

A PPM Phase Shifter Design

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Abstract

This note describes the design of a pure permanent magnet (PPM) phase shifter for LCLS-II. A PPM phase shifter has the advantage that it can be placed in the fringe field of an undulator without causing changes to the undulator field integrals. This is not the case with a phase shifter containing iron. Conversely, the fringe fields of the phase shifter must not enter iron elements, including the iron poles of the undulators. A phase shifter design is presented which meets strength, tuning range, and magnetic length requirements.

1 Introduction¹

Adjustable gap undulators need phase shifters between segments in order to keep the electron motion synchronized to the electromagnetic field. This note describes the design of a phase shifter which may be useful for LCLS-II. The note starts with a general discussion of phase shifters. It then discusses both undulator and phase shifter fringe fields, and why a pure permanent magnet (PPM) phase shifter is desirable. Details of a PPM phase shifter design are presented. Results of modeling the phase shifter are given.

2 Phase Shifter

2.1 Purpose

The LCLS undulator system is made of undulator segments with fixed distances between segments. The slippage in the break sections between segments is given by²

$$S_b = \frac{1}{2\gamma^2} L_b \quad (1)$$

where γ is the electron Lorentz factor and L_b is the break section length. The phase advance in the break section is given by

$$\phi_b = 2\pi \frac{S_b}{\lambda_r} \quad (2)$$

where λ_r is the radiation wavelength. λ_r is given by

$$\lambda_r = \lambda_u \frac{1}{2\gamma^2} \left(1 + \frac{1}{2} K^2 \right) \quad (3)$$

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²Z. Wolf, "Introduction To LCLS Undulator Tuning", LCLS-TN-04-7, June, 2004.

where λ_u is the fixed undulator period and K is the undulator parameter. K is proportional to the magnetic field in the undulator, which changes with gap. Combining factors, we find that the phase advance in a break section is given by

$$\phi_b = 2\pi \frac{L_b}{\lambda_u} \frac{1}{\left(1 + \frac{1}{2}K^2\right)} \quad (4)$$

When the undulator gap changes, changing the K value, the phase advance in the break section changes. Without correction, the electrons would enter the next undulator with the wrong phase for enhancing the FEL process. A phase shifter compensates for this changing phase advance.

2.2 Phase Integral

The phase change in a phase shifter is calculated by determining the slippage and then multiplying by $2\pi/\lambda_r$. We consider only planar phase shifters with one field component B_y .

The radiation in the phase shifter moves with the speed of light c in the z direction and the electron moves with speed v_z . In time dt , the slippage changes by

$$dS = (c - v_z) dt \quad (5)$$

Instead of using time as the independent variable, we use the z position of the electron. With this change of variable, we have $dz = v_z dt$. Then,

$$dS = \left(\frac{c}{v_z} - 1\right) dz \quad (6)$$

If the electron has velocity v with components v_x and v_z , $v^2 = v_z^2 + v_x^2$. We can write this as

$$v^2 = v_z^2 \left(1 + \frac{v_x^2}{v_z^2}\right) = v_z^2 (1 + x'^2) \quad (7)$$

where $x' = dx/dz$. The velocity can be expressed in terms of γ ,

$$v^2 = c^2 \left(1 - \frac{1}{\gamma^2}\right) \quad (8)$$

Combining these formulas, we have

$$c^2 \left(1 - \frac{1}{\gamma^2}\right) = v_z^2 (1 + x'^2) \quad (9)$$

This lets us solve for c/v_z in dS ,

$$\frac{c}{v_z} = \frac{\sqrt{(1 + x'^2)}}{\sqrt{(1 - \frac{1}{\gamma^2})}} \quad (10)$$

It is a good assumption for relativistic electrons that $\frac{1}{\gamma^2} \ll 1$, and $x'^2 \ll 1$. Using these approximations and keeping only first order terms in small quantities,

$$\frac{c}{v_z} \simeq 1 + \frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2 \quad (11)$$

The slippage differential can now be written as

$$dS = \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2\right) dz \quad (12)$$

Integrating from initial position z_0 to z , we find the change in slippage

$$\Delta S = \int_{z_0}^z \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2 \right) dz_1 \quad (13)$$

If we define the initial position to have zero slippage, $S(z_0) = 0$, then the slippage at z is

$$S(z) = \int_{z_0}^z \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2 \right) dz_1 \quad (14)$$

