

A Calculation Of The Fields In The Delta Undulator

Zachary Wolf
SLAC

January 27, 2014

Abstract

This note provides a calculation of the fields in the Delta undulator based on superposition of the fields from the four quadrants. The scalar potential for a single quadrant in a standard orientation is first calculated. Approximations are made to consider only the single dominant term. This approximate scalar potential is used to then calculate the scalar potential for a rotated quadrant in a fixed global coordinate system. The scalar potential for the quadrants of the undulator are calculated and the sum gives the scalar potential for the undulator. Arbitrary longitudinal positions for each quadrant are included. The fields in the assembled undulator for different polarization modes and different K settings are presented.

1 Introduction¹

SLAC is building a Delta undulator² which will be placed in the LCLS beam line to produce light with variable polarization. A 1 meter prototype has been constructed and a 3.2 meter full length device is being built. The undulator must be operated in different polarization modes and at different K values within each mode. This note presents calculations of the fields in the undulator in various polarization modes at arbitrary K values. It shows how to set the row positions in order to set the undulator mode and K value.

2 Scalar Potential For An Undulator Quadrant

2.1 Standard Orientation

We start by finding the magnetic scalar potential for a single undulator quadrant in a standard orientation. The quadrant in the standard orientation and the coordinate system are illustrated in figure 1. The origin of the coordinate system in x and y is at the position of the beam axis of the assembled undulator relative to the quadrant. The magnetic field above the quadrant obeys the following Maxwell equations.

$$\nabla \times B = 0 \tag{1}$$

$$\nabla \cdot B = 0 \tag{2}$$

Since the curl of B is zero, B can be expressed in terms of a scalar potential, $B = \nabla\phi$. The scalar potential obeys Laplace's equation.

$$\nabla^2\phi = 0 \tag{3}$$

¹Work supported in part by the DOE Contract DE-AC02-76SF00515. This work was performed in support of the LCLS project at SLAC.

²A. Temnykh, Physical Review Special Topics-Accelerators and Beams **11**, 120702 (2008).

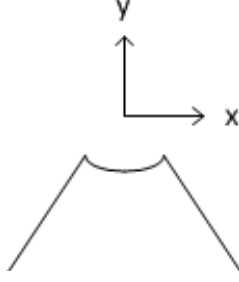


Figure 1: Undulator quadrant in the standard orientation.

We use separation of variables to solve this equation.

$$\phi = X(x)Y(y)Z(z) \quad (4)$$

Laplace's equation becomes

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = 0 \quad (5)$$

Since the sum of terms each containing a different independent variable is zero, each term must be a constant, and the sum of the constants must be zero.

$$\frac{X''}{X} = k_x^2 \quad (6)$$

$$\frac{Y''}{Y} = k_y^2 \quad (7)$$

$$\frac{Z''}{Z} = k_z^2 \quad (8)$$

$$k_x^2 + k_y^2 + k_z^2 = 0 \quad (9)$$

We only consider solutions periodic in z with multiples of the quadrant period, so k_z is imaginary.

$$Z_n(z) = c_3(n) \cos(nk_u z) + c_4(n) \sin(nk_u z) \quad (10)$$

where n is an integer, $c_3(n)$ and $c_4(n)$ are constants that depend on n , and $k_u = 2\pi/\lambda_u$ where λ_u is the undulator period. The X and Y terms have no periodicity requirements, however, the potential must be left-right symmetric, and the potential must decrease as one moves away from the quadrant. X and Y must then have the following form.

$$X_{k_x}(x) = c_1(k_x) \cosh(k_x x) \quad (11)$$

$$Y_{k_y}(y) = c_2(k_y) \exp(-k_y y) \quad (12)$$

k_x can be real or imaginary. k_y must be real and positive. The general form of the potential is

$$\begin{aligned} \phi = & \sum_{n=0}^{\infty} \iint_{\Omega} dk_x dk_y \{ [c_1(k_x) \cosh(k_x x)] [c_2(k_y) \exp(-k_y y)] \\ & \times [c_3(n) \cos(nk_u z) + c_4(n) \sin(nk_u z)] \} \end{aligned} \quad (13)$$

where the region of integration Ω is over all values of k_x and k_y , real and complex, that satisfy the constraints listed above, and that also satisfy the following constraint equation.

$$k_x^2 + k_y^2 - n^2 k_u^2 = 0 \quad (14)$$

We will only consider this solution near $x = 0$ and $y = 0$. We approximate the field by using only the dominant term in this region. It will contain the first harmonic with $n = 1$. So

$$Z(z) = c_3 \cos(k_u z) + c_4 \sin(k_u z) \quad (15)$$

The values of k_x and k_y are still free within the constraints. We assume, however, that the field variation in x is much smaller than the variation in y , or equivalently that $|k_x| \ll k_y$. In order to simplify the equations, we take $k_x = 0$. In this case, $k_y = k_u$. With these considerations, the scalar potential has the form

$$\phi = \phi_0 \exp(-k_u y) \cos(k_u (z - z_0)) \quad (16)$$

where ϕ_0 is a constant and we have combined the $\cos(k_u z)$ and $\sin(k_u z)$ terms into the single $\cos(k_u (z - z_0))$ term. We use this form of the potential to calculate the magnetic scalar potential in the undulator by rotating the quadrants and superposing their potentials.

