Delta Undulator End Design

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Abstract

This note describes the magnetic end design of the Delta undulator. Both three block ends and two block ends were studied. The chosen design uses three block ends and the block strengths are set by moving the blocks away from the beam axis. The options and reasons for this choice are discussed.

1 Introduction[1]

SLAC is building a Delta undulator[2] which will be placed in the LCLS beam line to produce light with variable polarization. A 1 meter prototype is being constructed and a 3.2 meter full length device will be built after the prototype is successfully tested.

This note discusses the magnetic design of the undulator ends. The ends were designed to be steering free and displacement free. The design technique which was followed is that of Schleuter et al.[3] Magnet block permeability is assumed to be μ_0, or equivalently, the relative permeability is assumed to be 1, making the problem linear so superposition holds. With this assumption, a steering free and displacement free end for a single quadrant will remain steering fee and displacement free for the assembled undulator, and for all row phases of the undulator. The relative permeability of 1 assumption allows a good physical understanding of the end design and leads to a good starting point for considering permeability effects. Permeability effects were considered independently, and only final conclusions will be given in this note.

2 End Design

In this section we first consider undulator end design in general by studying the most general three block design. We then consider two special cases: the first case is a three block end design where the blocks have the same size as the center blocks and there are no spaces between the blocks; the second is a two block end design, also with same size blocks and no spaces. Keeping the block dimensions the same as the center blocks and not having spaces between blocks is a design choice that makes construction of the undulator easier. Practical design considerations and final design choices will be considered in the next section.

[1] Work supported in part by the DOE Contract DE-AC02-76SF00515. This work was performed in support of the LCLS project at SLAC.

[2] A. Temnykh, Physical Review Special Topics-Accelerators and Beams **11**, 120702 (2008).

[3] R. Schlueter, S. Marks, and S. Prestemon, "Elliptically Polarizing Undulator End Designs", IEEE Trans. on App. Superconductivity, 16, June, 2006.

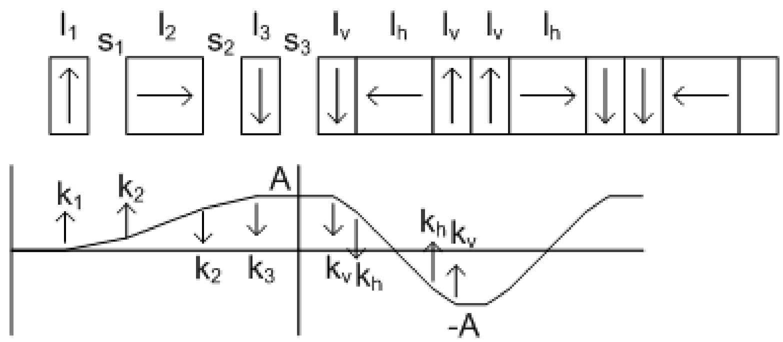

Figure 1: Vertical cross section of the end of the upper undulator magnet assembly and the approximate horizontal beam trajectory.

2.1 Three Block End Design

We now consider a general three block end design. Figure 1 shows a vertical cross section of the upper end of an undulator magnet assembly and a simplified form of the resulting horizontal trajectory. The trajectory has been simplified by modelling the effect of each vertically magnetized block as a kick (trajectory slope change) at the center of the block, and the effect of each horizontally magnetized block as a kick at each end of the block with the kicks of the same strength but in opposite directions.

We begin the analysis by considering the effect of blocks in the center of the undulator. We take the horizontal blocks to have length l_h and the vertical blocks to have length $2l_v$. The vertical blocks are conceptually divided in half for this analysis. The basic unit of the periodic structure is half a vertical block, followed by a horizontal block, followed by half a vertical block. This basic unit is steering free, i.e. it does not produce a net slope change to the trajectory. For every kick, there is an equal and opposite kick. When the incident slope is zero, the exit slope is zero. The basic unit takes the trajectory position from A in figure 1 to $-A$. A half vertical block changes the slope of the trajectory by k_v modeled as an impulse located at the center of the half block. Each end of a horizontal block changes the slope of the trajectory by k_h and the slope changes from the two ends have opposite sign. The second half vertical block in the basic unit changes the slope of the trajectory by $-k_v$. Given the slope changes and the lengths of the blocks, we calculate the trajectory change as follows.