To find x' , consider a relativistic electron in a magnetic field. It obeys the Lorentz force equation

$$\frac{d}{dt} (\gamma m \vec{v}) = q \vec{v} \times \vec{B} \quad (15)$$

where m is the electron rest mass, \vec{v} is the electron velocity with magnitude v , q is the electron charge (q is negative), and \vec{B} is the magnetic field strength. Since the energy of the electron is not changed by the magnetic field, γ is constant (we neglect the energy losses due to radiation) and can be taken outside the time derivative. The equation then becomes

$$\frac{d}{dt} \vec{v} = \frac{q}{\gamma m} \vec{v} \times \vec{B} \quad (16)$$

With only B_y in the phase shifter, and considering only first order effects, this equation reduces to

$$\frac{d}{dt} v_x = -\frac{q}{\gamma m} v_z B_y \quad (17)$$

Changing independent variables from t to z , and neglecting the small changes in v_z , we have

$$x'' = -\frac{q}{\gamma m v_z} B_y \quad (18)$$

We assume the trajectory has initial slope zero, and again neglect small changes in v_z . We find the slope of the trajectory by integration.

$$x'(z) = \int_{z_0}^z x''(z_1) dz_1 = -\frac{q}{\gamma m v_z} \int_{z_0}^z B_y(z_1) dz_1 \quad (19)$$

Inserting this formula for x' into the slippage equation gives

$$S(z) = \int_{z_0}^z \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} \left[-\frac{q}{\gamma m v_z} \int_{z_0}^{z_1} B_y(z_2) dz_2 \right]^2 \right) dz_1 \quad (20)$$

We define a cell to be a line on the phase shifter axis of length L_p which is centered on the phase shifter. The slippage in the phase shifter cell is

$$S_p = \frac{L_p}{2\gamma^2} + \frac{1}{2\gamma^2} \left(\frac{q}{m v_z} \right)^2 \int_{-L_p/2}^{L_p/2} \left(\int_{-L_p/2}^z B_y(z_1) dz_1 \right)^2 dz \quad (21)$$

The first term is the free space slippage in the phase shifter cell. The second term is the additional slippage from the magnetic field of the phase shifter. We define the phase integral PI as

$$PI = \int_{-L_p/2}^{L_p/2} \left(\int_{-L_p/2}^z B_y(z_1) dz_1 \right)^2 dz \quad (22)$$

In terms of the phase integral, the slippage in the phase shifter is

$$S_p = \frac{L_p}{2\gamma^2} + \frac{1}{2\gamma^2} \left(\frac{q}{mv_z} \right)^2 PI \quad (23)$$

To find the phase change in the phase shifter ϕ_p , we multiply the slippage by $2\pi/\lambda_r$. Expressing λ_r in terms of the undulator period and K value, we find

$$\phi_p = 2\pi \frac{L_p + \left(\frac{q}{mv_z} \right)^2 PI}{\lambda_u \left(1 + \frac{1}{2} K^2 \right)} \quad (24)$$

The phase change is independent of electron energy to first order ($v_z \simeq c$). It only depends on the phase shifter's and undulator's mechanical and magnetic properties. Typically, the undulator parameters are specified and the cell length of the phase shifter is specified, making the phase integral PI the key parameter describing the phase change in the phase shifter.

In order to compensate all possible phase advances from the break sections, the phase change of the phase shifter should be well over 2π for the largest λ_u and largest K value (longest radiation wavelength) of an undulator system. This sets the required phase integral. If

$$\phi_p > 2\pi \quad (25)$$

at λ_u and K_{\max} , then

$$2\pi \frac{L_p + \left(\frac{q}{mv_z} \right)^2 PI_{\max}}{\lambda_u \left(1 + \frac{1}{2} K_{\max}^2 \right)} > 2\pi \quad (26)$$

or

$$PI_{\max} > \left(\frac{mv_z}{q} \right)^2 \left[\lambda_u \left(1 + \frac{1}{2} K_{\max}^2 \right) - L_p \right] \quad (27)$$

We will discuss the required maximum phase integral more extensively below.

The phase shifter is turned off by opening the gap. In order to determine when the phase shifter is off, we compare the free space slippage term to the slippage term involving the phase integral. The phase shifter is off when

$$\frac{1}{2\gamma^2} \left(\frac{q}{mv_z} \right)^2 PI_{\min} < \frac{L_p}{2\gamma^2} \quad (28)$$

or

$$PI_{\min} < \left(\frac{mv_z}{q} \right)^2 L_p \quad (29)$$

2.3 Phase Shifter Range

The phase shifter must compensate the phase advance in the break sections at all undulator K values. In addition, it must do phase matching to compensate the end effects of the undulators in order for the electrons and radiation to have the proper phase when they enter the central region of the undulator. Below we consider the effects that the phase shifter must compensate in order to determine the required adjustment range of the phase shifter. All phases are understood to be at the longest radiation wavelength, requiring the largest slippage from the phase shifter. This is the worst case condition. In the following, $\mathbb{R}\phi_p$ refers to the phase shifter range. $\mathbb{R}\phi_p = \phi_{\max} - \phi_{\min}$, where ϕ_{\max} is the maximum phase advance in the phase shifter, and ϕ_{\min} is the minimum phase advance.