2.2 Scalar Potential For A Rotated Quadrant

In order to find the scalar potential in the undulator, we must rotate each quadrant to the proper orientation. A quadrant rotated about the beam axis is shown on the right side of figure 2. In

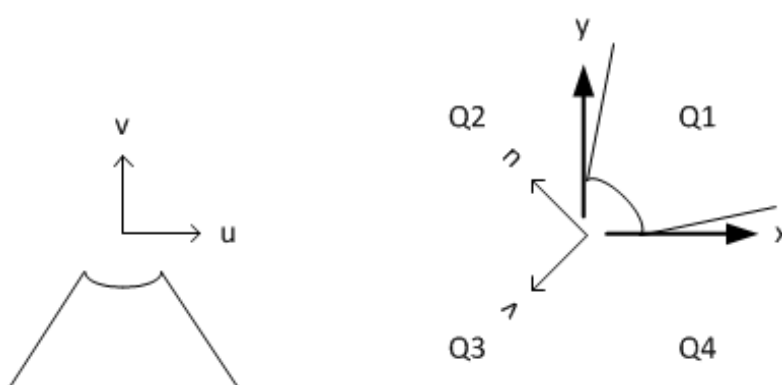


Figure 2: A quadrant in the standard orientation is shown on the left with coordinates attached to the quadrant. The scalar potential for the standard orientation is used to calculate the scalar potential when the quadrant is rotated as shown on the right for the quadrant 1 position.

the figure, the quadrant is shown in the standard orientation on the left. The coordinate axes have been relabeled u and v so that x and y can be used for the global system. From equation 16, the scalar potential for a quadrant in the standard orientation in terms of u and v is

$$\phi_{qs}(u, v, z) = \phi_{0q} \exp(-k_u v) \cos(k_u (z - z_0)) \quad (17)$$

When we rotate a quadrant from the standard orientation to the rotated orientation in the undulator, the scalar potential obeys the equation

$$\phi_q(x, y, z) = \phi_{qs}(u, v, z) \quad (18)$$

where x and y refer to the same location as u and v , and the subscript s refers to the standard orientation.

When the quadrant is rotated about the beam axis by an angle θ in the counter clockwise sense, the u, v coordinates rotate with it. The x, y coordinates are fixed and make a global system. The

relation between the two sets of coordinates is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \quad (19)$$

The inverse relation is

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (20)$$

The scalar potential for the rotated quadrant is then given by

$$\phi_q(x, y, z) = \phi_{0q} \exp(-k_u(-x \sin \theta + y \cos \theta)) \cos(k_u(z - z_0)) \quad (21)$$

2.3 Scalar Potentials For The Four Undulator Quadrants

The rotation angle to go from the standard orientation to the quadrant 1 orientation is $\theta = 135^\circ$. For quadrant 2, the rotation angle is $\theta = -135^\circ$. For quadrant 3, the rotation angle is $\theta = -45^\circ$. For quadrant 4, the rotation angle is $\theta = 45^\circ$. The scalar potentials for the 4 quadrants are

$$\phi_1(x, y, z) = \phi_{01} \exp\left(-\frac{k_u}{\sqrt{2}}(-x - y)\right) \cos(k_u(z - z_{01})) \quad (22)$$

$$\phi_2(x, y, z) = \phi_{02} \exp\left(-\frac{k_u}{\sqrt{2}}(x - y)\right) \cos(k_u(z - z_{02})) \quad (23)$$

$$\phi_3(x, y, z) = \phi_{03} \exp\left(-\frac{k_u}{\sqrt{2}}(x + y)\right) \cos(k_u(z - z_{03})) \quad (24)$$

$$\phi_4(x, y, z) = \phi_{04} \exp\left(-\frac{k_u}{\sqrt{2}}(-x + y)\right) \cos(k_u(z - z_{04})) \quad (25)$$

The quadrants are built identically except that quadrants 3 and 4 have the opposite polarity magnets compared to quadrants 1 and 2. So $\phi_{01} = \phi_{02} = -\phi_{03} = -\phi_{04} \equiv \phi_{0Q}$. The scalar potentials are then

$$\phi_1(x, y, z) = \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x - y)\right) \cos(k_u(z - z_{01})) \quad (26)$$

$$\phi_2(x, y, z) = \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x - y)\right) \cos(k_u(z - z_{02})) \quad (27)$$

$$\phi_3(x, y, z) = -\phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x + y)\right) \cos(k_u(z - z_{03})) \quad (28)$$

$$\phi_4(x, y, z) = -\phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x + y)\right) \cos(k_u(z - z_{04})) \quad (29)$$

It is noteworthy that quadrants 1 and 3 both have the same $(x + y)$ dependence, and quadrants 2 and 4 both depend on $(x - y)$. This leads us to first add the scalar potentials for quadrants 1 and 3, and then add the scalar potentials for quadrants 2 and 4, and then add the sums to get the scalar potential for the whole undulator. We will see that this can be interpreted as forming the entire undulator from two crossed planar undulators.

