Effect Of Quadrant Taper On Delta Undulator Phase Errors

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Abstract

The fields in the Delta undulator are very sensitive to the position of the magnet arrays. A taper in the arrays, a linear change in radial position along the undulator, causes very large phase errors. A taper can come either from assembly errors or from pitch of the quadrants on the tuning stand. The size of the phase errors resulting from taper is studied in this note and limits on taper are derived in order to limit phase errors.

1 Introduction[1]

After tuning the Delta undulator[2], it was noticed that all polarization modes had phase errors that had a roughly quadratic shape. This is indicative of a taper in the magnets, which is a linear radial change in magnet position relative to the beam axis along the undulator. The magnet arrays may also have angular errors in addition to radial position errors, but it is the radial errors that cause large field changes on the beam axis. In this note we explore the effects of radial magnet position errors on the undulator phase errors.

If the undulator is assembled with the magnet arrays getting closer to the beam axis as one moves down the undulator, the magnetic field in the undulator gets progressively larger and the slippage in the undulator gets progressively larger. As we will show, this causes a phase error with quadratic longitudinal position dependence. Alternatively, if the magnet arrays are tuned all with the same pitch on the test stand, a taper results in the assembled undulator.

The Delta undulator was tuned by individually tuning each of the four quadrants. The permanent magnet material of the quadrants has relative permeability approximately equal to 1, so the magnetic field of the assembled undulator is approximately the same as the superposition of the fields from the four quadrants. If each quadrant is tuned to give straight trajectories and small phase errors, the assembled undulator should give straight trajectories and small phase errors to good approximation.

The field from each quadrant decays exponentially with distance from the quadrant. The method of tuning each quadrant individually is very sensitive to the position of the Hall probe relative to the quadrant. The exponential dependence on radial position of the magnet arrays also makes the undulator very sensitive to assembly errors. No tuning can be done after assembly, so great care must be taken both during the tuning and during the assembly.

[1] Work supported in part by the DOE Contract DE-AC02-76SF00515. This work was performed in support of the LCLS project at SLAC.

[2] A. Temnykh, Physical Review Special Topics-Accelerators and Beams **11**, 120702 (2008).

2 Measured Phase Errors

The measured phase errors for linear polarization vertical field mode at maximum K are shown in figure 1. The shape of the phase error vs z is indicative of a decrease in the vertical gap between magnet arrays as one moves down the undulator. The magnetic field and therefore the slippage is too small at the undulator entrance and the phase decreases, the slippage is correct at the undulator center, and the slippage is too large at the undulator exit and the phase increases.

The phase errors in the four primary modes (linear polarization vertical magnetic field (LPVMF), linear polarization horizontal magnetic field (LPHMF), circular polarization right hand magnetic field (CPRMF), circular polarization left hand magnetic field (CPLMF)) all have the same shape and about the same size at a given K value. This is illustrated in figure 2. The fact that the other modes have the same phase error shape indicates that the horizontal gap between magnet arrays also decreases, like the vertical gap, as one moves down the undulator. There is a systematic decrease in the radial position of the magnet arrays longitudinally down the undulator, a taper. At maximum K there is no longitudinal force on the quadrants in the linear modes, so the phase errors are not from stretching of the quadrant assemblies due to longitudinal forces, as one could potentially suppose.

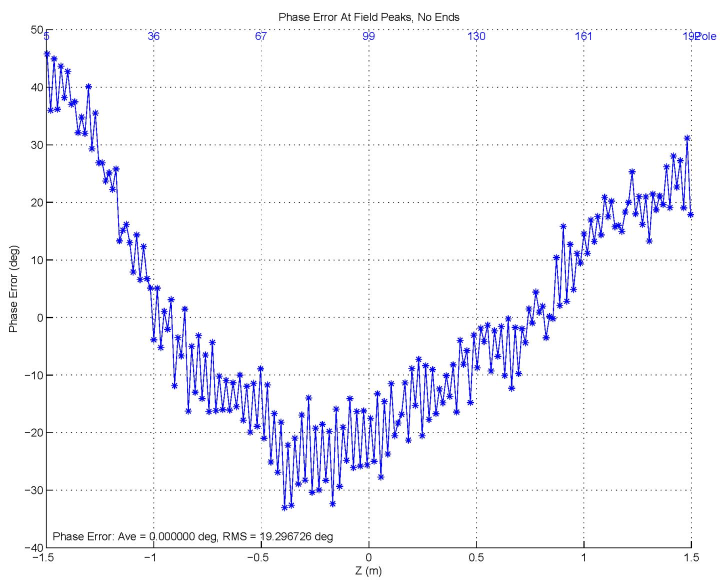

Figure 1: Measured phase error in the assembled undulator in planar vertical magnetic field mode at maximum K.

