(k_u(z_{01}-z_{04})) \\ &\quad + B_{03}b_{02}\cos(k_u(z_{03}-z_{02})) + B_{03}b_{04}\cos(k_u(z_{03}-z_{04})) \\ &\quad + \frac{1}{2}b_{02}^2 + \frac{1}{2}b_{04}^2 + b_{02}b_{04}\cos(k_u(z_{02}-z_{04}))\} \end{aligned} \tag{148}$$

$$\begin{aligned} \langle I_y^2 \rangle &= \frac{1}{k_u^2}\{\frac{1}{2}B_{02}^2 + \frac{1}{2}B_{04}^2 + B_{02}B_{04}\cos(k_u(z_{02}-z_{04})) \\ &\quad + B_{02}b_{01}\cos(k_u(z_{02}-z_{01})) + B_{02}b_{03}\cos(k_u(z_{02}-z_{03})) \\ &\quad + B_{04}b_{01}\cos(k_u(z_{04}-z_{01})) + B_{04}b_{03}\cos(k_u(z_{04}-z_{03})) \\ &\quad + \frac{1}{2}b_{01}^2 + \frac{1}{2}b_{03}^2 + b_{01}b_{03}\cos(k_u(z_{01}-z_{03}))\} \end{aligned} \tag{149}$$

Inserting these expressions into the equation for K^2, we get

$$\begin{aligned} K^2 &\sim \frac{1}{k_u^2}\{\frac{1}{2}B_{01}^2 + \frac{1}{2}B_{03}^2 + B_{01}B_{03}\cos(k_u(z_{01}-z_{03})) \\ &\quad + \frac{1}{2}B_{02}^2 + \frac{1}{2}B_{04}^2 + B_{02}B_{04}\cos(k_u(z_{02}-z_{04})) \\ &\quad + B_{01}b_{02}\cos(k_u(z_{01}-z_{02})) + B_{01}b_{04}\cos(k_u(z_{01}-z_{04})) \\ &\quad + B_{03}b_{02}\cos(k_u(z_{03}-z_{02})) + B_{03}b_{04}\cos(k_u(z_{03}-z_{04})) \\ &\quad + B_{02}b_{01}\cos(k_u(z_{02}-z_{01})) + B_{02}b_{03}\cos(k_u(z_{02}-z_{03})) \\ &\quad + B_{04}b_{01}\cos(k_u(z_{04}-z_{01})) + B_{04}b_{03}\cos(k_u(z_{04}-z_{03})) \\ &\quad + \frac{1}{2}b_{02}^2 + \frac{1}{2}b_{04}^2 + b_{02}b_{04}\cos(k_u(z_{02}-z_{04})) \\ &\quad + \frac{1}{2}b_{01}^2 + \frac{1}{2}b_{03}^2 + b_{01}b_{03}\cos(k_u(z_{01}-z_{03}))\} \end{aligned} \tag{150}$$

Simplifying, we have

$$\begin{aligned}
K^2 \quad \sim \quad & \frac{1}{k_u^2}\{\frac{1}{2}B_{01}^2 + \frac{1}{2}B_{02}^2 + \frac{1}{2}B_{03}^2 + \frac{1}{2}B_{04}^2 \\
& + \frac{1}{2}b_{01}^2 + \frac{1}{2}b_{02}^2 + \frac{1}{2}b_{03}^2 + \frac{1}{2}b_{04}^2 \\
& + (B_{01}B_{03} + b_{01}b_{03})\cos(k_u(z_{01} - z_{03})) + (B_{02}B_{04} + b_{02}b_{04})\cos(k_u(z_{02} - z_{04})) \\
& + (B_{01}b_{02} + B_{02}b_{01})\cos(k_u(z_{01} - z_{02})) + (B_{01}b_{04} + B_{04}b_{01})\cos(k_u(z_{01} - z_{04})) \\
& + (B_{03}b_{02} + B_{02}b_{03})\cos(k_u(z_{03} - z_{02})) + (B_{03}b_{04} + B_{04}b_{03})\cos(k_u(z_{03} - z_{04}))\} \quad (151)
\end{aligned}$$

Keeping only the largest term for each function involving Δ_{13}, Δ_{24}, or δ, we get

$$\begin{aligned}
K^2 \quad \sim \quad & \frac{1}{k_u^2}\{\frac{1}{2}B_{01}^2 + \frac{1}{2}B_{03}^2 + \frac{1}{2}B_{02}^2 + \frac{1}{2}B_{04}^2 \\
& + B_{01}B_{03}\cos(k_u(z_{01} - z_{03})) + B_{02}B_{04}\cos(k_u(z_{02} - z_{04})) \\
& + (B_{01}b_{02} + B_{02}b_{01})\cos(k_u(z_{01} - z_{02})) + (B_{01}b_{04} + B_{04}b_{01})\cos(k_u(z_{01} - z_{04})) \\
& + (B_{03}b_{02} + B_{02}b_{03})\cos(k_u(z_{03} - z_{02})) + (B_{03}b_{04} + B_{04}b_{03})\cos(k_u(z_{03} - z_{04}))\} \quad (152)
\end{aligned}$$

This expression has the same functional form as the expression for quadrant strength and transverse position errors. No improvements to the fit will result from following this expression further.