2.3.1 Initial Set Point

The phase shifter must compensate for any phase advance required. The phase shifter can be reset, however, in the sense that phase ϕ and $\phi + 2\pi n$ are equivalent when n is any integer. If the phase shifter has an adjustment range of 2π , in theory it can be set for all conditions. We take the initial set point contribution to the phase shifter range to be

$$\phi_i = 2\pi \quad (30)$$

2.3.2 Energy Scan

An additional contribution to the required range comes from the users' need to do scans over a photon energy range. The scans should be continuous, so the phase shifter must not be reset in the middle of a scan. The scan may be centered on any phase value, so the scan range must be added to the 2π phase range mentioned above. We will now consider how the energy scan range is converted into a K scan range, and in the following subsections we will consider the K scan contribution to the phase shifter range requirement.

Consider a scan in radiation energy, which is performed by changing the undulator K value. Since the radiation energy $E_r = hc/\lambda_r$, the scan range is

$$\frac{\Delta E_r}{E_r} = -\frac{\Delta \lambda_r}{\lambda_r} \quad (31)$$

Since

$$\lambda_r = \lambda_u \frac{1}{2\gamma^2} \left(1 + \frac{1}{2}K^2 \right) \quad (32)$$

we find

$$\Delta \lambda_r = \lambda_u \frac{1}{2\gamma^2} K \Delta K \quad (33)$$

and

$$\frac{\Delta \lambda_r}{\lambda_r} = \frac{K^2}{1 + \frac{1}{2}K^2} \frac{\Delta K}{K} \quad (34)$$

so

$$\frac{\Delta E_r}{E_r} = -\frac{K^2}{1 + \frac{1}{2}K^2} \frac{\Delta K}{K} \quad (35)$$

This formula gives the relative range of K values required for a relative range of radiation energy in a scan. When K is varied through this range, the phase change in the break sections will change and must be compensated. In addition, the phase matching from the neighboring undulators will change and must be compensated. In the following, we will consider these contributions to the phase shifter range requirement.

2.3.3 Break Section Phase Change During An Energy Scan

Modifying equation 4 to exclude the phase shifter, the phase change in the break sections is

$$\phi_b = 2\pi \frac{(L_b - L_p)}{\lambda_u \left(1 + \frac{1}{2}K^2 \right)} \quad (36)$$

In this expression, L_b is the break section length from the cell boundary of the previous undulator to the cell boundary of the next undulator. L_p is the phase shifter cell length. L_p is subtracted from L_b since the drift phase advance in L_p is part of the phase shifter correction.

When K is varied

$$\Delta\phi_b = -2\pi \frac{(L_b - L_p)}{\lambda_u \left(1 + \frac{1}{2}K^2\right)^2} K \Delta K \quad (37)$$

In terms of the energy scan range, this becomes

$$\Delta\phi_b = 2\pi \frac{(L_b - L_p)}{\lambda_u \left(1 + \frac{1}{2}K^2\right)} \frac{\Delta E_r}{E_r} \quad (38)$$

The phase shifter must compensate for this phase change.

2.3.4 Undulator Ends Phase Change During An Energy Scan

The phase advance from the undulator cell boundary to the central region of the undulator is given by the slippage in this region times $2\pi/\lambda_r$. From equations 14 and 3,

$$\phi_{end} = 2\pi \frac{\int_{end} \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2\right) dz}{\frac{1}{2\gamma^2} \lambda_u \left(1 + \frac{1}{2} K^2\right)} \quad (39)$$

We make the assumption that the undulator end field scales with the central field in the undulator. This implies that in the end region, x' at any point scales with K . We denote this as $x' = \alpha(z)K$. The phase change through the end region is then

$$\phi_{end} = 2\pi \frac{\int_{end} \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} (\alpha(z)K)^2\right) dz}{\frac{1}{2\gamma^2} \lambda_u \left(1 + \frac{1}{2} K^2\right)} \quad (40)$$

As K is varied, the phase change through the undulator end changes as

$$d\phi_{end} = 2\pi \frac{\int_{end} (\alpha(z))^2 dz}{\frac{1}{2\gamma^2} \lambda_u \left(1 + \frac{1}{2} K^2\right)} K dK - 2\pi \frac{\int_{end} \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} (\alpha(z)K)^2\right) dz}{\frac{1}{2\gamma^2} \lambda_u \left(1 + \frac{1}{2} K^2\right)^2} K dK \quad (41)$$

