# LCLS-II Undulator Tolerance Budget*

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## 1 Introduction

This document describes the methods used to calculate the main tolerance budget for the LCLS-II undulator sytems. The basic principles are explained in Ref. [1]. The approach uses the fact that the dependence of FEL performance on many individual parameters can be expressed as a Gaussian function of that parameter. The result of the tolerance budget analysis is shown in Table 1 and Table 2, which list the considered parameters in the second column (“Error Source”). The contents of the tables and the means of determining the entry values are described in this document.

## 2 Concept used for the tolerance budget analysis

The output power of an x-ray SASE FEL can be reduced if any of a number of parameters, $p_i$, deviates from its optimum value. To determine tolerances for these deviations, a budget approach was introduced during the LCLS construction period. The algorithm is based on the fact that the average reduction in output power, $P_i(p_i)$, can be modeled as a Gaussian function of the rms of the individual parameter distributions ($q_{i=}(p_i)_{rms}$) or simple functions $q_{i=}\left(f(p_i)\right)_{rms}$, thereof

$$\frac{P_i}{P_0} = e^{-\frac{q_i^2}{2\,\sigma_i^2}} \tag{1}$$

for $q_i < \sigma_i$. Once the rms performance dependences, $\sigma_i$, are determined, the allowable performance degradations, $P_i/P_0$, for that parameter can be used to determine the tolerance values

$$t_i = \mathrm{f}^{-1}\left(\sigma_\mathrm{i}\sqrt{-2\ln(P_i/P_0)}\right). \tag{2}$$

The individual levels of acceptable performance degradations can be fine-tuned in a performance budget such that the total performance reduction stays within a given limit

$$\frac{P}{P_0} = \prod \frac{P_i}{P_0} = e^{-\frac{1}{2}\Sigma\frac{q_i^2}{\sigma_i^2}} \stackrel{\mathrm{def}}{=} e^{-\frac{1}{2}\Sigma\, r_i^2}. \tag{3}$$

For LCLS, the budget analysis was done for a set of 8 parameters, for which the $\sigma_\mathrm{i}$ were calculated using a statistical analysis based on computer simulations using the FEL simulation code GENESIS 1.3 [2]. Some of the results were later verified with actual measurements at LCLS [3]. For the LCLS-II tolerance analysis, a method of establishing algebraic approximations for the $\sigma_\mathrm{i}$, as described in [1] is used The parameters used are described in the next section. The results of the budget analysis are presented in Table 1 and Table 2. The entries in the “Value” columns have been chosen to produce practical values in the “Tol” columns but, at the same time, keep the total loss as shown on the lower right corner of the table to 25%. As can be seen from the following section, the $\sigma_\mathrm{i}$ are dependent on a number of system parameters. The ones used for the results in

* Work supported in part by the DOE Contract DE-AC02-76SF00515.
This work was performed in support of the LCLS project at SLAC.

Table 1 and Table 2 are listed in Table 4. The analysis was done at electron and photon energies for which the greatest performance impact occurs within the baseline range.

**Table 1. Main tolerance budget for the HXR undulator beamline.**

| *n* | Error Source | rms values | | | budget calculations | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\sigma_t$ | Units | Corr | $r_i$ | Value | Tol | Units | $(\Delta P/P)_i$ |
| 1 | - Launch Angle $x'_0,y'_0$ | 1.88 | μrad | 0.71 | 0.360 | **0.48** | 0.48 | μrad | 93.7% |
| 2 | - $(\Delta K/K)_{rms}$ | 0.00060 | | 1.00 | 0.443 | 0.00026 | 0.00026 | | 90.6% |
| 3 | - Segment misalignment in x | 17527998 | $\mu m^2$ | 1.00 | 0.145 | 254048 | 504 | μm | 99.0% |
| 4 | - Segment misalignment in y | 30915.8 | $\mu m^2$ | 1.00 | 0.262 | **8100** | 90 | μm | 96.6% |
| 5 | - Jaw Pitch [μrad] | 201.7 | μrad | 1.00 | 0.099 | **20** | 20 | μrad | 99.5% |
| 6 | - Quad Position Stability x,y | 4.77 | μm | 0.71 | 0.074 | **0.25** | 0.25 | μm | 99.7% |
| 7 | - Quad Positioning Error x,y | 4.77 | μm | 0.71 | 0.297 | **1.00** | 1.00 | μm | 95.7% |
| 8 | - Break Length Error | 16.8 | mm | 1.00 | 0.059 | **1.00** | 1.00 | mm | 99.8% |
| 9 | - Strongback deflection | 79.**0** | μm | 1.00 | 0.139 | **11.0** | 11.0 | μm | 99.0% |
| 10 | - Phase Shake Error | 16.6 | degXray | 1.00 | 0.181 | **3.0** | 3.0 | degXray | 98.4% |
| 11 | - Phase Shifter Error | 45.4 | degXray | 1.00 | 0.066 | **3.0** | 3.0 | degXray | 99.8% |
| 12 | - Cell Phase Error | 45.4 | degXray | 1.00 | 0.066 | **3.0** | 3.0 | degXray | 99.8% |
| | | | | | | | Total | $\Delta P/P$: | 74.7% |
| | | | | | | | Total Loss | $1-\Delta P/P$: | 25.3% |