4.3.2 Phase Error Sort

A strong pair of magnets in a period was followed by a weak pair in a period in an effort to avoid phase error accumulation. We model this by putting a modulation on the main field.

$$\overrightarrow{B_1} = \hat{x}\left(B_{01} + b_{01}\cos(\frac{1}{2}k_u(z - z_{01}))\right)\cos(k_u(z - z_{01})) \quad (153)$$

$$\overrightarrow{B_2} = \hat{y}\left(B_{02} + b_{02}\cos(\frac{1}{2}k_u(z - z_{02}))\right)\cos(k_u(z - z_{02})) \quad (154)$$

$$\overrightarrow{B_3} = \hat{x}\left(B_{03} + b_{03}\cos(\frac{1}{2}k_u(z - z_{03}))\right)\cos(k_u(z - z_{03})) \quad (155)$$

$$\overrightarrow{B_4} = \hat{y}\left(B_{04} + b_{04}\cos(\frac{1}{2}k_u(z - z_{04}))\right)\cos(k_u(z - z_{04})) \quad (156)$$

The horizontal and vertical fields are given by the sum of the fields from the quadrants.

$$\begin{aligned}
B_x \quad = \quad & B_{01}\cos(k_u(z - z_{01})) + B_{03}\cos(k_u(z - z_{03})) \\
& + \frac{1}{2}b_{01}\cos(\frac{3}{2}k_u(z - z_{01})) + \frac{1}{2}b_{01}\cos(\frac{1}{2}k_u(z - z_{01})) \\
& + \frac{1}{2}b_{03}\cos(\frac{3}{2}k_u(z - z_{03})) + \frac{1}{2}b_{03}\cos(\frac{1}{2}k_u(z - z_{03})) \quad (157) \\
B_y \quad = \quad & B_{02}\cos(k_u(z - z_{02})) + B_{04}\cos(k_u(z - z_{04})) \\
& + \frac{1}{2}b_{02}\cos(\frac{3}{2}k_u(z - z_{02})) + \frac{1}{2}b_{02}\cos(\frac{1}{2}k_u(z - z_{02})) \\
& + \frac{1}{2}b_{04}\cos(\frac{3}{2}k_u(z - z_{04})) + \frac{1}{2}b_{04}\cos(\frac{1}{2}k_u(z - z_{04})) \quad (158)
\end{aligned}$$

The K value is proportional to

$$K^2 \sim \langle I_x^2 \rangle + \langle I_y^2 \rangle \quad (159)$$

where the average is taken along z. Performing the squares and averages, we have

$$\begin{aligned}\langle I_x^2\rangle &= \frac{1}{k_u^2}\{\frac{1}{2}B_{01}^2+\frac{1}{2}B_{03}^2+B_{01}B_{03}\cos(k_u(z_{01}-z_{03}))\\&+\frac{1}{8}b_{01}^2+\frac{1}{8}b_{03}^2+\frac{1}{4}b_{01}b_{03}\cos(\frac{3}{2}k_u(z_{01}-z_{03}))\\&+\frac{1}{8}b_{01}^2+\frac{1}{8}b_{03}^2+\frac{1}{4}b_{01}b_{03}\cos(\frac{1}{2}k_u(z_{01}-z_{03}))\}\end{aligned} \tag{160}$$

$$\begin{aligned}\langle I_y^2\rangle &= \frac{1}{k_u^2}\{\frac{1}{2}B_{02}^2+\frac{1}{2}B_{04}^2+B_{02}B_{04}\cos(k_u(z_{02}-z_{04}))\\&+\frac{1}{8}b_{02}^2+\frac{1}{8}b_{04}^2+\frac{1}{4}b_{02}b_{04}\cos(\frac{3}{2}k_u(z_{02}-z_{04}))\\&+\frac{1}{8}b_{02}^2+\frac{1}{8}b_{04}^2+\frac{1}{4}b_{02}b_{04}\cos(\frac{1}{2}k_u(z_{02}-z_{04}))\}\end{aligned} \tag{161}$$

Inserting these expressions into the equation for K^2, we get

$$\begin{aligned}K^2 \sim\ & \frac{1}{2}B_{01}^2+\frac{1}{2}B_{03}^2+\frac{1}{2}B_{02}^2+\frac{1}{2}B_{04}^2\\&+\frac{1}{4}b_{01}^2+\frac{1}{4}b_{03}^2+\frac{1}{4}b_{02}^2+\frac{1}{4}b_{04}^2\\&+B_{01}B_{03}\cos(k_u(z_{01}-z_{03}))+B_{02}B_{04}\cos(k_u(z_{02}-z_{04}))\\&+\frac{1}{4}b_{01}b_{03}\cos(\frac{3}{2}k_u(z_{01}-z_{03}))+\frac{1}{4}b_{01}b_{03}\cos(\frac{1}{2}k_u(z_{01}-z_{03}))\\&+\frac{1}{4}b_{02}b_{04}\cos(\frac{3}{2}k_u(z_{02}-z_{04}))+\frac{1}{4}b_{02}b_{04}\cos(\frac{1}{2}k_u(z_{02}-z_{04}))\end{aligned} \tag{162}$$