In order to produce these phase errors, either assembly errors make the radial distance between the magnet arrays have a taper, or the quadrants were tuned with a systematic pitch on the measurement bench. In the case of quadrant pitch on the tuning bench, the distance from the magnets to the beam axis would increase when the quadrant was put on the bench. Then during tuning, the magnets would be moved to correct the magnet positions. Finally, when the undulator was assembled, the magnets would have a systematic linear decrease in distance from the beam axis as one moved down the undulator. This is illustrated in figure 3. It should be noted, however, that

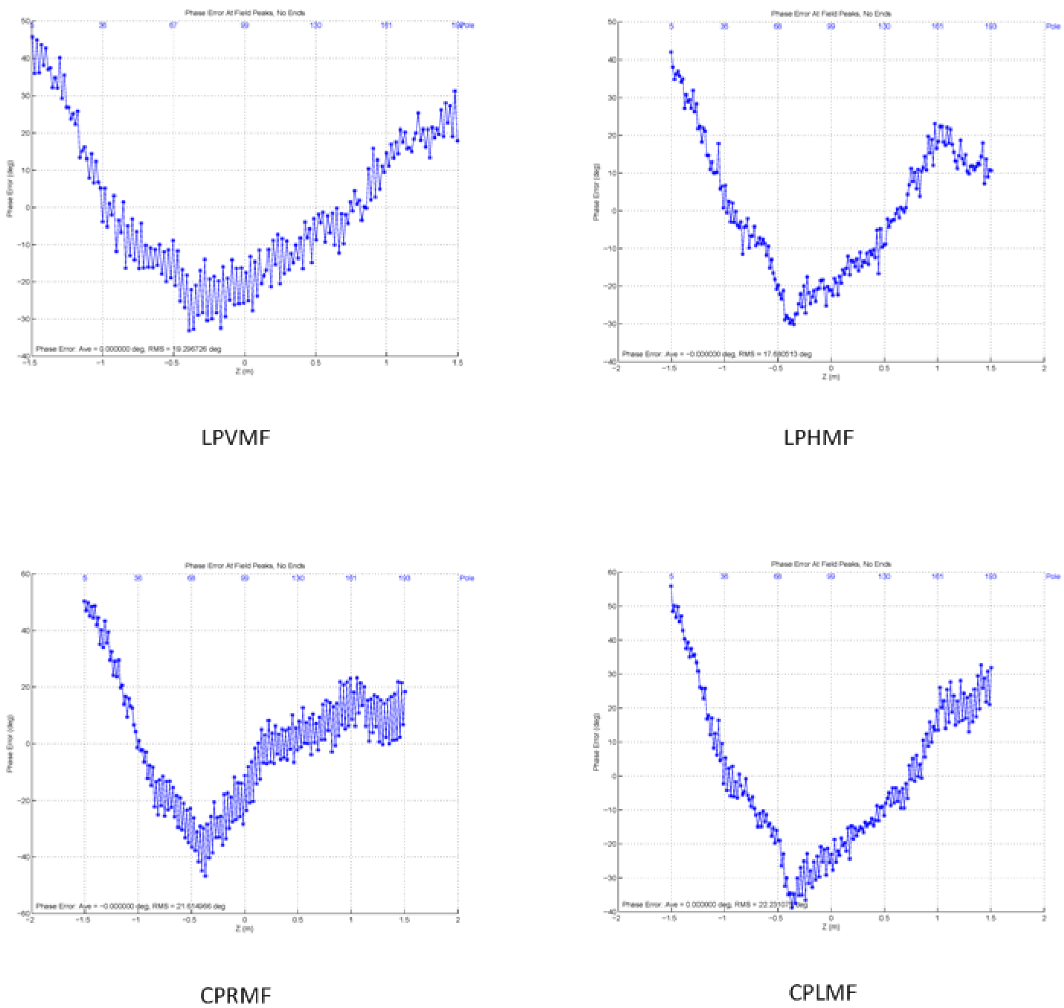

Figure 2: Phase errors in the four primary modes at maximum K. Note that the scales are different in the different plots.

two of the quadrants are reversed in the assembled undulator compared to their orientation on the tuning bench, making it seem less likely that there was a systematic pitch during tuning.