$$2A = k_v\left(\frac{l_v}{2} + l_h + \frac{l_v}{2}\right) + k_h\left(l_h + \frac{l_v}{2}\right) - k_h\left(\frac{l_v}{2}\right) - k_v\left(0\right) \tag{1}$$

Simplifying, we find

$$A = \frac{1}{2}k_v\left(l_v + l_h\right) + \frac{1}{2}k_h\left(l_h\right) \tag{2}$$

We now consider the blocks at the end of the undulator. The end magnets are numbered 1, 2, and 3. Their lengths are l_1, l_2, and l_3 as indicated in figure 1. The slope changes from end blocks are k_1 centered in the first block, k_2 and $-k_2$ at each end of the second block, and k_3 centered in the third block. There is a space s_1 between blocks 1 and 2, space s_2 between blocks 2 and 3, and space s_3 between the end blocks and the undulator center blocks. The purpose of the three end blocks is to bring the trajectory to amplitude A with slope zero. This happens at the position of the center of the third block, so space s_3 has no impact on the design of the end. We take $s_3 = 0$ and in practice, combine block 3 with the first vertical half block of the center section.

To make the slope zero after block 3, we require

$$k_1 + k_2 - k_2 - k_3 = 0 \tag{3}$$

or

$$k_1 = k_3 \tag{4}$$

To reach amplitude A, we require

$$A = k_1 \left(\frac{l_1}{2} + s_1 + l_2 + s_2 + \frac{l_3}{2} \right) + k_2 \left(l_2 + s_2 + \frac{l_3}{2} \right) - k_2 \left(s_2 + \frac{l_3}{2} \right) - k_1 (0) \tag{5}$$

Simplifying, we find

$$A = k_1 \left(\frac{l_1}{2} + s_1 + l_2 + s_2 + \frac{l_3}{2} \right) + k_2 (l_2) \tag{6}$$

We now equate the trajectory amplitude from the end blocks to the trajectory amplitude in the undulator center. This determines a relation between the end blocks and the center blocks.

$$k_1 \left(\frac{l_1}{2} + s_1 + l_2 + s_2 + \frac{l_3}{2} \right) + k_2 (l_2) = \frac{1}{2} k_v (l_v + l_h) + \frac{1}{2} k_h (l_h) \tag{7}$$

The slope changes from the vertical and horizontal blocks depend on block dimensions in different ways. In undulators where the gap is varied, in order for this equation to be always valid, we must break this equation into two equations, one for the horizontal blocks and one for the vertical blocks. While not required for the Delta undulator which has a fixed gap, treating the horizontal blocks and vertical blocks independently greatly simplifies the construction of the ends. We break equation 7 into separate equations for horizontal and vertical blocks. The two equations are

$$k_2 (l_2) = \frac{1}{2} k_h (l_h) \tag{8}$$

$$k_1 \left(\frac{l_1}{2} + s_1 + l_2 + s_2 + \frac{l_3}{2} \right) = \frac{1}{2} k_v (l_v + l_h) \tag{9}$$

At this point, there is much design flexibility both in terms of block dimensions and space sizes, and in terms of construction methods. Practical construction methods influence the design choices. For instance, one method to make the ends is to change the block strengths and spacings by changing their dimensions, such as by cutting blocks to change their lengths. An alternate method is to demagnetize the end blocks to change their strengths while keeping the block dimensions the same as the center blocks. A third method is to move the end blocks away from the beam axis to change their strengths while keeping their dimensions the same. It was determined that construction of the Delta undulator would be most simple if all blocks had the same footprint so the block holders would be made in a periodic, repetitive pattern. As a further simplification, we thus take kick locations to be in the same places as a periodic extension of the central region. We take

$$l_1 = 2l_v \tag{10}$$

$$s_1 = 0 \tag{11}$$

$$l_2 = l_h \tag{12}$$

$$s_2 = 0 \tag{13}$$

$$l_3 = l_v \tag{14}$$

$$s_3 = 0 \tag{15}$$

In this case, the equations for the end blocks become

$$k_2\left(l_h\right) = \frac{1}{2}k_h\left(l_h\right) \tag{16}$$

$$k_1\left(l_v + l_h + \frac{l_v}{2}\right) = \frac{1}{2}k_v\left(l_v + l_h\right) \tag{17}$$