The functional form of this fit is

$$\begin{aligned}K^2 &= c_1+c_2\cos(k_u(z_{01}-z_{03}))+c_3\cos(k_u(z_{02}-z_{04}))\\&+d_1\cos(\frac{3}{2}k_u(z_{01}-z_{03}))+d_2\cos(\frac{1}{2}k_u(z_{01}-z_{03}))\\&+d_3\cos(\frac{3}{2}k_u(z_{02}-z_{04}))+d_4\cos(\frac{1}{2}k_u(z_{02}-z_{04}))\end{aligned} \tag{163}$$

The residuals from the fit are shown in figure 12.

If we add these new terms to the fit formula including all previous errors, the functional form of the fit is

$$\begin{aligned}K^2 &= c_1+c_2\cos(k_u(z_{01}-z_{03}))+c_3\cos(k_u(z_{02}-z_{04}))\\&+d_1\cos(k_u(z_{01}-z_{02}))+d_2\cos(k_u(z_{01}-z_{04}))\\&+d_3\cos(k_u(z_{03}-z_{02}))+d_4\cos(k_u(z_{03}-z_{04}))\\&+d_5\sin(k_u(z_{01}-z_{03}))+d_6\sin(k_u(z_{02}-z_{04}))\\&+d_7\cos(\frac{3}{2}k_u(z_{01}-z_{03}))+d_8\cos(\frac{1}{2}k_u(z_{01}-z_{03}))\\&+d_9\cos(\frac{3}{2}k_u(z_{02}-z_{04}))+d_{10}\cos(\frac{1}{2}k_u(z_{02}-z_{04}))\end{aligned} \tag{164}$$

With this fit including quadrant strength and transverse position errors, quadrant longitudinal position errors, and errors from the magnet sorting algorithm, the residuals are shown in figure 13. The residuals are improved, but not significantly.

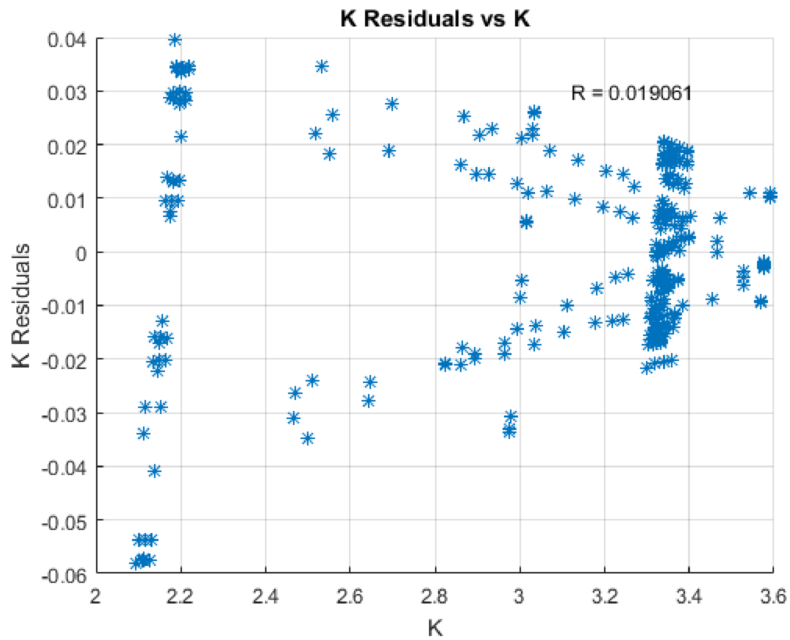

Figure 12: Residuals from the fit with phase error sorting effects.

4.4 Discussion

In this section the form of the fitting function was motivated by various errors in the undulator. The fit is adequate to set the Delta undulator, but is problematic in that the coefficients don't behave as expected. The relative strengths of the 1-3 and 2-4 undulators depends on the cuts applied to the data. The $\sin\left(k_u \frac{\Delta_{13}}{2}\right) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \cos(\delta)$ is again very important to the fit, yet the coefficient was expected to be second order. This is shown in appendix A1. The fits can be used to set the undulator, but should probably not be used to draw conclusions about errors in the undulator.

5 Conclusion

Fits were found that are adequate to set the K value of the Delta undulator in all polarization modes and all K values larger than 2.0. The form of the fit function was derived from several models of the undulator. The size of the fit coefficients, however, could not be fully explained by the models.

Acknowledgements
I am grateful to Heinz-Dieter Nuhn for many discussions about this work.