2.4 Scalar Potential For The Quadrant 1 And Quadrant 3 Pair

Consider the first undulator subset made of quadrants 1 and 3. The scalar potential for this combination is given by

$$\phi_{13} = \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x - y)\right) \cos(k_u(z - z_{01})) - \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x + y)\right) \cos(k_u(z - z_{03})) \quad (30)$$

Let

$$z_{01} = Z_{13} + \frac{\Delta_{13}}{2} \quad (31)$$

$$z_{03} = Z_{13} - \frac{\Delta_{13}}{2} \quad (32)$$

So

$$Z_{13} = \frac{z_{01} + z_{03}}{2} \quad (33)$$

is the average z-position of the quadrants, and

$$\Delta_{13} = z_{01} - z_{03} \quad (34)$$

is the z-shift between the quadrants.

With these definitions, the scalar potential for the pair of quadrants becomes

$$\begin{aligned} \phi_{13} = & \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x-y)\right) \cos\left(k_u(z-Z_{13}) - k_u \frac{\Delta_{13}}{2}\right) \\ & - \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x+y)\right) \cos\left(k_u(z-Z_{13}) + k_u \frac{\Delta_{13}}{2}\right) \end{aligned} \quad (35)$$

Expanding the cosine terms, this becomes

$$\begin{aligned} \phi_{13} = & \left[\phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x-y)\right) - \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x+y)\right) \right] \\ & \times \cos(k_u(z-Z_{13})) \cos\left(k_u \frac{\Delta_{13}}{2}\right) \end{aligned} \quad (36)$$

$$+ \left[\phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x-y)\right) + \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x+y)\right) \right] \quad (37)$$

$$\times \sin(k_u(z-Z_{13})) \sin\left(k_u \frac{\Delta_{13}}{2}\right) \quad (38)$$

Simplifying, we get

$$\begin{aligned} \phi_{13} = & 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u(z-Z_{13})) \cos\left(k_u \frac{\Delta_{13}}{2}\right) \\ & + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u(z-Z_{13})) \sin\left(k_u \frac{\Delta_{13}}{2}\right) \end{aligned} \quad (39)$$

This is the scalar potential for a rotated, planar, adjustable phase undulator.

2.5 Scalar Potential For The Quadrant 2 And Quadrant 4 Pair

The scalar potential for the combination of quadrants 2 and 4 is given by

$$\phi_{24} = \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u(z-z_{02})) - \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x+y)\right) \cos(k_u(z-z_{04})) \quad (40)$$

Let

$$z_{02} = Z_{24} + \frac{\Delta_{24}}{2} \quad (41)$$

$$z_{04} = Z_{24} - \frac{\Delta_{24}}{2} \quad (42)$$

So

$$Z_{24} = \frac{z_{02} + z_{04}}{2} \quad (43)$$

is the average z-position of the quadrants, and

$$\Delta_{24} = z_{02} - z_{04} \quad (44)$$

is the z-shift between the quadrants.

With these definitions, the scalar potential for the pair of quadrants becomes

$$\begin{aligned} \phi_{24} &= \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x-y)\right) \cos\left(k_u(z-Z_{24}) - k_u \frac{\Delta_{24}}{2}\right) \\ &\quad - \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x+y)\right) \cos\left(k_u(z-Z_{24}) + k_u \frac{\Delta_{24}}{2}\right) \end{aligned} \quad (45)$$

Expanding the cosine terms, this becomes

$$\begin{aligned} \phi_{24} &= \left[\phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x-y)\right) - \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x+y)\right) \right] \\ &\quad \times \cos(k_u(z-Z_{24})) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \end{aligned} \quad (46)$$

$$\begin{aligned} &+ \left[\phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(x-y)\right) + \phi_{0Q} \exp\left(-\frac{k_u}{\sqrt{2}}(-x+y)\right) \right] \\ &\quad \times \sin(k_u(z-Z_{24})) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \end{aligned} \quad (47)$$

Simplifying, we get

$$\begin{aligned} \phi_{24} &= -2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u(z-Z_{24})) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \\ &\quad + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u(z-Z_{24})) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \end{aligned} \quad (48)$$

This is the scalar potential for a rotated, planar, adjustable phase undulator.

2.6 Scalar Potential For The Assembled Undulator

The scalar potential for the undulator is the sum of the scalar potentials for the quadrant pairs.

$$\phi = \phi_{13} + \phi_{24} \quad (49)$$

Performing the sum, we find

$$\begin{aligned} \phi &= 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u(z-Z_{13})) \cos\left(k_u \frac{\Delta_{13}}{2}\right) \\ &\quad + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u(z-Z_{13})) \sin\left(k_u \frac{\Delta_{13}}{2}\right) \\ &\quad - 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u(z-Z_{24})) \cos\left(k_u \frac{\Delta_{24}}{2}\right) \\ &\quad + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u(z-Z_{24})) \sin\left(k_u \frac{\Delta_{24}}{2}\right) \end{aligned} \quad (50)$$

By putting in the various values for Z_{13} , Z_{24} , Δ_{13} , and Δ_{24} , we get the scalar potential for the various undulator modes set to different K values.