**Table 2. Main tolerance budget for the SXR undulator beamline.**

| *n* | Error Source | rms values | | | budget calculations | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\sigma_t$ | Units | Corr | $r_i$ | Value | Tol | Units | $(\Delta P/P)_i$ |
| 1 | - Launch Angles $x'_0,y'_0$ | 4.5 | μrad | 0.71 | 0.311 | **1.00** | 1.00 | μrad | 95.3% |
| 2 | - $(\Delta K/K)_{rms}$ | 0.00131 | | 1.00 | 0.345 | 0.00045 | 0.00045 | | 94.2% |
| 3 | - Segment misalignment in x | 1932472 | $\mu m^2$ | 1.00 | 0.118 | 228168 | 478 | μm | 99.3% |
| 4 | - Segment misalignment in y | 264225 | $\mu m^2$ | 1.00 | 0.151 | **40000** | 200 | μm | 98.9% |
| 5 | - Jaw Pitch [μrad] | 85.4 | μrad | 1.00 | 0.293 | **25** | 25 | μrad | 95.8% |
| 6 | - Quad Position Stability x,y | 11.88 | μm | 0.71 | 0.238 | **2.00** | 2.00 | μm | 97.2% |
| 7 | - Quad Positioning Error x,y | 11.88 | μm | 0.71 | 0.119 | **1.00** | 1.00 | μm | 99.3% |
| 8 | - Break Length Error | 90.4 | mm | 1.00 | 0.044 | **4.0** | 4.0 | mm | 99.9% |
| 9 | - Strongback deflection | 310.0 | μm | 1.00 | 0.142 | **44.0** | 44.0 | μm | 99.0% |
| 10 | - Phase Shake Error | 16.6 | degXray | 1.00 | 0.301 | **5.0** | 5.0 | degXray | 95.6% |
| 11 | - Phase Shifter Error | 47.0 | degXray | 1.00 | 0.170 | **8.0** | 8.0 | degXray | 98.6% |
| 12 | - Cell Phase Error | 47.0 | degXray | 1.00 | 0.170 | **8.0** | 8.0 | degXray | 98.6% |
| | | | | | | | Total | $\Delta P/P$: | 74.9% |
| | | | | | | | Total Loss | $1-\Delta P/P$: | 25.1% |

**Table 3. Basic LCLS-II undulator parameters considered in this document.**

| Parameter | HXR | SXR | Units | Remarks |
|---|---|---|---|---|
| $E_{electron}$ | 7 – 13.5 | 7 – 13.5 | GeV | Tolerances are evaluated at worst case energy |
| $E_{ph}$ | 2.000-13.000 | 0.250 – 2.500 | keV | Maximum photon energy range |
| $K$ | 3.68-0.54 | 11.83-1.98 | | Undulator K parameter range |

**Table 4: Parameters used for tolerance determination.**

| Parameter | HXU Values | SXU Values | Units |
|---|---|---|---|
| $E_{electron}$ | 13.500 | 13.500 | GeV |
| $\Delta E_{electron}$ *(rms)* | 1430 | 1430 | keV |
| $E_{ph}$ | 13.000 | 2.500 | keV |
| $\lvert G \rvert$ | 4.5 | 4.5 | T |
| $\lambda_u$ | 32 | 63 | mm |
| $N_{seg}$ | 20 | 15 | |
| $L_{seg}$ | 3.4 | 3.4 | m |
| $L_{break}$ | 1 | 1 | m |
| $\varepsilon_n$ | 0.36 | 0.36 | µm |
| $I_{pk}$ | 3000 | 3000 | A |
| $m_s$ | 0.00034 | 0.00068 | $1/mm^2$ |
| $\Delta x_{rms}$ | 0.40000 | 0.40000 | mm |
| $\delta x_{rms}$ | 0.40000 | 0.40000 | mm |
| $\Delta\varphi_{rms}$ | 1.00000 | 1.00000 | mrad |

## 3 Individual tolerance analysis

This section describes the analysis of each individual parameter. The formulae use constants from Table 4.