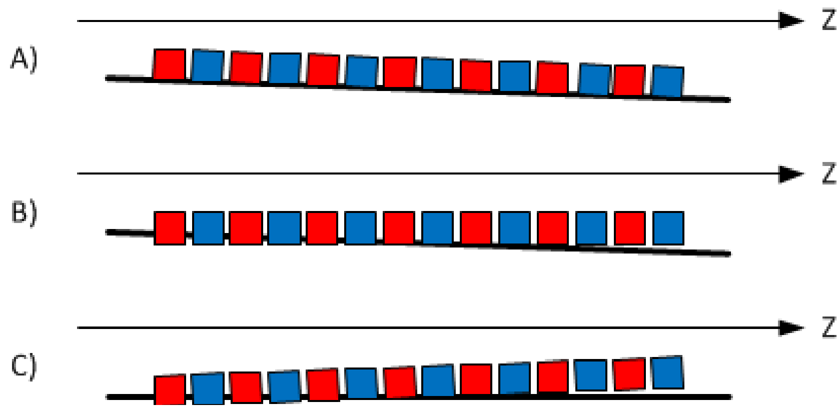

Figure 3: A) Quadrant was put on the bench with pitch. B) Magnets were moved during tuning to correct the pitch. C) When the undulator is assembled, the magnet keeper is parallel to the beam axis, but the magnets move toward the beam axis as one moves down the undulator.

3 Phase Errors From Taper In The Assembled Undulator

In this section we calculate the phase errors resulting from a taper in the magnet arrays of the assembled undulator. The results are the same whether the taper comes from assembly errors or from quadrant pitch on the tuning bench. We consider the case of a planar undulator with vertical field, but the results are easily extended to other polarization modes.

The slippage between two points a and b in a planar undulator is given by[3]

$$S_{ab} = \int_a^b \frac{1}{2\gamma^2} + \frac{1}{2} x'^2 dz \tag{1}$$

where γ is the Lorentz factor and x' is the slope of the horizontal trajectory in the vertical undulator field. The horizontal trajectory slope is given by

$$x' = \frac{\widetilde{K}}{\gamma} \cos(k_u z) \tag{2}$$

where $\widetilde{K}$ is the local undulator parameter at the z-position of interest, and $k_u = 2\pi/\lambda_u$, where λ_u is the undulator period. If we consider two points a and b spaced apart by one period, the slippage in the period is given by

$$S_{\lambda_u} = \frac{1}{2\gamma^2} \left(1 + \frac{1}{2} \widetilde{K}^2 \right) \lambda_u \tag{3}$$

The vertical magnetic field can be considered as an ideal field B_y plus small errors ΔB_y that change with position. We assume the magnetic field keeps its sinusoidal z-dependence to good approximation and the small errors make field changes over long distances compared to a period.

[3] Z. Wolf, "Introduction To LCLS Undulator Tuning", LCLS-TN-04-7, June, 2004.

The local undulator parameter $\widetilde{K}$ can be written as an ideal value K, plus a small position dependent change ΔK.

$$\widetilde{K} = K + \Delta K \tag{4}$$

The error in the field causes a change in the slippage per period compared to the ideal case of

$$\Delta S_{\lambda_u} = \frac{\lambda_u}{2\gamma^2} K \Delta K \tag{5}$$

Since K is proportional to the field, we have

$$\frac{\Delta K}{K} = \frac{\Delta B_y}{B_y} \tag{6}$$

So

$$\Delta S_{\lambda_u} = \frac{\lambda_u}{2\gamma^2} K^2 \frac{\Delta B_y}{B_y} \tag{7}$$