We rewrite this as

$$\begin{aligned} d\phi_{end} = & 2\pi \frac{\int_{end} \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} (\alpha(z)K)^2\right) dz}{\frac{1}{2\gamma^2} \lambda_u \left(1 + \frac{1}{2} K^2\right)} 2 \frac{dK}{K} - 2\pi \frac{\int_{end} \frac{1}{2} \frac{1}{\gamma^2} dz}{\frac{1}{2\gamma^2} \lambda_u \left(1 + \frac{1}{2} K^2\right)} 2 \frac{dK}{K} \\ & - 2\pi \frac{\int_{end} \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} (\alpha(z)K)^2\right) dz}{\frac{1}{2\gamma^2} \lambda_u \left(1 + \frac{1}{2} K^2\right)} \frac{K dK}{\left(1 + \frac{1}{2} K^2\right)} \end{aligned} \quad (42)$$

Expressing this in terms of ϕ_{end} , we find

$$d\phi_{end} = \phi_{end} \left[2 - \frac{K^2}{\left(1 + \frac{1}{2} K^2\right)} \right] \frac{dK}{K} - 2\pi \frac{L_{end}}{\lambda_u \left(1 + \frac{1}{2} K^2\right)} 2 \frac{dK}{K} \quad (43)$$

which simplifies to

$$d\phi_{end} = \phi_{end} \frac{2}{\left(1 + \frac{1}{2} K^2\right)} \frac{dK}{K} - 2\pi \frac{L_{end}}{\lambda_u \left(1 + \frac{1}{2} K^2\right)} 2 \frac{dK}{K} \quad (44)$$

Expressing dK/K in terms of the energy scan range, this becomes

$$d\phi_{end} = \left[\phi_{end} \frac{2}{\left(1 + \frac{1}{2} K^2\right)} - 2\pi \frac{L_{end}}{\lambda_u \left(1 + \frac{1}{2} K^2\right)} 2 \right] \left(-\frac{1 + \frac{1}{2} K^2}{K^2} \frac{\Delta E_r}{E_r} \right) \quad (45)$$

which simplifies to

$$d\phi_{end} = -\phi_{end} \frac{2}{K^2} \frac{\Delta E_r}{E_r} + 2\pi \frac{L_{end}}{\lambda_u} \frac{2}{K^2} \frac{\Delta E_r}{E_r} \quad (46)$$

The phase shifter must compensate for both the exit end of the previous undulator and the entrance end of the next undulator. So the phase shifter must compensate for twice the above change.

$$\Delta\phi_e = -\phi_{end} \frac{4}{K^2} \frac{\Delta E_r}{E_r} + 2\pi \frac{L_{end}}{\lambda_u} \frac{4}{K^2} \frac{\Delta E_r}{E_r} \quad (47)$$

2.3.5 Phase Range Of Phase Shifter

The phase shifter range must include the initial set point, the phase change in the break section during a scan, and the phase change in the undulator ends during a scan. We add the absolute values of all terms to ensure an adequate range limit.

$$\mathbb{R}\phi_p = \phi_i + |\Delta\phi_b| + |\Delta\phi_e| \quad (48)$$

Inserting the values calculated above and taking the absolute value of each term in the undulator ends phase change as a worst case, we have

$$\mathbb{R}\phi_p = 2\pi + 2\pi \frac{(L_b - L_p)}{\lambda_u (1 + \frac{1}{2}K^2)} \left| \frac{\Delta E_r}{E_r} \right| + \phi_{end} \frac{4}{K^2} \left| \frac{\Delta E_r}{E_r} \right| + 2\pi \frac{L_{end}}{\lambda_u} \frac{4}{K^2} \left| \frac{\Delta E_r}{E_r} \right| \quad (49)$$

or

$$\mathbb{R}\phi_p = 2\pi \left[1 + \left(\frac{(L_b - L_p)}{\lambda_u (1 + \frac{1}{2}K^2)} + \frac{\phi_{end}}{2\pi} \frac{4}{K^2} + \frac{L_{end}}{\lambda_u} \frac{4}{K^2} \right) \left| \frac{\Delta E_r}{E_r} \right| \right] \quad (50)$$

This formula gives the required phase adjustment range of the phase shifter.

2.3.6 Phase Integral Range Of Phase Shifter

We wish to also find the range of phase integral required. As given above, the phase change in the phase shifter is

$$\phi_p = 2\pi \frac{L_p + \left(\frac{q}{mv_z}\right)^2 PI}{\lambda_u (1 + \frac{1}{2}K^2)} \quad (51)$$

The minimum phase change occurs when the phase shifter is off with $PI \simeq 0$.

$$\phi_{p \min} = 2\pi \frac{L_p}{\lambda_u (1 + \frac{1}{2}K^2)} \quad (52)$$

The maximum phase change of the phase shifter must give the required phase compensation range.

$$\phi_{p \min} + \mathbb{R}\phi_p = 2\pi \frac{L_p + \left(\frac{q}{mv_z}\right)^2 PI_{\max}}{\lambda_u (1 + \frac{1}{2}K^2)} \quad (53)$$

This implies

$$\mathbb{R}\phi_p = 2\pi \frac{\left(\frac{q}{mv_z}\right)^2 PI_{\max}}{\lambda_u (1 + \frac{1}{2}K^2)} \quad (54)$$