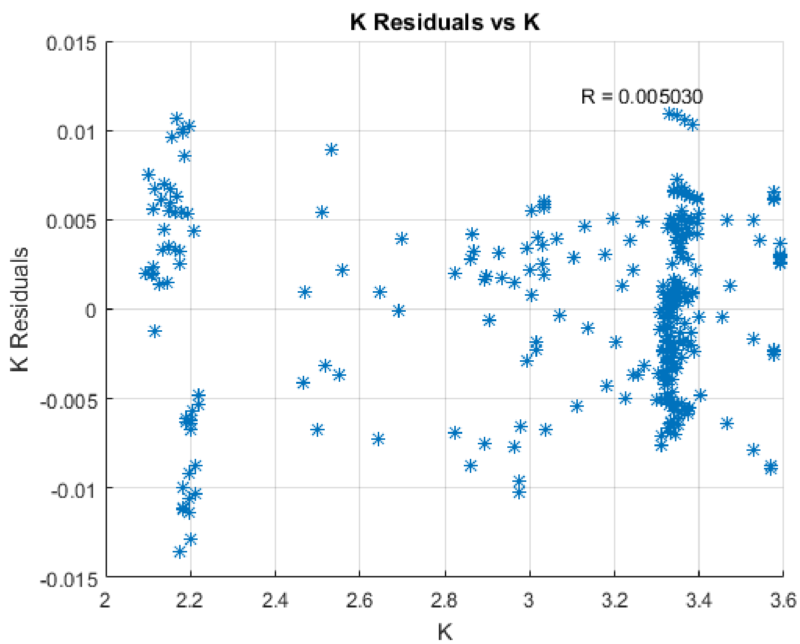

Figure 13: Residuals from the fit including quadrant strength and transverse position errors, quadrant longitudinal row shift errors, and magnet sorting errors.

Appendix

A1 Compare The Undulator Error Fit To The Ideal Undulator Fit

The residuals from the model with quadrant strength and transverse position errors look very similar to the residuals from the model of an ideal undulator measured off the magnetic axis. The fitting functions are the same as we will now show.

To make explicit the effect of the field strength differences between the quadrants, let

$$B_{01} = B_0 + \beta_1 \tag{165}$$
$$B_{02} = B_0 + \beta_2 \tag{166}$$
$$B_{03} = B_0 + \beta_3 \tag{167}$$
$$B_{04} = B_0 + \beta_4 \tag{168}$$

where B_0 is the ideal field strength from each quadrant and β_i is the field strength error for quadrant i. Starting from equation 100, we expand K^2 making sure to keep the dominant term for each function involving Δ_{13}, Δ_{24}, or δ.

$$\begin{aligned} K^2 \sim\; & 2B_0^2 + B_0(\beta_1 + \beta_2 + \beta_3 + \beta_4) \\ & + \frac{1}{2}\left(\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_4^2\right) + \frac{1}{2}B_0^2\left(\epsilon_1^2 + \epsilon_3^2 + \epsilon_2^2 + \epsilon_4^2\right) \\ & + (B_0^2 + B_0(\beta_1 + \beta_3))\cos(k_u(z_{01} - z_{03})) \\ & + (B_0^2 + B_0(\beta_2 + \beta_4))\cos(k_u(z_{02} - z_{04})) \\ & + (\epsilon_1 - \epsilon_2)(B_0^2 + B_0(\beta_1 + \beta_2))\cos(k_u(z_{01} - z_{02})) \\ & + (\epsilon_1 - \epsilon_4)(B_0^2 + B_0(\beta_1 + \beta_4))\cos(k_u(z_{01} - z_{04})) \\ & + (\epsilon_3 - \epsilon_2)(B_0^2 + B_0(\beta_3 + \beta_2))\cos(k_u(z_{03} - z_{02})) \\ & + (\epsilon_3 - \epsilon_4)(B_0^2 + B_0(\beta_3 + \beta_4))\cos(k_u(z_{03} - z_{04})) \end{aligned} \tag{169}$$

The identity $\cos(2x) = 2\cos^2(x) - 1$ can be used to give

$$\begin{aligned} K^2 \sim\; & \frac{1}{2}\left(\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_4^2\right) + \frac{1}{2}B_0^2\left(\epsilon_1^2 + \epsilon_3^2 + \epsilon_2^2 + \epsilon_4^2\right) \\ & + 2(B_0^2 + B_0(\beta_1 + \beta_3))\cos^2\left(\frac{k_u}{2}(z_{01} - z_{03})\right) \\ & + 2(B_0^2 + B_0(\beta_2 + \beta_4))\cos^2\left(\frac{k_u}{2}(z_{02} - z_{04})\right) \\ & + (\epsilon_1 - \epsilon_2)(B_0^2 + B_0(\beta_1 + \beta_2))\cos(k_u(z_{01} - z_{02})) \\ & + (\epsilon_1 - \epsilon_4)(B_0^2 + B_0(\beta_1 + \beta_4))\cos(k_u(z_{01} - z_{04})) \\ & + (\epsilon_3 - \epsilon_2)(B_0^2 + B_0(\beta_3 + \beta_2))\cos(k_u(z_{03} - z_{02})) \\ & + (\epsilon_3 - \epsilon_4)(B_0^2 + B_0(\beta_3 + \beta_4))\cos(k_u(z_{03} - z_{04})) \end{aligned} \tag{170}$$