The change in phase corresponding to the slippage change is given by

$$\Delta P = \frac{2\pi}{\lambda_r} \Delta S \tag{8}$$

where, to good approximation, we can use the radiation wavelength from the ideal undulator, which is given by

$$\lambda_r = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{1}{2} K^2\right) \tag{9}$$

So the change in phase per period compared to the ideal case is

$$\Delta P_{\lambda_u} = 2\pi \frac{K^2}{(1 + \frac{1}{2} K^2)} \frac{\Delta B_y}{B_y} \tag{10}$$

The fields from a Delta undulator quadrant are given by[4]

$$B_r = B_{0q} \exp(-k_u r) \cos(k_u (z - z_0)) \tag{11}$$

where r is measured away from the quadrant and z is measured along the quadrant. If the quadrant moves away from the beam axis by Δr_q, the field changes by

$$\frac{\Delta B_r}{B_r} = -k_u \Delta r_q \tag{12}$$

Positive position changes of the quadrant make the magnets move farther from the beam axis, decreasing the field from the quadrant.

In the assembled undulator in planar, vertical field mode, the vertical field is given by

$$B_y = \frac{4 B_r}{\sqrt{2}} \tag{13}$$

so

$$\left(\frac{\Delta B_y}{B_y}\right)_{undulator} = \frac{\Delta B_r}{B_r} = -k_u \Delta r_q \tag{14}$$

assuming the field contribution is the same for all quadrants.

[4] Z. Wolf, "A Calculation Of The Fields In The Delta Undulator", LCLS-TN-14-1, January, 2014.

The change in slippage per period in the assembled undulator compared to the case when the quadrants have no taper is then

$$\Delta S_{\lambda_u} = -\frac{\lambda_u}{2\gamma^2} K^2 k_u \Delta r_q \tag{15}$$

and the change in phase per period is

$$\Delta P_{\lambda_u} = -2\pi \frac{K^2}{(1+\frac{1}{2}K^2)} k_u \Delta r_q \tag{16}$$

For the Delta undulator, the period is 0.032 m. Inserting a value of $K = 3$ and a period position change of $\Delta r_q = 10$ microns, we get a phase change of -1.16 degree per period. This difference will accumulate as one moves down the undulator.

Suppose the undulator has length L and we take $z = 0$ at the undulator center. The accumulated slippage change along the undulator from initial position $z = -L/2$ where the slippage change is zero, to an arbitrary z is given by

$$\Delta S(z) = \int_{-L/2}^{z} -\frac{\lambda_u}{2\gamma^2} K^2 k_u \Delta r_q(z') \frac{dz'}{\lambda_u} \tag{17}$$

We assume the quadrants taper in a constant manner. We take the position to be correct at the undulator center, and to be large by $\Delta r_{q\,\max}$ at the entrance end. The radial position of the magnet array in the undulator is given by

$$\Delta r_q(z') = -\Delta r_{q\,\max} \left(\frac{z'}{L/2}\right) \tag{18}$$

From the undulator entrance to an arbitrary location z, the taper causes a slippage change in the undulator compared to the untapered case of

$$\Delta S(z) = \frac{\lambda_u}{2\gamma^2} K^2 \frac{k_u}{\lambda_u} \Delta r_{q\,\max} \frac{1}{L} \left(z^2 - \left(\frac{L}{2}\right)^2\right) \tag{19}$$

and the corresponding phase change is

$$\Delta P(z) = 2\pi \frac{K^2}{(1+\frac{1}{2}K^2)} \frac{k_u}{\lambda_u} \Delta r_{q\,\max} \frac{1}{L} \left(z^2 - \left(\frac{L}{2}\right)^2\right) \tag{20}$$

Without errors, the undulator has constant K value along its length. The slippage as a function of z is given by

$$S_0(z) = \frac{1}{2\gamma^2} \left(1 + \frac{1}{2}K^2\right) z \tag{21}$$

With the errors, the slippage as a function of z becomes $S = S_0 + \Delta S$:

$$S(z) = \frac{1}{2\gamma^2} \left(1 + \frac{1}{2}K^2\right) z + \frac{\lambda_u}{2\gamma^2} K^2 \frac{k_u}{\lambda_u} \Delta r_{q\,\max} \frac{1}{L} \left(z^2 - \left(\frac{L}{2}\right)^2\right) \tag{22}$$

and the phase as a function of z is

$$P(z) = \frac{2\pi}{\lambda_u} z + 2\pi \frac{K^2}{(1+\frac{1}{2}K^2)} \frac{k_u}{\lambda_u} \Delta r_{q\,\max} \frac{1}{L} \left(z^2 - \left(\frac{L}{2}\right)^2\right) \tag{23}$$