Inserting the expression for the required phase range of the phase shifter, we find the expression for the required phase integral.

$$PI_{\max} = \left(\frac{mv_z}{q}\right)^2 \lambda_u \left(1 + \frac{1}{2}K^2\right) \left[1 + \left(\frac{(L_b - L_p)}{\lambda_u (1 + \frac{1}{2}K^2)} + \frac{\phi_{end}}{2\pi} \frac{4}{K^2} + \frac{L_{end}}{\lambda_u} \frac{4}{K^2} \right) \left| \frac{\Delta E_r}{E_r} \right| \right] \quad (55)$$

As noted above, this requirement on the phase integral should be calculated using the maximum undulator K value since this requires the largest PI from the phase shifter.

We wish to make a numerical estimate of PI_{\max} for the LCLS-II phase shifter. We set $v_z \simeq c$. The constant $\frac{mc}{q}$ has a value $\frac{mc}{q} = 1.7 \times 10^{-3} \text{ Tm}$. For the longest radiation wavelength (maximum PI required), $\lambda_u = 0.055 \text{ m}$ and $K = K_{\max} = 9.9$. We take $L_b = 1 \text{ m}$, $L_p = 0 \text{ m}$ (worst case), and $L_{\text{end}} = 0.1 \text{ m}$. We take $\phi_{\text{end}} = 2\pi$ which is a reasonable value, but with some uncertainty; however the result is insensitive to this choice. The maximum energy scan range is expected to be $\frac{\Delta E_r}{E_r} = 0.1$. Using these values,

$$PI_{\max} = (1.7 \times 10^{-3} \text{ Tm})^2 (0.055 \text{ m}) \left(1 + \frac{1}{2} (9.9)^2\right) \times \left[1 + \left(\frac{1 \text{ m}}{(0.055 \text{ m}) \left(1 + \frac{1}{2} (9.9)^2\right)} + \frac{4}{(9.9)^2} + \frac{0.1 \text{ m}}{0.055 \text{ m}} \frac{4}{(9.9)^2}\right) (0.1)\right] \quad (56)$$

Keeping the terms separated to see the size of each effect, we find

$$PI_{\max} = (7.9 \times 10^{-6} \text{ T}^2\text{m}^3) [1 + (0.36 + 0.04 + 0.07) (0.1)] \quad (57)$$

The undulator ends make a smaller contribution to PI_{\max} than the break section length. With these values, we determine the required phase integral to be $PI_{\max} = 8.3 \times 10^{-6} \text{ T}^2\text{m}^3$. Larger values allow and even larger phase range before the phase shifter must be reset.

2.4 Undulator Fringe Fields

The magnetic field of an undulator extends past the physical end of the undulator and is called the fringe field. The distance the fringe field extends past the end of the undulator depends on the undulator gap. The fringe fields must be accounted for when the undulator is tuned and when it is used since the fringe fields change the trajectory of the electron beam. If a permeable material is placed in the fringe field region, the field that the electron beam sees is changed. This is illustrated in figure 1. If an iron electromagnet near the undulator is used as a phase shifter, a similar object

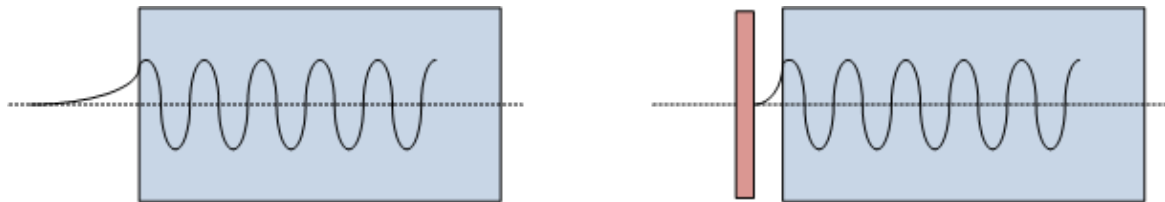


Figure 1: The fringe field extends past the physical end of the undulator. Permeable material (right) changes the fringe field.

must be included when the undulator is tuned, or the electron trajectories will be in error. On the other hand, a PPM phase shifter near the end of the undulator is essentially transparent to the undulator fringe field, so the undulator can be tuned as an independent object.

2.5 Phase Shifter Fringe Fields

As noted above, a PPM phase shifter can be placed in the undulator fringe field without the need to account for it during undulator tuning. Similarly, the fringe fields of the phase shifter must

not enter a permeable material without accounting for it during the phase shifter tuning. When using hybrid undulators, the fringe field of the phase shifter can interact with the steel poles of the undulator to affect the electron beam trajectory. This problem is addressed by designing the phase shifter to have as small fringe fields as possible, and then placing the phase shifter sufficiently far from permeable material. This is illustrated in figure 2.