We can solve for the row positions in terms of the undulator row setting parameters with the added information that the measurements were made with the sum of the quadrant row shifts equal to zero. We find

$$z_{01} = \frac{1}{2}\left(\frac{\delta}{k_u} + \Delta_{13}\right) \tag{171}$$

$$z_{02} = \frac{1}{2}\left(-\frac{\delta}{k_u} + \Delta_{24}\right) \tag{172}$$

$$z_{03} = \frac{1}{2}\left(\frac{\delta}{k_u} - \Delta_{13}\right) \tag{173}$$

$$z_{04} = \frac{1}{2}\left(-\frac{\delta}{k_u} - \Delta_{24}\right) \tag{174}$$

Using these row setting parameters, K^2 has the form

$$\begin{aligned} K^2 \sim\ & \frac{1}{2}\left(\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_4^2\right) + \frac{1}{2}B_0^2\left(\epsilon_1^2 + \epsilon_3^2 + \epsilon_2^2 + \epsilon_4^2\right) \\ & + 2(B_0^2 + B_0(\beta_1 + \beta_3))\cos^2\left(\frac{k_u}{2}\Delta_{13}\right) \\ & + 2(B_0^2 + B_0(\beta_2 + \beta_4))\cos^2\left(\frac{k_u}{2}\Delta_{24}\right) \\ & + (\epsilon_1 - \epsilon_2)(B_0^2 + B_0(\beta_1 + \beta_2))\cos\left(\frac{k_u}{2}(\Delta_{13} - \Delta_{24}) + \delta\right) \\ & + (\epsilon_1 - \epsilon_4)(B_0^2 + B_0(\beta_1 + \beta_4))\cos\left(\frac{k_u}{2}(\Delta_{13} + \Delta_{24}) + \delta\right) \\ & + (\epsilon_3 - \epsilon_2)(B_0^2 + B_0(\beta_3 + \beta_2))\cos\left(\frac{k_u}{2}(\Delta_{13} + \Delta_{24}) - \delta\right) \\ & + (\epsilon_3 - \epsilon_4)(B_0^2 + B_0(\beta_3 + \beta_4))\cos\left(\frac{k_u}{2}(\Delta_{13} - \Delta_{24}) - \delta\right) \end{aligned} \tag{175}$$

In order make the form similar to the ideal undulator, we use the identity

$$\begin{aligned} & \cos\left(\frac{k_u}{2}(\Delta_{13} \pm \Delta_{24}) \pm \delta\right) \\ = & \cos\left(\frac{k_u}{2}(\Delta_{13} \pm \Delta_{24})\right)\cos(\delta) \mp \sin\left(\frac{k_u}{2}(\Delta_{13} \pm \Delta_{24})\right)\sin(\delta) \\ = & \left[\cos\left(\frac{k_u}{2}\Delta_{13}\right)\cos\left(\frac{k_u}{2}\Delta_{24}\right) \mp \sin\left(\frac{k_u}{2}\Delta_{13}\right)\sin\left(\frac{k_u}{2}\Delta_{24}\right)\right]\cos(\delta) \\ & \mp \left[\sin\left(\frac{k_u}{2}\Delta_{13}\right)\cos\left(\frac{k_u}{2}\Delta_{24}\right) \pm \cos\left(\frac{k_u}{2}\Delta_{13}\right)\sin\left(\frac{k_u}{2}\Delta_{24}\right)\right]\sin(\delta) \end{aligned} \tag{176}$$

Using this identity, equation 175 can be rewritten to give

$$
\begin{aligned}
K^2 \sim\ & 2(B_0^2 + B_0(\beta_1+\beta_3))\cos^2\left(\frac{k_u}{2}\Delta_{13}\right) + 2(B_0^2 + B_0(\beta_2+\beta_4))\cos^2\left(\frac{k_u}{2}\Delta_{24}\right) \\
& + B_0^2\left[(\epsilon_1-\epsilon_2)+(\epsilon_1-\epsilon_4)+(\epsilon_3-\epsilon_2)+(\epsilon_3-\epsilon_4)\right]\cos\left(\frac{k_u}{2}\Delta_{13}\right)\cos\left(\frac{k_u}{2}\Delta_{24}\right)\cos(\delta) \\
& + B_0^2\left[(\epsilon_1-\epsilon_2)-(\epsilon_1-\epsilon_4)-(\epsilon_3-\epsilon_2)+(\epsilon_3-\epsilon_4)\right]\sin\left(\frac{k_u}{2}\Delta_{13}\right)\sin\left(\frac{k_u}{2}\Delta_{24}\right)\cos(\delta) \\
& + B_0\left[(\epsilon_1-\epsilon_2)(\beta_1+\beta_2)-(\epsilon_1-\epsilon_4)(\beta_1+\beta_4)-(\epsilon_3-\epsilon_2)(\beta_3+\beta_2)+(\epsilon_3-\epsilon_4)(\beta_3+\beta_4)\right] \\
& \times\sin\left(\frac{k_u}{2}\Delta_{13}\right)\sin\left(\frac{k_u}{2}\Delta_{24}\right)\cos(\delta) \\
& + B_0^2\left[-(\epsilon_1-\epsilon_2)-(\epsilon_1-\epsilon_4)+(\epsilon_3-\epsilon_2)+(\epsilon_3-\epsilon_4)\right]\sin\left(\frac{k_u}{2}\Delta_{13}\right)\cos\left(\frac{k_u}{2}\Delta_{24}\right)\sin(\delta) \\
& + B_0^2\left[(\epsilon_1-\epsilon_2)-(\epsilon_1-\epsilon_4)+(\epsilon_3-\epsilon_2)-(\epsilon_3-\epsilon_4)\right]\cos\left(\frac{k_u}{2}\Delta_{13}\right)\sin\left(\frac{k_u}{2}\Delta_{24}\right)\sin(\delta) \\
& + \frac{1}{2}\left(\beta_1^2+\beta_2^2+\beta_3^2+\beta_4^2\right) + \frac{1}{2}B_0^2\left(\epsilon_1^2+\epsilon_3^2+\epsilon_2^2+\epsilon_4^2\right) \qquad (177)
\end{aligned}
$$