In order to determine an effective K value for the undulator, and also the rms phase error in the undulator, a linear fit is made to the slippage as a function of z. The linear fit has slope M and the slope gives the effective K value, K_{eff}, as follows:

$$M = \frac{1}{2\gamma^2}\left(1 + \frac{1}{2}K_{eff}^2\right) \tag{24}$$

and

$$K_{eff} = \sqrt{2\left(2\gamma^2 M - 1\right)} \tag{25}$$

The residuals of the fit give the phase errors.

$$\epsilon_\phi = \frac{2\pi}{\lambda_r} \times residuals \tag{26}$$

where in this case we use the measured effective K value to determine the radiation wavelength

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{1}{2}K_{eff}^2\right) \tag{27}$$

The Delta undulator has $\lambda_u = .032$ m, and total length $L = 96\lambda_u$ which is approximately 3.1 m. Take $K = 3$, and $\gamma = 10^4$ although the value of γ drops out of the phase calculations. We take $\Delta r_{q\,\max} = 20$ microns, so the taper is 40 μm over the 3.1 m undulator length. This taper gives phase errors similar to the measured values. The phase in the undulator both without errors and with scaled errors multiplied by a factor of 10 for illustration is shown in figure 4. As you can see,

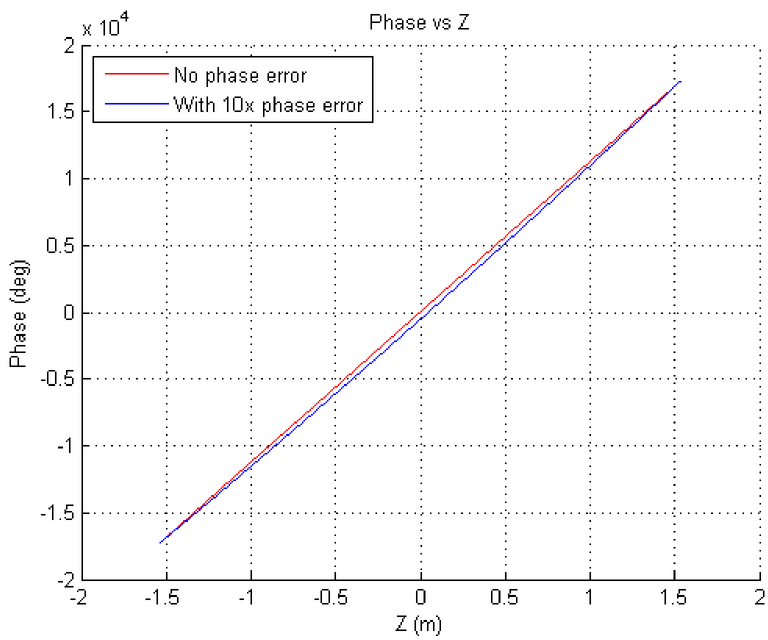

Figure 4: Phase in the undulator both without errors and with errors multiplied by a factor of 10.

the phase change from the taper is small on the scale of the total phase change, but it is visible at the undulator center. The change in phase is shown by itself in figure 5. These are the actual changes and the previous factor of 10 scaling is not included here or from this point on. When a