Simplifying,

$$k_2 = \frac{1}{2}k_h \tag{18}$$

$$k_1\left(\frac{3}{2}l_v + l_h\right) = \frac{1}{2}k_v\left(l_v + l_h\right) \tag{19}$$

As a further simplification, we take the horizontal blocks and the vertical blocks to have the same dimensions.

$$2l_v = l_h = l \tag{20}$$

The equations for the end blocks now become

$$k_2 = \frac{1}{2}k_h \tag{21}$$

$$k_1\left(\frac{3}{2}\frac{1}{2}l + l\right) = \frac{1}{2}k_v\left(\frac{1}{2}l + l\right) \tag{22}$$

or

$$k_2 = \frac{1}{2}k_h \tag{23}$$

$$k_1\left(\frac{7}{4}l\right) = \frac{1}{2}k_v\left(\frac{3}{2}l\right) \tag{24}$$

and finally

$$k_2 = \frac{1}{2}k_h \tag{25}$$

$$k_1 = \frac{3}{7}k_v \tag{26}$$

From equation 4

$$k_3 = k_1 = \frac{3}{7}k_v \tag{27}$$

The three block end design is shown in figure 2.

We now consider the kicks from entire blocks by adding together the kicks from partial blocks. We use capital K to denote the kick from a whole magnet block.

The third block in the magnet array consist of block 3 and the first vertical half block of the periodic part of the undulator. Similarly, the vertical magnet blocks in the periodic part of the undulator consist of two half blocks. Let K_v be the kick from a full vertical magnet block.

$$K_v = 2k_v \tag{28}$$

The kick from the third block in the magnet array is

$$K_3 = k_3 + k_v \tag{29}$$

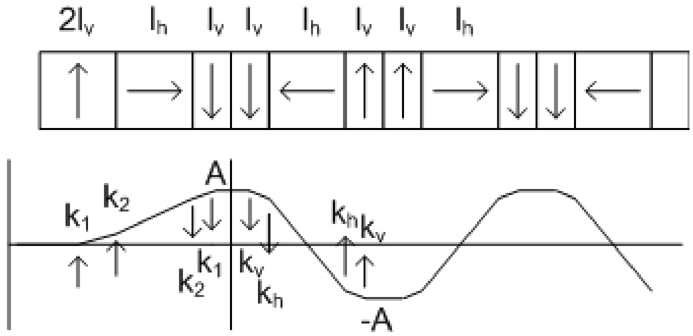

Figure 2: The three block end design is illustrated above, and the resulting beam trajectory is illustrated below.

Solving for K_3, we find

$$K_3 = \frac{3}{7}\frac{1}{2}K_v + \frac{1}{2}K_v \tag{30}$$

or

$$K_3 = \frac{5}{7}K_v \tag{31}$$

For the second block in the magnet array, $K_2 = k_2$. Similarly, $K_h = k_h$. So

$$K_2 = \frac{1}{2}K_h \tag{32}$$

For the first block in the magnet array,

$$K_1 = \frac{3}{7}\frac{1}{2}K_v \tag{33}$$

or

$$K_1 = \frac{3}{14}K_v \tag{34}$$

We now summarize the results for the full blocks:

$$K_1 = \frac{3}{14}K_v \tag{35}$$

$$K_2 = \frac{1}{2}K_h \tag{36}$$

$$K_3 = \frac{5}{7}K_v \tag{37}$$

These equations tell us how to set the kicks from the first three blocks of the undulator in order to produce a steering free and centered trajectory. This result applies to blocks of the same size as the center blocks of the undulator with no spaces between the blocks.

We can produce these kicks by demagnetizing the blocks so their magnetic moments are according to these fractions. Alternatively we can offset the blocks away from the beam axis so the kicks have these strengths. Cutting the blocks and placing them at the right location would work for block 1 and 3 in this scheme, but not for block 2 since the kicks from the ends of block 2 would be at the wrong positions, and cutting block 2 would not change its kick strengths.