3 Scalar Potential In The Various Undulator Modes

3.1 Linear Polarization Vertical Field

Consider the linear polarization vertical field mode. The magnet arrays in the maximum K configuration are illustrated in figure 3. The undulator is built so that when all the quadrants are in their zero position, the undulator is in this mode at maximum K . We wish to consider the general

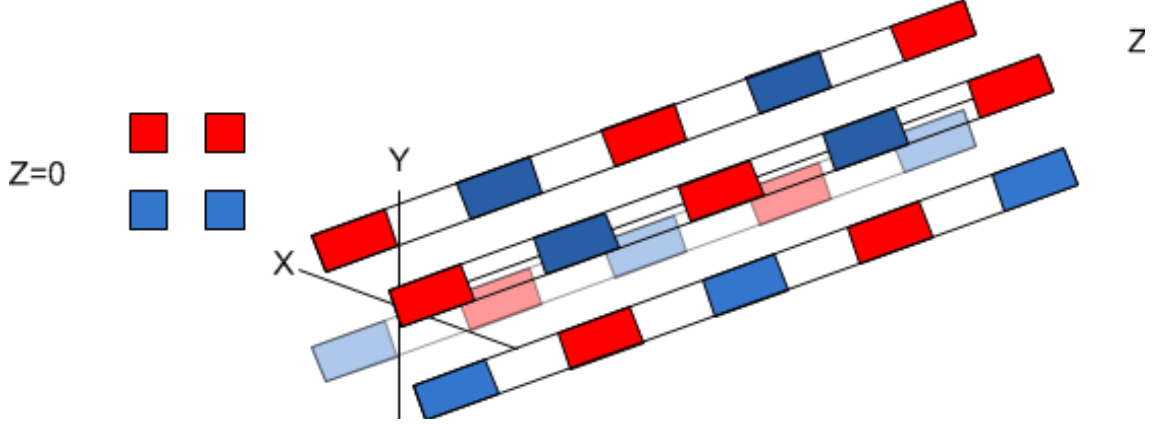


Figure 3: Linear polarization vertical field mode. The undulator is in the maximum K configuration.

case where we change the undulator K value. We keep the average row positions at zero, but we put in an offset in the quadrant 1 and 3 pair, and an equal offset in the quadrant 2 and 4 pair. In this case, $Z_{13} = 0$, $Z_{24} = 0$, $\Delta_{13} = \Delta_{24} = \Delta$. The individual rows are at

$$z_{01} = \frac{\Delta}{2} \quad (51)$$

$$z_{02} = \frac{\Delta}{2} \quad (52)$$

$$z_{03} = -\frac{\Delta}{2} \quad (53)$$

$$z_{04} = -\frac{\Delta}{2} \quad (54)$$

The scalar potential is

$$\begin{aligned} \phi = & 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\ & + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \\ & - 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\ & + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \end{aligned} \quad (55)$$

Expand this expression using the following identities.

$$\sinh(a+b) = \sinh(a) \cosh(b) + \cosh(a) \sinh(b) \quad (56)$$

$$\cosh(a+b) = \cosh(a) \cosh(b) + \sinh(a) \sinh(b) \quad (57)$$

Expanding the scalar potential gives

$$\begin{aligned} \phi &= 2\phi_{0Q} \left[\sinh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) + \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \right. \\ &\quad \left. - \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) + \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \right] \\ &\quad \times \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \end{aligned} \quad (58)$$

$$\begin{aligned} &+ 2\phi_{0Q} \left[\cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) + \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \right. \\ &\quad \left. + \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) - \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \right] \end{aligned} \quad (59)$$

$$\times \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \quad (60)$$

Simplifying, we get

$$\begin{aligned} \phi &= 4\phi_{0Q} \cos\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \cos(k_u z) \\ &\quad + 4\phi_{0Q} \sin\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \sin(k_u z) \end{aligned} \quad (61)$$

Letting $\phi_0 = 4\phi_{0Q}$, the fields are

$$\begin{aligned} B_x &= \phi_0 \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \cos(k_u z) \\ &\quad + \phi_0 \frac{k_u}{\sqrt{2}} \sin\left(k_u \frac{\Delta}{2}\right) \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \sin(k_u z) \end{aligned} \quad (62)$$

$$\begin{aligned} B_y &= \phi_0 \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \cos(k_u z) \\ &\quad + \phi_0 \frac{k_u}{\sqrt{2}} \sin\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \sin(k_u z) \end{aligned} \quad (63)$$

$$\begin{aligned} B_z &= -\phi_0 k_u \cos\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \sin(k_u z) \\ &\quad + \phi_0 k_u \sin\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \cos(k_u z) \end{aligned} \quad (64)$$

On the magnetic axis where $x = 0$ and $y = 0$, the fields are

$$B_x = 0 \quad (65)$$

$$B_y = \phi_0 \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \cos(k_u z) \quad (66)$$

$$B_z = \phi_0 k_u \sin\left(k_u \frac{\Delta}{2}\right) \cos(k_u z) \quad (67)$$

This is the field of a planar undulator with the magnetic field in the y-direction. B_y has maximum value with $\Delta = 0$ and it decreases with increasing Δ . B_z is zero when $\Delta = 0$, and it increases with increasing Δ . The K value is proportional to B_y , so it varies with Δ as $\cos\left(k_u \frac{\Delta}{2}\right)$.