The second order terms were only included for the $\sin\left(\frac{k_u}{2}\Delta_{13}\right)\sin\left(\frac{k_u}{2}\Delta_{24}\right)\cos(\delta)$ term since the first order coefficient went to zero. This expression simplifies to

$$
\begin{aligned}
K^2 \sim\ & 2(B_0^2 + B_0(\beta_1+\beta_3))\cos^2\left(\frac{k_u}{2}\Delta_{13}\right) + 2(B_0^2 + B_0(\beta_2+\beta_4))\cos^2\left(\frac{k_u}{2}\Delta_{24}\right) \\
& + 2B_0^2\left[(\epsilon_1+\epsilon_3)-(\epsilon_2+\epsilon_4)\right]\cos\left(\frac{k_u}{2}\Delta_{13}\right)\cos\left(\frac{k_u}{2}\Delta_{24}\right)\cos(\delta) \\
& + B_0\left[(\epsilon_1-\epsilon_3)(\beta_2-\beta_4)+(\epsilon_4-\epsilon_2)(\beta_1-\beta_3)\right]\sin\left(\frac{k_u}{2}\Delta_{13}\right)\sin\left(\frac{k_u}{2}\Delta_{24}\right)\cos(\delta) \\
& + 2B_0^2\left[-\epsilon_1+\epsilon_3\right]\sin\left(\frac{k_u}{2}\Delta_{13}\right)\cos\left(\frac{k_u}{2}\Delta_{24}\right)\sin(\delta) \\
& + 2B_0^2\left[-\epsilon_2+\epsilon_4\right]\cos\left(\frac{k_u}{2}\Delta_{13}\right)\sin\left(\frac{k_u}{2}\Delta_{24}\right)\sin(\delta) \\
& + \frac{1}{2}\left(\beta_1^2+\beta_2^2+\beta_3^2+\beta_4^2\right) + \frac{1}{2}B_0^2\left(\epsilon_1^2+\epsilon_3^2+\epsilon_2^2+\epsilon_4^2\right) \qquad (178)
\end{aligned}
$$

This expression is of the form

$$
\begin{aligned}
K^2 =\ & c_1\cos^2\left(k_u\frac{\Delta_{13}}{2}\right) + c_2\cos^2\left(k_u\frac{\Delta_{24}}{2}\right) \\
& + d_1\cos\left(k_u\frac{\Delta_{13}}{2}\right)\cos\left(k_u\frac{\Delta_{24}}{2}\right)\cos(\delta) + d_2\sin\left(k_u\frac{\Delta_{13}}{2}\right)\sin\left(k_u\frac{\Delta_{24}}{2}\right)\cos(\delta) \\
& d_3\sin\left(k_u\frac{\Delta_{13}}{2}\right)\cos\left(k_u\frac{\Delta_{24}}{2}\right)\sin(\delta) + d_4\cos\left(k_u\frac{\Delta_{13}}{2}\right)\sin\left(k_u\frac{\Delta_{24}}{2}\right)\sin(\delta) \\
& + e_1 \qquad (179)
\end{aligned}
$$

This is the same functional form as the fit for the ideal undulator. The interpretation of the coefficients is different, however. The explicit form of the fit coefficients ignoring the factor of $2\left(\frac{q}{mk_uc}\right)^2$ common to all the coefficients is

$$
\begin{aligned}
c_1 &= 2(B_0^2 + B_0(\beta_1 + \beta_3)) && (180)\\
c_2 &= 2(B_0^2 + B_0(\beta_2 + \beta_4)) && (181)\\
d_1 &= 2B_0^2\left[(\epsilon_1 + \epsilon_3) - (\epsilon_2 + \epsilon_4)\right] && (182)\\
d_2 &= B_0\left[(\epsilon_1 - \epsilon_3)(\beta_2 - \beta_4) + (\epsilon_4 - \epsilon_2)(\beta_1 - \beta_3)\right] && (183)\\
d_3 &= 2B_0^2\left[-\epsilon_1 + \epsilon_3\right] && (184)\\
d_4 &= 2B_0^2\left[-\epsilon_2 + \epsilon_4\right] && (185)\\
e_1 &= \frac{1}{2}\left(\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_4^2\right) + \frac{1}{2}B_0^2\left(\epsilon_1^2 + \epsilon_3^2 + \epsilon_2^2 + \epsilon_4^2\right) && (186)
\end{aligned}
$$