### 3.1 Launch Angles $x_0'$, $y_0'$

As described in Ref. [1], the rms of the FEL dependence on launch angle in the horizontal or vertical plane is given by

$$\sigma_{t,x_0',} = \sigma_{t,y_0',} = \frac{1}{3.3\,\gamma}\sqrt{\frac{\lambda_u\,(1+K^2/2)}{2\,L_{G,1D}}} \tag{4}$$

with the 1-D FEL power gain length

$$L_{G,1D} = \frac{\lambda_u}{4\pi\,\sqrt{3}\,\rho_{1D}} \tag{5}$$

the relativistic Lorentz parameter

$$\gamma = \frac{E_{electron}}{m_e c^2}, \tag{6}$$

The FEL parameter

$$\rho_{1D} = \frac{1}{\gamma}\left(\frac{1}{16}\frac{\mu_0}{4\pi}\frac{e}{m_e c}\frac{I_{pk}K^2 JJ^2}{\langle\beta_{x,y}\rangle \frac{\varepsilon_n}{\gamma} k_u^2}\right)^{\frac{1}{3}}, \tag{7}$$

the Bessel function correction

$$JJ = J_0\left(\frac{K^2/4}{1+K^2/2}\right) - J_1\left(\frac{K^2/4}{1+K^2/2}\right), \tag{8}$$

the undulator wave number

$$k_u = \frac{2\,\pi}{\lambda_u}, \tag{9}$$

the average beta function (ignoring the small vertical focusing of the undulator segments)

$$\langle\beta_{x,y}\rangle = \frac{8\,\left|f_Q\right|^2}{\sqrt{16\,\left|f_Q\right|^2 - L_{FODO}^2}}, \tag{10}$$

the quadruple focal length

$$f_Q = \frac{B\rho}{|IG|}, \tag{11}$$

and the magnetic rigidity

$$B\rho = \frac{E_{electron}}{e\ c}. \tag{12}$$

## 3.2 The undulator *K* parameter

The rms of the FEL dependence on the undulator parameter *K* is equal to the FEL parameter (Eq. (7))

$$\sigma_{t,\Delta K/_K} = \rho_{1D}. \tag{13}$$

The actual K value, as seen by the passing electron beam depends on a number of other parameters and is treated in a sub-budget approach (see section 4).

## 3.3 Horizontal misalignment of the undulator segments

The rms of the FEL dependence on the horizontal deviation (misalignment) of the electron beam path from the undulator center is proportional to the rms spread of the undulator parameter *K*

$$\sigma_{t,f(\Delta x)} = \frac{1}{|m_s|}\sigma_{t,\Delta K/_K}. \tag{14}$$

The tolerance is for a combination of four parameters as described in [1]: the horizontal misalignment of the undulator with respect to the beam, $\Delta x_{align}$, the horizontal rotation of one jaw with respect to the opposite jaw around the vertical axis at the undulator center, $\Delta\varphi_{jaw}$, the horizontal displacement of the two undulator jaws with respect to each other, $\delta x_{jaw}$, and the horizontal field rolloff, $m_s = \frac{1}{2\,K_0}\frac{\partial^2 K}{\partial x^2}\Big|_{x=0}$.