3.2 Linear Polarization Horizontal Field

The magnet arrays for linear polarization horizontal field mode in the maximum K configuration are illustrated in figure 4. The quadrant 2 and 4 pair are shifted by half a period to make the field

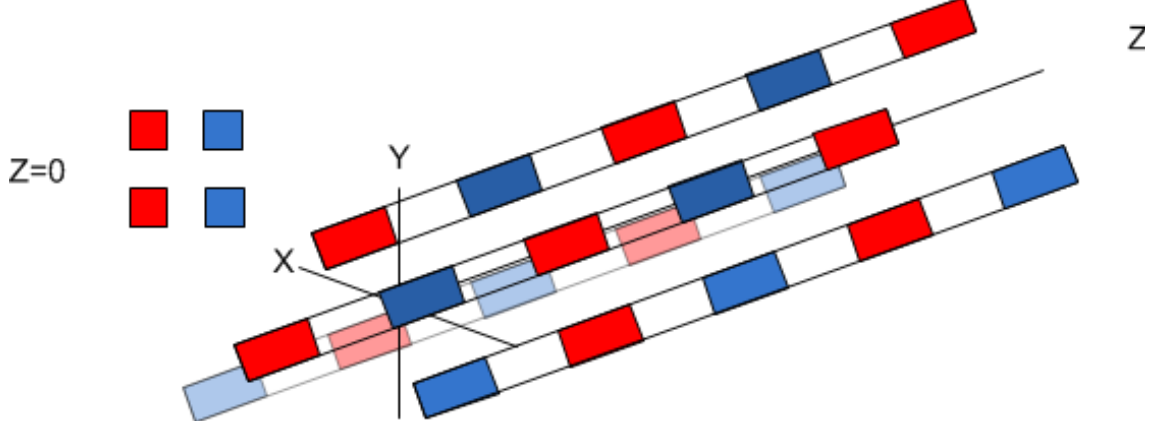


Figure 4: Linear polarization horizontal field mode. The undulator is in the maximum K configuration.

horizontal at $z = 0$. To change K , we put in an offset in the quadrant 1 and 3 pair, and an equal offset in the quadrant 2 and 4 pair. In this case we want quadrant 1 and quadrant 4 to shift in the same direction. With these considerations, $Z_{13} = 0$, $Z_{24} = -\frac{\lambda_u}{2}$, $\Delta_{13} = \Delta$, and $\Delta_{24} = -\Delta$. The individual rows are at

$$z_{01} = \frac{\Delta}{2} \quad (68)$$

$$z_{02} = -\frac{\lambda_u}{2} - \frac{\Delta}{2} \quad (69)$$

$$z_{03} = -\frac{\Delta}{2} \quad (70)$$

$$z_{04} = -\frac{\lambda_u}{2} + \frac{\Delta}{2} \quad (71)$$

From equation 50, the scalar potential is

$$\begin{aligned} \phi = & 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\ & + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \\ & - 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos\left(k_u \left(z + \frac{\lambda_u}{2}\right)\right) \cos\left(-k_u \frac{\Delta}{2}\right) \\ & + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin\left(k_u \left(z + \frac{\lambda_u}{2}\right)\right) \sin\left(-k_u \frac{\Delta}{2}\right) \end{aligned} \quad (72)$$

Since $k_u \frac{\lambda_u}{2} = \pi$, the scalar potential becomes

$$\begin{aligned}
\phi &= 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\
&\quad + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \\
&\quad + 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\
&\quad + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right)
\end{aligned} \tag{73}$$

Expand this expression using the following identities.

$$\sinh(a+b) = \sinh(a) \cosh(b) + \cosh(a) \sinh(b) \tag{74}$$

$$\cosh(a+b) = \cosh(a) \cosh(b) + \sinh(a) \sinh(b) \tag{75}$$

Expanding the scalar potential gives

$$\begin{aligned}
\phi &= 2\phi_{0Q} \left[\sinh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) + \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \right. \\
&\quad \left. + \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) - \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \right] \\
&\quad \times \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right)
\end{aligned} \tag{76}$$

$$\begin{aligned}
&+ 2\phi_{0Q} \left[\cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) + \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \right. \\
&\quad \left. + \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) - \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \right]
\end{aligned} \tag{77}$$

$$\times \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \tag{78}$$

Simplifying, we get

$$\begin{aligned}
\phi &= 4\phi_{0Q} \cos\left(k_u \frac{\Delta}{2}\right) \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \cos(k_u z) \\
&\quad + 4\phi_{0Q} \sin\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \sin(k_u z)
\end{aligned} \tag{79}$$

Letting $\phi_0 = 4\phi_{0Q}$, the fields are

$$\begin{aligned} B_x &= \phi_0 \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \cos(k_u z) \\ &\quad + \phi_0 \frac{k_u}{\sqrt{2}} \sin\left(k_u \frac{\Delta}{2}\right) \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \sin(k_u z) \end{aligned} \quad (80)$$

$$\begin{aligned} B_y &= \phi_0 \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \cos(k_u z) \\ &\quad + \phi_0 \frac{k_u}{\sqrt{2}} \sin\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \sinh\left(\frac{k_u}{\sqrt{2}}y\right) \sin(k_u z) \end{aligned} \quad (81)$$

$$\begin{aligned} B_z &= -\phi_0 k_u \cos\left(k_u \frac{\Delta}{2}\right) \sinh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \sin(k_u z) \\ &\quad + \phi_0 k_u \sin\left(k_u \frac{\Delta}{2}\right) \cosh\left(\frac{k_u}{\sqrt{2}}x\right) \cosh\left(\frac{k_u}{\sqrt{2}}y\right) \cos(k_u z) \end{aligned} \quad (82)$$

On the magnetic axis where $x = 0$ and $y = 0$, the fields are

$$B_x = \phi_0 \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \cos(k_u z) \quad (83)$$

$$B_y = 0 \quad (84)$$

$$B_z = \phi_0 k_u \sin\left(k_u \frac{\Delta}{2}\right) \cos(k_u z) \quad (85)$$

This is the field of a planar undulator with the magnetic field in the x-direction. B_x has maximum value with $\Delta = 0$ and it decreases with increasing Δ . B_z is zero when $\Delta = 0$, and it increases with increasing Δ . The K value is proportional to B_x , so it varies with Δ as $\cos\left(k_u \frac{\Delta}{2}\right)$.