The three block end design was simulated using a charge model for the blocks[4]. The trajectory of a 13.6 GeV beam is shown in figure 3. The ends are steering free as anticipated. There is a small

[4] Z. Wolf, "Variable Phase PPM Undulator Study", LCLS-TN-11-1, May, 2011.

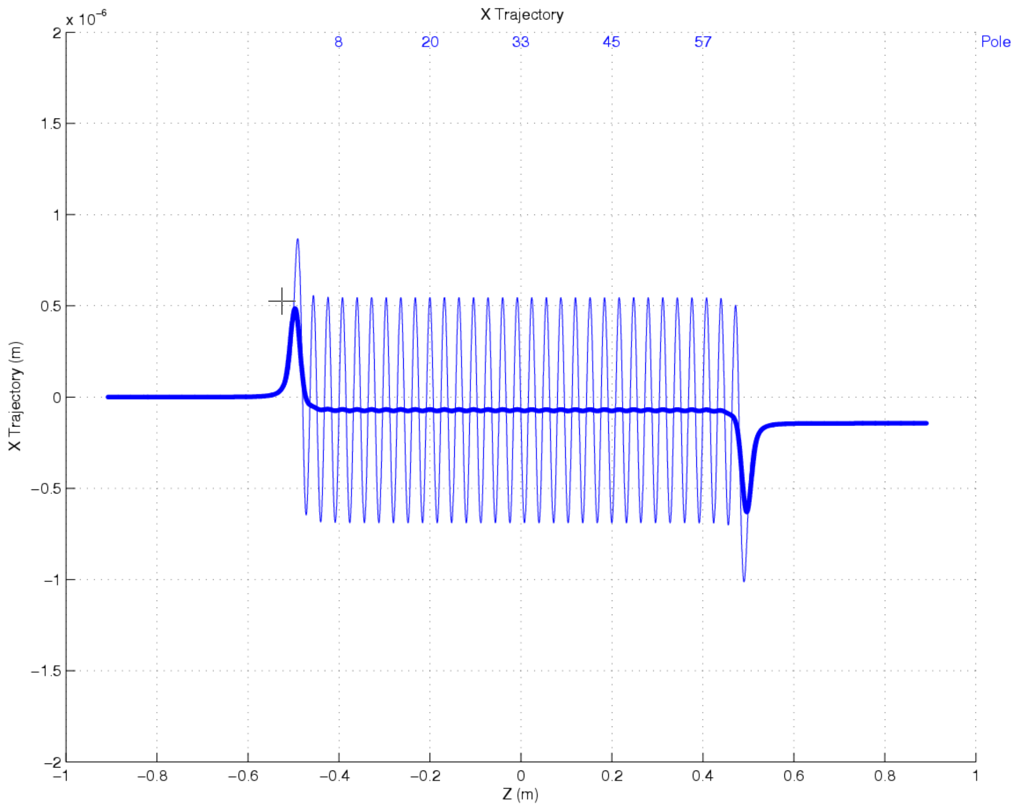

Figure 3: The three block end design produces steering free ends but introduces an offset due to combining different strength sub-blocks into the third block.

offset to the trajectory, however, of 0.07 microns in the center of the undulator. It was determined that this is caused by combining block 3 with the first half block of the center section in the above model. The displaced kick of the combined average block compared to the two different kicks of the individual blocks was responsible for the offset. Using the simulation, it was determined that the offset could be corrected by making block 2 stronger by 4%. This can be done during the tuning of the undulator quadrants.

2.2 Two Block End Design

When studying end designs for the Delta undulator, we also considered a two block design. This could be advantageous because it leaves more space for the center portion of the undulator, so the undulator will produce more light.

Consider the two block end design shown in figure 4. As we did for the three block end design, we choose end blocks which are the same size as the center blocks. The first block we take to be horizontal, the second vertical, and then the center part of the undulator starts. To produce the centered periodic trajectory of the undulator center, we design the end to make the trajectory go to zero at the center of the third block, and to have the appropriate slope of the central trajectory at the center of the third block so that the periodic part of the trajectory motion can begin. We now find the required block strengths to meet these requirements.