The parameter that is to be balanced is

$$f(\Delta x) = (\Delta X)_{rms}^2 = \sqrt{\left(\Delta x_{align}\right)_{rms}^4 + \left(\delta x_{jaw}\right)_{rms}^4 + \left(\frac{\pi}{\lambda_u}\frac{\left(\varphi_{jaw}\right)_{rms}}{\langle m_s\rangle}\left(\Delta x_{align}\right)_{rms}\right)^2}. \tag{15}$$

3.4 Vertical misalignment of the undulator segments

The rms of the FEL dependence on the vertical deviation (misalignment) of the electron beam path from the undulator center is proportional to the rms spread of the undulator parameter *K*

$$\sigma_{t,f(\Delta y)} = \frac{\sigma_{t,\Delta K}/_K}{(k_u)^2}. \tag{16}$$

The parameter that is to be balanced is

$$f(\Delta y) = \Delta y^2. \tag{17}$$

3.5 Pitch angle between the two undulator jaws

The rms of the FEL dependence on the pitch between the two undulator jaws has been approximated with a 2nd order polynomial of K, based on simulations

$$\sigma_{t,Jaw\ Pitch} = \left|\frac{\sigma_{t,\Delta K}/_K}{0.005\ K^2 - 0.0985\ K + 0.1865}\right|. \tag{18}$$

3.6 The quadrupole position stability in horizontal or vertical direction

The rms of the FEL dependence on the quadrupole stability in one plane is proportional to the launch angle error

$$\sigma_{t,x,quad} = \frac{\sigma_{t,x_0',}}{\sqrt{L_{sat,mag}/L_{seg}}} f_Q. \tag{19}$$

3.7 The quadrupole positioning errror in horizontal or vertical direction

The rms of the FEL dependence on the quadrupole positioning error is the same as that for the quadrupole position stability.

3.8 The break length error

The rms of the FEL dependence on the distance between undulator segments is proportional to the undulator period

$$\sigma_{t,L,break} = \frac{\lambda_u\ (1 + K^2/2)}{2\ \sqrt{L_{sat,mag}/L_{seg}}}. \tag{20}$$

3.9 The strongback deflection error

The rms of the FEL dependence on the strongback deflection error is derived in a separate analysis.

3.10 The phase shake error

The rms of the FEL dependence on the phase shake throughout an undulator segment derived in a separate analysis.

3.11 The phase shift error

The rms of the FEL dependence on the error of the phase shifter setting is

$$\sigma_{t,\varphi,PS} = \frac{180}{\sqrt{L_{sat,mag}/L_{seg}}}. \tag{21}$$

3.12 The cell phase error

The rms of the FEL dependence on the error of the phase shifter setting is

$$\sigma_{t,\Delta\varphi,seg} = \frac{180}{\sqrt{L_{sat,mag}/L_{seg}}}. \quad (22)$$

For parameters 1 and 6 an equal contributions from the horizontal planes are assumed. This has been included through the correction factor column.

## 4 Sub-Budget for the undulator *K* values

The *K* parameter, as seen by the beam, depends on several parameters and can be written in as

$$K = K_{MMF}(1 + \Delta T\,\alpha_K + \Delta g\,\beta_K), \quad (23)$$

where $K_{MMF}$ is the undulator parameter as measured in the MMF, $\Delta T$ is the temperature deviation between MMF and Undulator Hall reduced by the difference in the temperature set points, $\alpha_K$ is the temperature-to-*K* coefficient, $\Delta g$ is the difference in gap setting between the MMF and the undulator Hall, $\beta_K$ is the gap-to-*K* coefficient. The temperature-to-K coefficient can be written as

$$\alpha_K = \frac{1}{K_{MMF}}\frac{dK}{dT} \approx \frac{1}{K}\frac{dK}{dT} = \frac{1}{B_r}\frac{dB_r}{dT} = \alpha_{NdFeB} \quad (24)$$

which uses using Eq.(26). The gap-to-*K* coefficient can be written as

$$\beta_K = \frac{1}{K_{MMF}}\frac{dK}{dg} \approx \frac{1}{K}\frac{dK}{dg} = \frac{a_{Hybd}}{\lambda_u} + 2\frac{b_{Hybd}}{\lambda_u}\frac{g}{\lambda_u} \quad (25)$$

which uses Eq.(27), and **Table 5** for the definition of the undulator K vs. gap modeling. The last two transformations use

$$\frac{1}{B_{r,NdBFe}}\frac{dB_{r,NdBFe}}{dT} \approx 10^{-3}\frac{1}{°\mathrm{C}} \equiv \alpha_{NdFeB.} \quad (26)$$

The relation between the on-axis undulator field and the undulator gap is given by the Halbach formula for a hybrid undulator

$$B = B_0\, e^{a_{Hybd}\frac{g}{\lambda_u} + b_{Hybd}\left(\frac{g}{\lambda_u}\right)^2}. \quad (27)$$

The coefficients for this equation that are used to model the LCLS-II undulators are given in Table 5.