3.3 Right Hand Circular Polarization

In right hand circular polarization mode at maximum K , the arrays are arranged as shown in figure 5. The field rotates in a right handed sense as one moves down the undulator. At maximum K , the quadrant 2 and quadrant 4 pair are shifted in z by $\lambda_u/4$. In the general case where we change the undulator K value, we make relative shifts in the quadrant 1 and quadrant 3 pair, and also in the quadrant 2 and quadrant 4 pair. The relative shifts for the pairs are equal, and we move quadrant 1 and quadrant 2 in the same direction. With these considerations, $Z_{13} = 0$, $Z_{24} = \lambda_u/4$, $\Delta_{13} = \Delta$, and $\Delta_{24} = \Delta$. The individual rows are at

$$z_{01} = \frac{\Delta}{2} \quad (86)$$

$$z_{02} = \frac{\lambda_u}{4} + \frac{\Delta}{2} \quad (87)$$

$$z_{03} = -\frac{\Delta}{2} \quad (88)$$

$$z_{04} = \frac{\lambda_u}{4} - \frac{\Delta}{2} \quad (89)$$

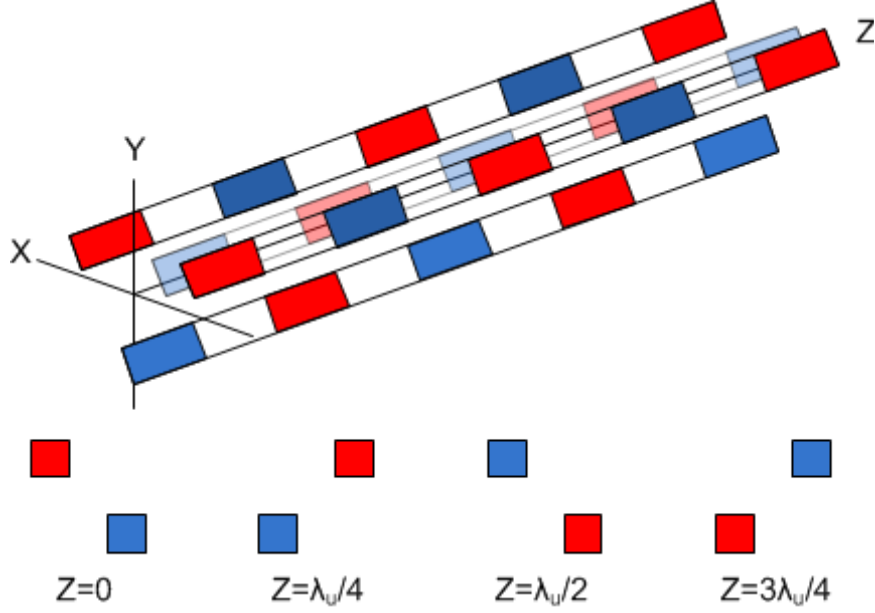


Figure 5: Configuration of the undulator in right hand circular polarization mode.

From equation 50, the scalar potential is

$$\begin{aligned}
\phi &= 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\
&+ 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \\
&- 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos\left(k_u \left(z - \frac{\lambda_u}{4}\right)\right) \cos\left(k_u \frac{\Delta}{2}\right) \\
&+ 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin\left(k_u \left(z - \frac{\lambda_u}{4}\right)\right) \sin\left(k_u \frac{\Delta}{2}\right)
\end{aligned} \tag{90}$$

Since $k_u \frac{\lambda_u}{4} = \frac{\pi}{2}$, the scalar potential becomes

$$\begin{aligned}
\phi &= 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\
&+ 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \\
&- 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\
&- 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \sin\left(k_u \frac{\Delta}{2}\right)
\end{aligned} \tag{91}$$

Let $\phi_0 = 4\phi_{0Q}$ as for the linear modes. Dividing the scalar potential into a term that goes as

$\cos(k_u \frac{\Delta}{2})$ and a term that goes as $\sin(k_u \frac{\Delta}{2})$, we get.

$$\begin{aligned} \phi &= \frac{\phi_0}{2} \cos\left(k_u \frac{\Delta}{2}\right) \left[\sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) - \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \right] \\ &+ \frac{\phi_0}{2} \sin\left(k_u \frac{\Delta}{2}\right) \left[\cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) - \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \right] \end{aligned} \quad (92)$$