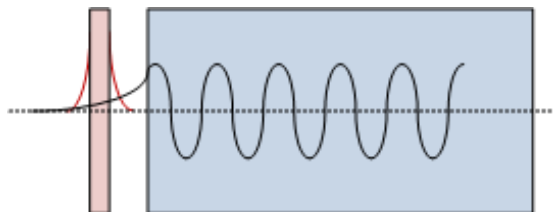


Figure 2: The PPM phase shifter is designed to have rapidly decaying fringe fields. The phase shifter must be separated from permeable material by sufficient distance so that the fringe fields are not altered.

2.6 Phase Shifter Requirements

In order to design the phase shifter, we must list the requirements that the design must meet. The requirements needed are the minimum gap, PI_{\max} when the gap is minimum, PI_{\min} when the gap is maximally open, and maximum magnetic length under any gap conditions.

It is desirable for the beam pipe size to remain roughly constant in the machine. Minimizing the phase shifter gap maximizes the phase integral, so we wish to make the gap as small as possible. This is limited, however, since we wish the minimum gap to contain a beam pipe of similar size to that in other components. We take the minimum gap to be 7 mm.

When the phase shifter gap is closed, the phase shift is maximum. We use the value calculated in equation 57 to set the limit on the maximum phase shift. In terms of the phase integral, $PI_{\max} > 8.3 \times 10^{-6} \text{ T}^2\text{m}^3$.

It is also desirable to provide a large range of phase adjustment, and possibly turn the phase shifter off completely. When the phase shifter gap is open, the phase shift is minimum. As noted above, we set as our criterion that when the gap is maximally open

$$PI < \left(\frac{mv_z}{q} \right)^2 L_p$$

Using $\frac{mc}{q} = 1.7 \times 10^{-3} \text{ Tm}$ and setting $L_p = 0.4 \text{ m}$, we determine that the phase shifter is off when $PI < 1.2 \times 10^{-6} \text{ T}^2\text{m}^3$. We set the limit at $PI_{\min} = 1.2 \times 10^{-7} \text{ T}^2\text{m}^3$. We do not specify the value of the maximum gap.

For LCLS-II, the break lengths will be approximately 1 meter long. As a worst case estimate for the phase shifter fringe fields, suppose the quadrupoles are placed in the middle of the breaks so that their steel does not truncate the undulator fringe fields. (If the quadrupole was positioned off center, the phase shifter could be placed in the larger space between the quadrupole and undulator, and have a longer magnetic length.) The quadrupole length is 164 mm from mirror plate to mirror plate. This leaves 418 mm between the quadrupole steel and the undulator end pole. We plan to place the phase shifter in the center of this space. We do not want the fringe fields from the phase shifter to go into the steel of the quadrupole or into the steel of the undulator, otherwise the truncated field integral will change the slope of the beam trajectory. Thus, we require the magnetic length of the phase shifter to be less than 418 mm at all gap settings. We define the magnetic

length to include all magnetic fields except for $3 \mu\text{Tm}$ of first field integral at each end of the phase shifter. If the excluded field were truncated, it would give an acceptable slope change to the beam trajectory.

A summary of the phase shifter requirements is given in the table below:

Requirement	Value	Units
Minimum gap	7	mm
PI_{\max} at minimum gap	$> 8.3 \times 10^{-6}$	T^2m^3
PI_{\min} at maximum gap	$< 1.2 \times 10^{-7}$	T^2m^3
Maximum magnetic length	418	mm

3 PPM Phase Shifter Design

The phase shifter design is begun by using a simplified model of the effect of the permanent magnet blocks on the beam³. The effect of each vertically magnetized block is modeled as a kick (trajectory slope change) at the center of the block, and the effect of each horizontally magnetized block is modeled as a kick at each end of the block, with the kicks in opposite directions. Figure 3 illustrates the first half of the phase shifter and its effect on the beam. The top of the figure shows the upper array of magnets and their equivalent magnetic charges. The bottom of the figure shows the effect of the magnets on the electron beam. The phase shifter is designed to be antisymmetric, so the second half is similar to the first, but producing the opposite sign magnetic field.

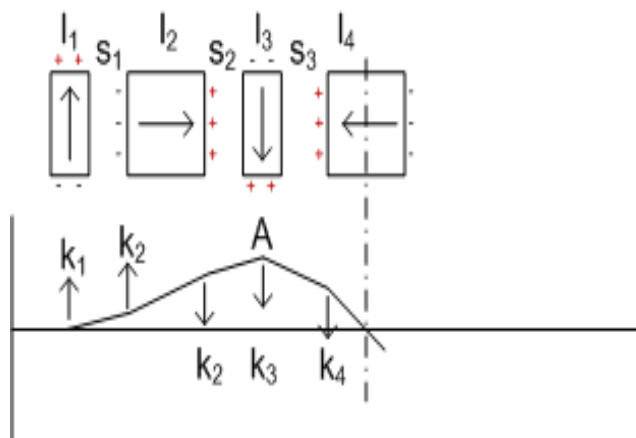


Figure 3: The top of the figure shows the upper array of magnets in the phase shifter and their equivalent magnetic charges. The bottom of the figure shows the effect of the magnets on the beam.