The error analysis for Eq.(23) gives

$$(\delta K)^2 = \left(\frac{dK}{dK_{MMF}}\delta K_{MMF}\right)^2 + \left(\frac{dK}{\Delta T}\delta \Delta T\right)^2 + \left(\frac{dK}{d\alpha_K}\delta \alpha_K\right)^2 + \left(\frac{dK}{d\Delta g}\delta \Delta g\right)^2 + \left(\frac{dK}{d\beta_K}\delta \beta_K\right)^2 \quad (28)$$

The derivative in the first term of Eq.(28) is

$$\frac{dK}{dK_{MMF}} = 1 + \Delta T\ \alpha_K + \Delta g\ \beta_K. \quad (29)$$

**Table 5. Field modeling parameters (Halbach equation, valid for $\frac{g}{\lambda_u} < 1$ ).**

| Parameter | SXR | HXR | Units | |
|---|---|---|---|---|
| $B_0$ | 3.522 | 3.577 | T | Field extrapolation to zero gap (proportional to $B_r$) |
| $K_0$ | 20.720 | 10.687 | | $K_0 = B_0\ (e/m_e c)\ (\lambda_u/2\pi)$ |
| $a_{Hybd}$ | -5.08 | -5.08 | | Linear gap coefficient |
| $b_{Hybd}$ | 1.54 | 1.54 | | Quadratic gap coefficient |

The derivative in the second term of Eq.(23) is

$$\frac{dK}{\Delta T} = K_{MMF}\ \alpha_K. \quad (30)$$

The derivative in the third term of Eq.(23) is

$$\frac{dK}{d\alpha_K} = K_{MMF}\ \Delta T. \quad (31)$$

The derivative in the fourth term of Eq.(23) is

$$\frac{dK}{d\Delta g} = K_{MMF}\ \beta_K. \quad (32)$$

The derivative in the fifth term of Eq.(23) is

$$\frac{dK}{d\beta_K} = K_{MMF}\ \Delta g. \quad (33)$$

The combination of the last equations gives

$$\frac{(\delta K)^2}{K_{MMF}^2} = \left(\frac{(1 + \Delta T \alpha_K + \Delta g\ \beta_K)}{K_{MMF}}\delta K_{MMF}\right)^2 + (\alpha_K \delta \Delta T)^2 + (\Delta T \delta \alpha_K)^2 + (\beta_K \delta \Delta g)^2 + (\Delta g \delta \beta_K)^2 \quad (34)$$

**Table 6. Tolerance sub-budget for SXU. The equation have been evaluatated for an undulator gap at which the largest error contribution occurs.**

| Parameter | Typical | RMS, $\delta$ | Units | |
|---|---|---|---|---|
| $K_{MMF}$ | 4.4699 | 0.00114 | | ± 0.044% uniform max allowed MMF ***K*** error |
| $\alpha_K$ | 0.000125 | 0.000025 | ºC$^{-1}$ | relative termal coefficient |
| $\Delta T$ | 2.0 | 0.29 | ºC | ± 0.50 ºC uniform without compensation |
| $\beta_K$ | -0.0000642 | 0.0000064 | µm$^{-1}$ | gap coefficient |
| $\Delta g$ | 0.0 | 5.77 | µm | ± 10 µm uniform, gap accuracy |
| $K$ | 4.47102 | 0.00203 | | |

**Table 7. Tolerance sub-budget for HXU. The equation have been evaluatated for an undulator gap at which the largest error contribution occurs.**

| Parameter | Typical | RMS, $\delta$ | Units | |
|---|---|---|---|---|
| $K_{MMF}$ | 2.3910 | 0.00032 | | ± 0.023% uniform max allowed MMF *K* error |
| $\alpha_K$ | 0.000125 | 0.000025 | ºC$^{-1}$ | relative termal coefficient |
| $\Delta T$ | 2.0 | 0.12 | ºC | ± 0.20 ºC uniform without compensation |
| $\beta_K$ | -0.000127 | 0.0000127 | µm$^{-1}$ | gap coefficient |
| $\Delta g$ | 0.0 | 1.73 | µm | ± 3.0 µm uniform, gap accuracy |
| $K$ | 2.39157 | 0.00063 | | |

The relative tolerance is $\frac{\delta K}{K}$=0.00045 (SXU) and $\frac{\delta K}{K}$=0.00026 (HXU).

## References

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