The fields are

$$\begin{aligned} B_x &= \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \left[\cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) - \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \right] \\ &+ \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \sin\left(k_u \frac{\Delta}{2}\right) \left[\sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) - \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \right] \end{aligned} \quad (93)$$

$$\begin{aligned} B_y &= \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \left[\cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) + \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \right] \\ &+ \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \sin\left(k_u \frac{\Delta}{2}\right) \left[\sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) + \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \right] \end{aligned} \quad (94)$$

$$\begin{aligned} B_z &= \frac{\phi_0}{2} k_u \cos\left(k_u \frac{\Delta}{2}\right) \left[-\sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) - \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \right] \\ &+ \frac{\phi_0}{2} k_u \sin\left(k_u \frac{\Delta}{2}\right) \left[\cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) + \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \right] \end{aligned} \quad (95)$$

The fields on the $x = 0, y = 0$ axis are

$$B_x = \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) [\cos(k_u z) - \sin(k_u z)] \quad (96)$$

$$B_y = \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) [\cos(k_u z) + \sin(k_u z)] \quad (97)$$

$$B_z = \frac{\phi_0}{2} k_u \sin\left(k_u \frac{\Delta}{2}\right) [\cos(k_u z) + \sin(k_u z)] \quad (98)$$

Simplifying, we find

$$B_x = \frac{\phi_0}{2} k_u \cos\left(k_u \frac{\Delta}{2}\right) \cos\left(k_u z + \frac{\pi}{4}\right) \quad (99)$$

$$B_y = \frac{\phi_0}{2} k_u \cos\left(k_u \frac{\Delta}{2}\right) \sin\left(k_u z + \frac{\pi}{4}\right) \quad (100)$$

$$B_z = \frac{\phi_0}{\sqrt{2}} k_u \sin\left(k_u \frac{\Delta}{2}\right) \sin\left(k_u z + \frac{\pi}{4}\right) \quad (101)$$

The field rotates in a right handed sense as one moves down the undulator. B_x and B_y have the same peak value, so the polarization mode is circular. B_x and B_y go as $\cos(k_u \frac{\Delta}{2})$, so the undulator K value goes as $\cos(k_u \frac{\Delta}{2})$.

3.4 Left Hand Circular Polarization

In left hand circular polarization mode at maximum K , the arrays are arranged as shown in figure 6. The field rotates in a left handed sense as one moves down the undulator. At maximum K , the quadrant 2 and quadrant 4 pair are shifted in z by $-\lambda_u/4$. In the general case where we change the undulator K value, we make relative shifts in the quadrant 1 and quadrant 3 pair, and also in the

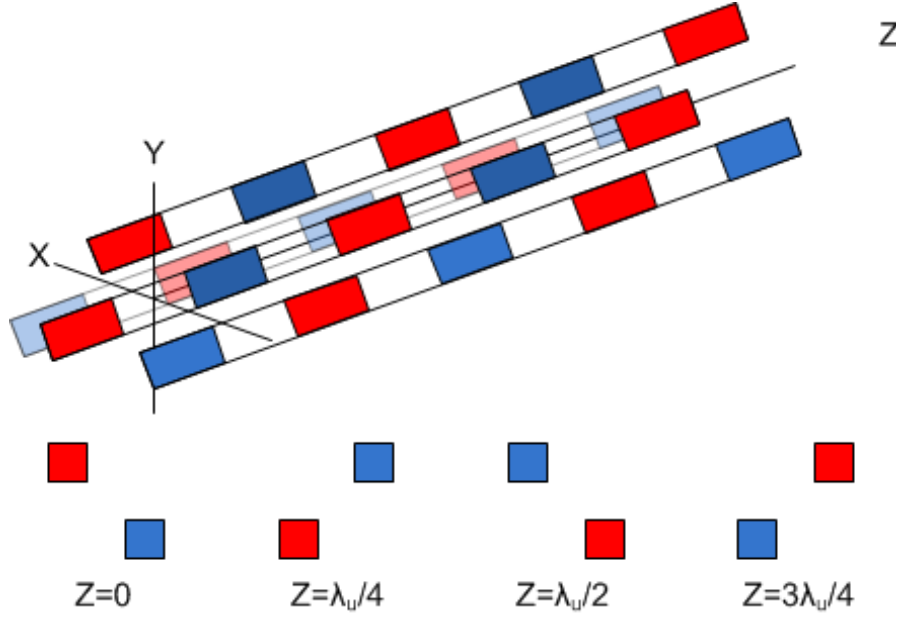


Figure 6: Configuration of the undulator in left hand circular polarization mode.

quadrant 2 and quadrant 4 pair. The relative shifts for the pairs are equal, and we move quadrant 1 and quadrant 2 in the same direction. With these considerations, $Z_{13} = 0$, $Z_{24} = -\lambda_u/4$, $\Delta_{13} = \Delta$, and $\Delta_{24} = \Delta$. The individual rows are at

$$z_{01} = \frac{\Delta}{2} \quad (102)$$

$$z_{02} = -\frac{\lambda_u}{4} + \frac{\Delta}{2} \quad (103)$$

$$z_{03} = -\frac{\Delta}{2} \quad (104)$$

$$z_{04} = -\frac{\lambda_u}{4} - \frac{\Delta}{2} \quad (105)$$

From equation 50, the scalar potential is

$$\begin{aligned} \phi = & 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\ & + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \\ & - 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos\left(k_u \left(z + \frac{\lambda_u}{4}\right)\right) \cos\left(k_u \frac{\Delta}{2}\right) \\ & + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin\left(k_u \left(z + \frac{\lambda_u}{4}\right)\right) \sin\left(k_u \frac{\Delta}{2}\right) \end{aligned} \quad (106)$$