With this functional form, the residuals of the fit are as shown in figure 14. The fit coefficients

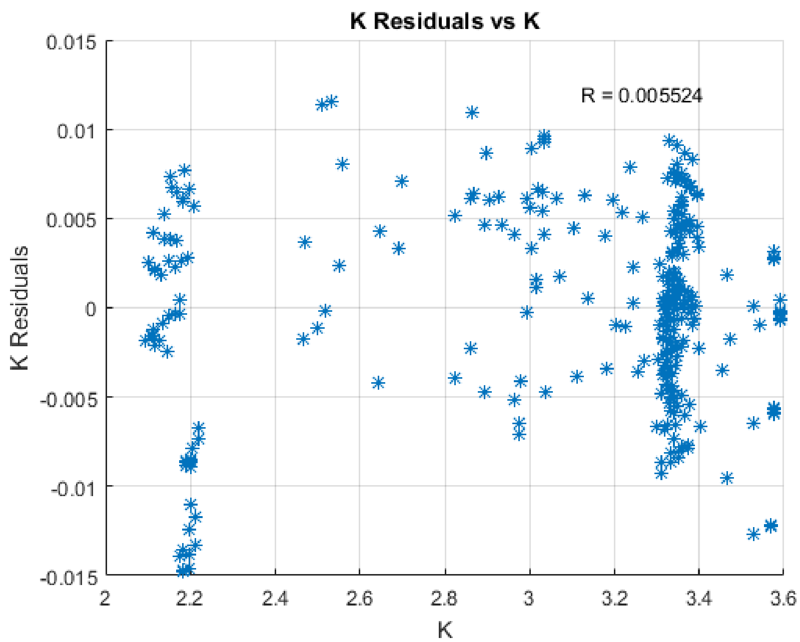

Figure 14: Residuals from the fit including quadrant strength and angle errors, but with the form of the ideal undulator fit.

are

$$
\begin{aligned}
c_1 &= 6.503 && (187)\\
c_2 &= 6.159 && (188)\\
d_1 &= -0.056 && (189)\\
d_2 &= 0.298 && (190)\\
d_3 &= -0.194 && (191)\\
d_4 &= -0.073 && (192)\\
e_1 &= 0.169 && (193)
\end{aligned}
$$

The relative strength of the 1-3 undulator compared to the 2-4 undulator is

$$\frac{(B_{01}+B_{03})-(B_{02}+B_{04})}{B_0} = \frac{(\beta_1+\beta_3)-(\beta_2+\beta_4)}{B_0} \tag{194}$$

$$= \frac{c_1-c_2}{c_1+c_2}2 \tag{195}$$

The angle errors of each quadrant relative to quadrant 1 are

$$\epsilon_2-\epsilon_1 \simeq \frac{d_3-d_1-d_4}{2c_1} \tag{196}$$

$$\epsilon_3-\epsilon_1 \simeq \frac{-d_3}{c_1} \tag{197}$$

$$\epsilon_4-\epsilon_1 \simeq \frac{d_3+d_4-d_1}{2c_1} \tag{198}$$

We can calculate the errors from the fit coefficients.

$$\frac{(B_{01}+B_{03})-(B_{02}+B_{04})}{B_0} = 0.054 \tag{199}$$

$$\epsilon_2-\epsilon_1 = -0.005 \tag{200}$$

$$\epsilon_3-\epsilon_1 = 0.030 \tag{201}$$

$$\epsilon_4-\epsilon_1 = -0.016 \tag{202}$$

The 1-3 undulator is 5.4% stronger than the 2-4 undulator. The difference in the quadrant angles is up to 0.03 radians. This corresponds to 120 μm quadrant position errors at a radius of 4 mm. These errors are reasonable. The coefficient of the $\sin\left(k_u\frac{\Delta_{13}}{2}\right)\sin\left(k_u\frac{\Delta_{24}}{2}\right)\cos(\delta)$ term, d_2, is second order in the fit function, yet a large value is returned from the fit.

A2 Fields At Nominal $K=0$

The behavior of K when K is small has been problematic for the fits. K has remained fairly large when it was expected to go to zero. The reason is visible in the fields. Plot 15 shows the B_x and B_y fields when $\Delta_{13}=\lambda_u/2$ and $\Delta_{24}=\lambda_u/2$ in the linear vertical magnetic field mode. The fields are on the order of 300 G, or about 3% of the maximum field strength. They were expected to be much smaller.