The phase shifter starts with a vertical block of length l_1 followed by a space of length s_1 . This is followed by a horizontal block of length l_2 followed by a space of length s_2 . The third block is vertical of length l_3 followed by space s_3 . The middle of the phase shifter is a horizontal block and the center of that block is the center of the phase shifter. The half length of the horizontal block is l_4 .

The first block changes the slope of the beam by k_1 and the slope change is modeled by an impulse in the longitudinal center of the block. The horizontal second block makes slope changes

³R. Schlueter, S. Marks, and S. Prestemon, "Elliptically Polarizing Undulator End Designs", IEEE Trans. on App. Superconductivity, 16, June, 2006.

k_2 and $-k_2$ at the ends. The vertical third block makes a slope change $-k_3$ centered longitudinally along the block. The horizontal fourth half block makes a slope change $-k_4$ at the end of the block.

The trajectory amplitude, through the trajectory slope, will play a role in maximizing the slippage from the phase shifter. The trajectory amplitude is given by

$$A = k_1 \left(\frac{l_1}{2} + s_1 + l_2 + s_2 + \frac{l_3}{2} \right) + k_2 \left(l_2 + s_2 + \frac{l_3}{2} \right) - k_2 \left(s_2 + \frac{l_3}{2} \right) \quad (58)$$

or

$$A = k_1 \left(\frac{l_1}{2} + s_1 + l_2 + s_2 + \frac{l_3}{2} \right) + k_2 (l_2) \quad (59)$$

This can be rewritten as

$$A = k_1 \left(\frac{l_1}{2} + \frac{l_3}{2} + s_1 + s_2 \right) + (k_1 + k_2) (l_2) \quad (60)$$

The amplitude A and the trajectory slope can be increased several ways. For example, making k_1 larger by making l_1 longer is one way. Increasing l_3 , s_1 , or s_2 is another way. We will come back to this topic after the relations between the various lengths are determined.

Since the phase shifter is antisymmetric, at the center of the middle block, we make the trajectory go to zero. This is expressed by the following equation.

$$\begin{aligned} & k_1 \left(\frac{l_1}{2} + s_1 + l_2 + s_2 + l_3 + s_3 + l_4 \right) + k_2 (l_2 + s_2 + l_3 + s_3 + l_4) \\ & - k_2 (s_2 + l_3 + s_3 + l_4) - k_3 \left(\frac{l_3}{2} + s_3 + l_4 \right) - k_4 (l_4) = 0 \end{aligned} \quad (61)$$

Simplifying, we find

$$k_1 \left(\frac{l_1}{2} + s_1 + l_2 + s_2 + l_3 + s_3 + l_4 \right) - k_3 \left(\frac{l_3}{2} + s_3 + l_4 \right) + k_2 (l_2) - k_4 (l_4) = 0 \quad (62)$$

The slope changes from the vertical and horizontal blocks depend on block dimensions and phase shifter gap in different ways. In order for this equation to be valid at all gap settings, we break this equation into two equations, one for the horizontal blocks and one for the vertical blocks.

$$k_1 \left(\frac{l_1}{2} + s_1 + l_2 + s_2 + l_3 + s_3 + l_4 \right) - k_3 \left(\frac{l_3}{2} + s_3 + l_4 \right) = 0 \quad (63)$$

$$k_2 (l_2) - k_4 (l_4) = 0 \quad (64)$$

The slope change from the horizontal blocks depends on the block's transverse dimensions and not on its length, i.e. it depends on the magnetic charge at the ends of the block. We want all blocks to have the same transverse dimensions and same B_r , so $k_2 = k_4$. Equation 64 then gives

$$l_2 = l_4 \quad (65)$$

The slope change from a vertical block depends on the length of the block since its magnetic charges are along the beam direction. Let α be the proportionality factor between slope change and block length. We assume α is the same for all blocks since we require all blocks to have the same B_r . Then $k_1 = \alpha l_1$ and $k_3 = \alpha l_3$. Setting $l_2 = l_4$, the equation for the vertical blocks becomes

$$l_1 \left(\frac{l_1}{2} + 2l_2 + l_3 + s_1 + s_2 + s_3 \right) = l_3 \left(\frac{l_3}{2} + l_2 + s_3 \right) \quad (66)$$