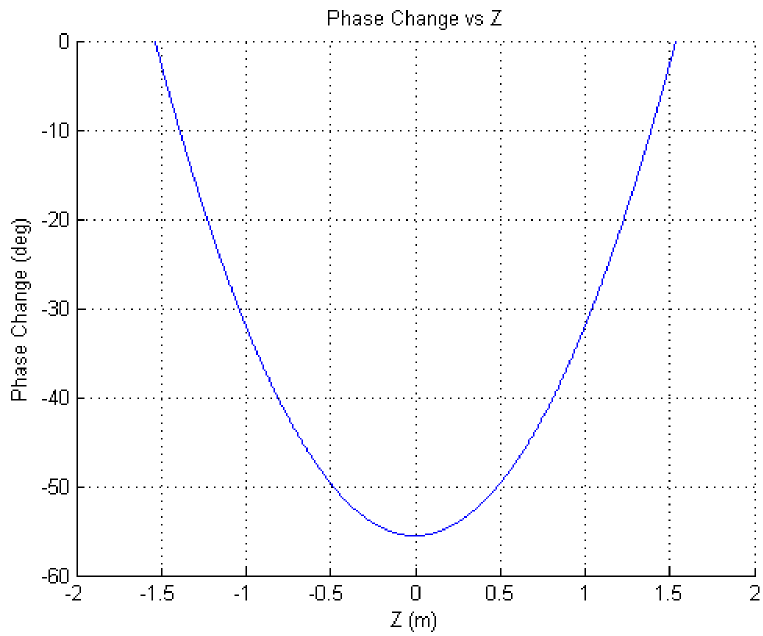

Figure 5: Change in phase from a taper in the magnet arrays of 40 μm over the 3.1 m undulator length.

linear fit is made to the slippage as a function of z, the K value does not change from its value of 3, and the residuals of the fit give the phase errors. The phase errors are shown in figure 6. The rms phase error is 16.6 degrees. The rms phase error scales linearly with the size of the taper. If we wish to limit the rms phase errors from taper to 5 degrees, the quadrants must have a taper of less than 12 μm per 3.1 m. This is a very tight tolerance. The magnet keepers have a reference surface machined into them. That surface must be straight and level with the bench during tuning to better than 12 μm per 3.1 m. Then the reference surface must be placed parallel to the beam axis in the radial direction in the assembled undulator to better than 12 μm per 3.1 m. We assume the quadrant pitch error during tuning and the quadrant pitch error during assembly are uncorrelated and their errors add in quadrature. In this case both errors should be reduced by $1/\sqrt{2}$ to 8 μm per 3.1 m.

4 Conclusion

The Delta undulator is very susceptible to pitch of the quadrants both on the tuning bench and radial pitch of the quadrants in the assembled undulator. If we are to keep rms phase errors from taper below 5 degrees, we must keep the quadrant pitch below 8 μm over the undulator length on the tuning bench, and the radial pitch of the quadrants below 8 μm over the undulator length in the assembled undulator.

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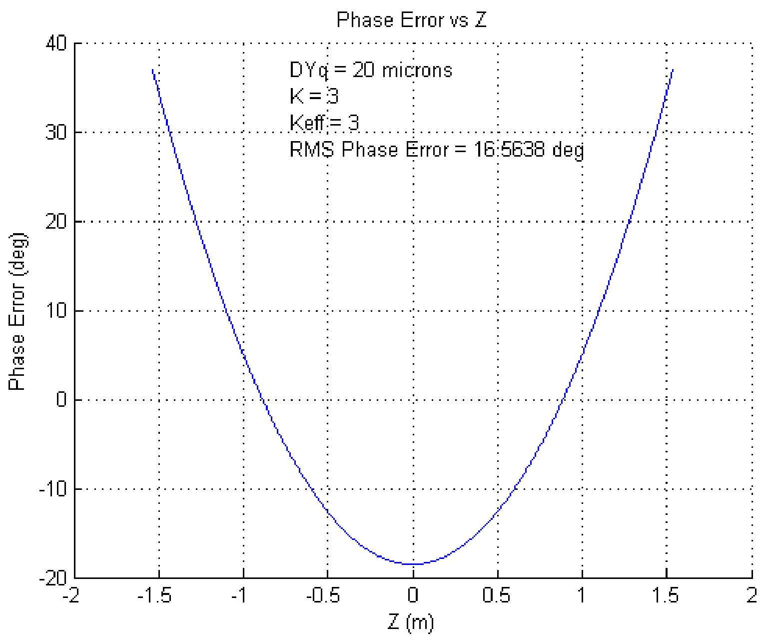

Figure 6: Phase error and K shift when the undulator is assembled with magnet array tapers of 40 μm over the 3.1 m undulator length.