The problem is not that a large field is feeding into B_x and B_y due to measurement errors. B_x and B_y are not feeding into each other since both fields are small. B_z is large, but the peaks of B_x and B_y are at the zero crossing of B_z as shown in figure 16. B_z is not feeding into the B_x and B_y measurements. The large size of B_x and B_y at nominal $K=0$ remains a mystery.

Unfortunately, the fits did not explain the mystery. When the cuts are removed, the fits diverge at small K as shown in figure 17. The fitting functions provide acceptable fits at large K, but their form is not correct to explain the behavior of K over its entire range.

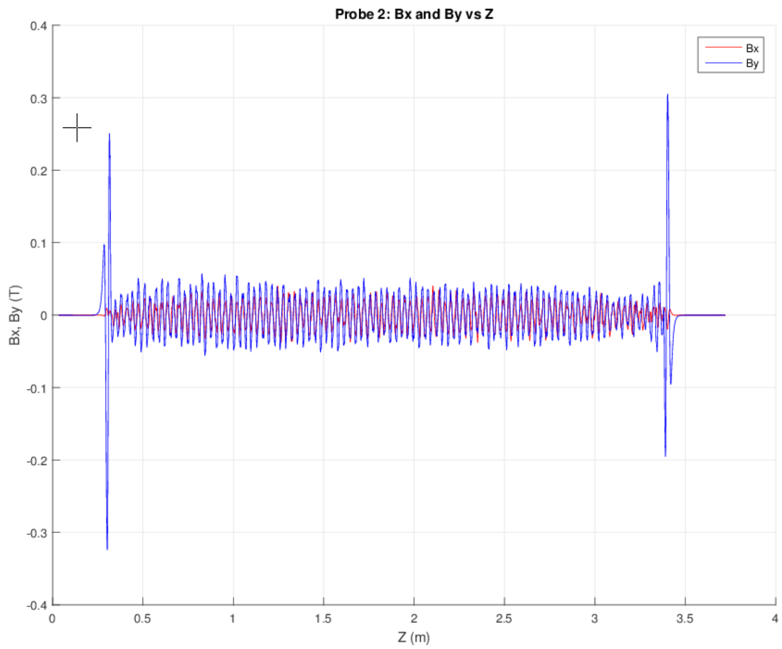

Figure 15: Measured B_x and B_y at nominal $K = 0$.

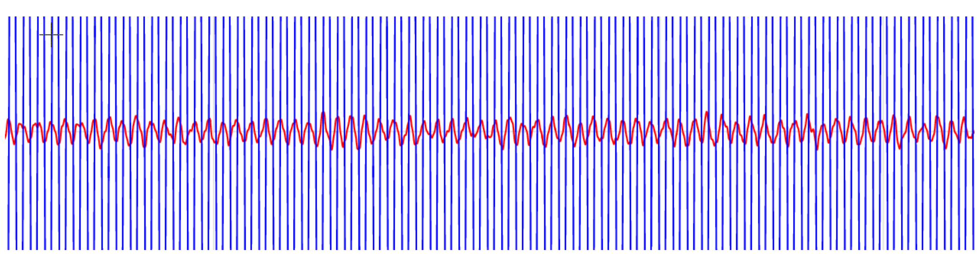

Figure 16: B_x (red) and B_z (blue) for nominal $K = 0$.

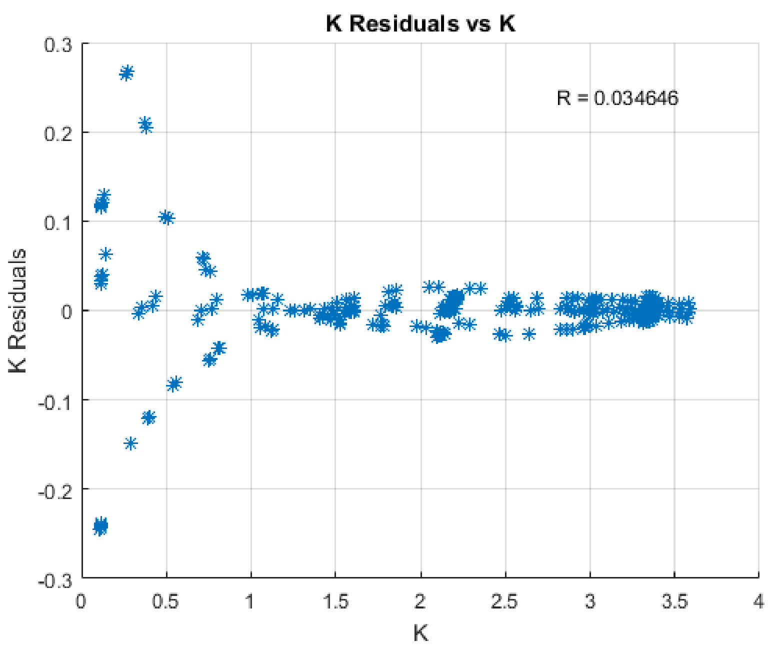

Figure 17: Residuals with no cuts applied when using the fit including quadrant strength and transverse position errors, quadrant longitudinal row position errors, and possible magnet sorting effect errors.