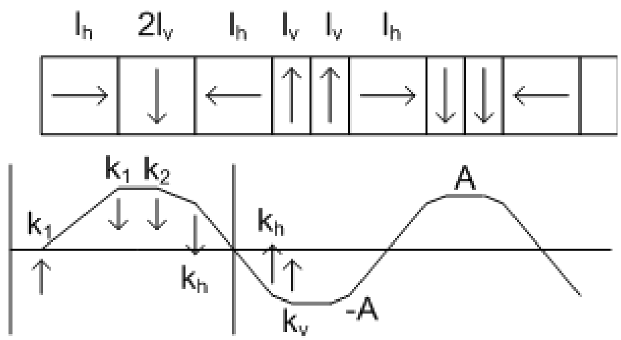

Figure 4: This figure illustrates the two block end design and the associated trajectory.

To make the trajectory go to zero at the center of the third block, we require

$$k_1 l_h - k_2 l_v - (k_2 + k_h) \frac{l_h}{2} = 0 \tag{38}$$

The downward slope at the center of the third block is

$$m_{end} = k_2 + k_h \tag{39}$$

The slope in the center of a horizontal block in the periodic part of the undulator is

$$m_{ctr} = k_v + k_h \tag{40}$$

Equating m_{end} to m_{ctr}, we find

$$k_2 = k_v \tag{41}$$

Equation 38 now becomes

$$k_1 l_h - k_v l_v - (k_v + k_h) \frac{l_h}{2} = 0 \tag{42}$$

or

$$k_1 l_h = k_v l_v + (k_v + k_h) \frac{l_h}{2} \tag{43}$$

Taking $l_v = \frac{1}{2} l_h$, we have

$$k_1 = k_v + \frac{1}{2} k_h \tag{44}$$

Denoting the kicks from full blocks by capitol letters, we have $K_1 = k_1$, $K_2 = k_2$, $K_h = k_h$, and $K_v = 2k_v$. For the full blocks, equations 44 and 41 give

$$K_1 = \frac{K_v + K_h}{2} \tag{45}$$

$$K_2 = \frac{1}{2} K_v \tag{46}$$

Note that setting the strength of the first horizontal block is now more difficult because it is not a simple fraction of the strength of a horizontal block or a vertical block, but rather a combination of the two. Magnetic moment measurements are not sufficient to determine K_1. Rather the field integral of a center vertical block and one end of a center horizontal block must be determined.

3 Practical Design Considerations

3.1 Permeability Effects

The analysis above considered blocks with relative permeability 1. In practice, the relative permeability of the blocks is approximately 1.05, so the end designs given above are only approximately correct. Permeability effects were independently studied using Radia, and it was determined that the three block end design produced smaller field integrals in the undulator than the two block end design[5]. We thus chose the three block end design for the Delta undulator prototype. Some modification to the three block design might make the field integrals smaller, and we will study modifications on the prototype.

3.2 Construction Method

A study was made to determine the best way to implement the three block end design[6]. End blocks were build using three methods: cutting blocks, demagnetizing blocks, and offsetting blocks.

Magnet blocks were cut using wire EDM. This worked very well for the vertical blocks and the fractional block strengths were accurately set by the fractional size of the blocks. The blocks must be sealed after cutting so that they are not exposed to the elements.

Blocks were also demagnetized in our pulse magnetizer. This was difficult to do accurately. It was also noticed that the fractionally magnetized blocks were not stable. There was a concern that the ends would change as the row phases were adjusted in the Delta undulator.

Offsetting the blocks was the method chosen for use. The field integrals of the blocks were measured as a function of offset distance. The correct offset was determined by fitting the measured field integrals and interpolating to get the correct fractional field integral. The blocks remain fully magnetized so the stability concerns are minimized.

4 Conclusion

A three block end design was chosen for the Delta undulator. The strengths of the end blocks were determined using the analytical methods described in this note. They are

$$K_1 = \frac{3}{14} K_v \tag{47}$$

$$K_2 = \frac{1}{2} K_h \tag{48}$$

$$K_3 = \frac{5}{7} K_v \tag{49}$$

The design was confirmed and permeability effects were studied using Radia. The method chosen to implement the design was to change the block strengths by moving the blocks away from the beam axis. Tuning is done by moving the blocks. Modifications to the design to reduce the field integrals from permeability effects will be tested on the prototype.

Acknowledgements
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[5] Heinz-Dieter Nuhn, private communication.
[6] This work was performed by Scott Anderson.