Since $k_u \frac{\lambda_u}{4} = \frac{\pi}{2}$, the scalar potential becomes

$$\begin{aligned}
\phi &= 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\
&\quad + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) \sin\left(k_u \frac{\Delta}{2}\right) \\
&\quad + 2\phi_{0Q} \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \cos\left(k_u \frac{\Delta}{2}\right) \\
&\quad + 2\phi_{0Q} \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \sin\left(k_u \frac{\Delta}{2}\right)
\end{aligned} \tag{107}$$

Let $\phi_0 = 4\phi_{0Q}$ as for the linear modes. Dividing the scalar potential into a term that goes as $\cos\left(k_u \frac{\Delta}{2}\right)$ and a term that goes as $\sin\left(k_u \frac{\Delta}{2}\right)$, we get.

$$\begin{aligned}
\phi &= \frac{\phi_0}{2} \cos\left(k_u \frac{\Delta}{2}\right) \left[\sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) + \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \right] \\
&\quad + \frac{\phi_0}{2} \sin\left(k_u \frac{\Delta}{2}\right) \left[\cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) + \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \right]
\end{aligned} \tag{108}$$

The fields are

$$\begin{aligned}
B_x &= \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \left[\cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) + \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \right] \\
&\quad + \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \sin\left(k_u \frac{\Delta}{2}\right) \left[\sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) + \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \right]
\end{aligned} \tag{109}$$

$$\begin{aligned}
B_y &= \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) \left[\cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) - \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \right] \\
&\quad + \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \sin\left(k_u \frac{\Delta}{2}\right) \left[\sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) - \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \right]
\end{aligned} \tag{110}$$

$$\begin{aligned}
B_z &= \frac{\phi_0}{2} k_u \cos\left(k_u \frac{\Delta}{2}\right) \left[-\sinh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \sin(k_u z) + \sinh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \cos(k_u z) \right] \\
&\quad + \frac{\phi_0}{2} k_u \sin\left(k_u \frac{\Delta}{2}\right) \left[\cosh\left(\frac{k_u}{\sqrt{2}}(x+y)\right) \cos(k_u z) - \cosh\left(\frac{k_u}{\sqrt{2}}(x-y)\right) \sin(k_u z) \right]
\end{aligned} \tag{111}$$

The fields on the $x = 0, y = 0$ axis are

$$B_x = \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) [\cos(k_u z) + \sin(k_u z)] \tag{112}$$

$$B_y = \frac{\phi_0}{2} \frac{k_u}{\sqrt{2}} \cos\left(k_u \frac{\Delta}{2}\right) [\cos(k_u z) - \sin(k_u z)] \tag{113}$$

$$B_z = \frac{\phi_0}{2} k_u \sin\left(k_u \frac{\Delta}{2}\right) [\cos(k_u z) - \sin(k_u z)] \tag{114}$$

Simplifying, we find

$$B_x = \frac{\phi_0}{2} k_u \cos\left(k_u \frac{\Delta}{2}\right) \sin\left(k_u z + \frac{\pi}{4}\right) \tag{115}$$

$$B_y = \frac{\phi_0}{2} k_u \cos\left(k_u \frac{\Delta}{2}\right) \cos\left(k_u z + \frac{\pi}{4}\right) \tag{116}$$

$$B_z = \frac{\phi_0}{\sqrt{2}} k_u \sin\left(k_u \frac{\Delta}{2}\right) \cos\left(k_u z + \frac{\pi}{4}\right) \tag{117}$$

The field rotates in a left handed sense as one moves down the undulator. B_x and B_y have the same peak value, so the polarization mode is circular. B_x and B_y go as $\cos\left(k_u \frac{\Delta}{2}\right)$, so the undulator K value goes as $\cos\left(k_u \frac{\Delta}{2}\right)$.

4 Summary

The following table summarizes how to set the undulator quadrants for each polarization mode and for each K value. The K value goes as

$$\frac{K}{K_{\max}} = \cos\left(k_u \frac{\Delta}{2}\right) \quad (118)$$

Solving for Δ , we find

$$\Delta = \frac{2}{k_u} \cos^{-1}\left(\frac{K}{K_{\max}}\right) \quad (119)$$

	z_{01}	z_{02}	z_{03}	z_{04}
Linear Vertical	$\Delta/2$	$\Delta/2$	$-\Delta/2$	$-\Delta/2$
Linear Horizontal	$\Delta/2$	$-\lambda_u/2 - \Delta/2$	$-\Delta/2$	$-\lambda_u/2 + \Delta/2$
Circular Right	$\Delta/2$	$\lambda_u/4 + \Delta/2$	$-\Delta/2$	$\lambda_u/4 - \Delta/2$
Circular Left	$\Delta/2$	$-\lambda_u/4 + \Delta/2$	$-\Delta/2$	$-\lambda_u/4 - \Delta/2$

These quadrant positions are useful for an initial setting of the undulator quadrants. In practice, a detailed map of K value vs row phase in each polarization mode must be used to accurately set the rows.

5 Conclusion

A simple model of the scalar potential for an undulator quadrant was used to calculate the scalar potential for the assembled undulator. This model leads to an intuitive understanding of how to set the row phases to get each polarization mode and to set the K value for each mode.