All quantities in this equation are free parameters. As a design choice, we wish to limit the interdependence of the vertically and horizontally magnetized blocks. This lets us optimize the design by changing vertical and horizontal blocks independently if we wish to. To do this, we set $l_1 2l_2 = l_3 l_2$ so the terms with l_2 cancel. This requires

$$l_3 = 2l_1 \quad (67)$$

With this constraint, we have

$$l_1 \left(\frac{l_1}{2} + 2l_1 + s_1 + s_2 + s_3 \right) = 2l_1 (l_1 + s_3) \quad (68)$$

This simplifies to

$$\frac{l_1}{2} + s_1 + s_2 - s_3 = 0 \quad (69)$$

Since s_1 and s_2 have the wrong sign to solve this equality without unnecessarily increasing the overall length, we make

$$s_1 = s_2 = 0 \quad (70)$$

In this case,

$$s_3 = \frac{l_1}{2} \quad (71)$$

The design choices of the lengths of all the magnets and spaces is now complete. There are two free parameters, l_1 and l_2 . Once these values are chosen, all other block lengths and space lengths are determined. The design is summarized in figure 4 and in the equations below. Let l_v be the

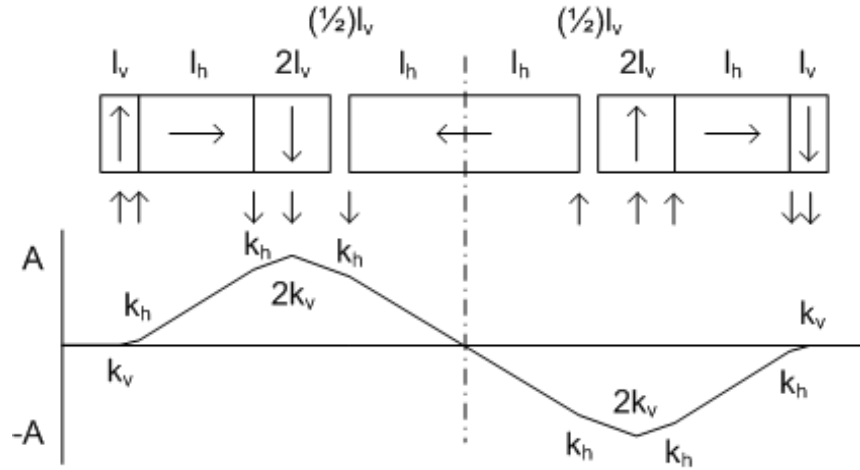


Figure 4: Design of the phase shifter.

length of vertical block 1. Let l_h be the length of horizontal block 2. Then the design of the first half of the antisymmetric phase shifter is summarized by the following relations.

$$l_1 = l_v \quad (72)$$

$$l_2 = l_h \quad (73)$$

$$l_3 = 2l_v \quad (74)$$

$$l_4 = l_h \quad (75)$$

$$s_1 = 0 \quad (76)$$

$$s_2 = 0 \quad (77)$$

$$s_3 = \frac{l_v}{2} \quad (78)$$

Now that the relations between the magnet blocks are determined, we wish to return to the question of how to maximize the strength of the phase shifter. As noted above, the amplitude of the trajectory in the phase shifter is

$$A = k_1 \left(\frac{l_1}{2} + \frac{l_3}{2} + s_1 + s_2 \right) + (k_1 + k_2) (l_2) \quad (79)$$

Inserting the appropriate lengths of the magnets and spaces, this expression becomes

$$A = k_1 \left(\frac{3}{2} l_v \right) + (k_1 + k_2) (l_h) \quad (80)$$

The slope change k_1 is proportional to l_v , $k_1 = \alpha l_v$. The slope change k_2 is proportional to the height of the horizontal block h_h , $k_2 = \beta h_h$. Inserting these relations we find

$$A = \alpha l_v \left(\frac{3}{2} l_v \right) + (\alpha l_v + \beta h_h) (l_h) \quad (81)$$

We will want to minimize the block heights to minimize the fringe fields, so we do not want to use h_h as a way to increase A . This expression tells us that the trajectory amplitude goes as l_v^2 and $l_v l_h$.

The length of the phase shifter is

$$L = 2(l_1 + s_1 + l_2 + s_2 + l_3 + s_3 + l_4) \quad (82)$$

Inserting the appropriate lengths

$$L = 2 \left(l_v + l_h + 2l_v + \frac{l_v}{2} + l_h \right) \quad (83)$$

or

$$L = 2 \left(\frac{7}{2} l_v + 2l_h \right) \quad (84)$$

This expression says that there is a slightly larger effect on the length when the vertical blocks are lengthened, as opposed to the horizontal blocks.

We wish to maximize the slippage in the phase shifter. The expression for the slippage is

$$S(z) = \int_{z_0}^z \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2 \right) dz_1 \quad (85)$$

Consider the effect of increasing both l_v and l_h together. Since x' goes as A/L , which scales as the block length, and x'^2 scales as the block length squared, and the integral scales as the block length, we expect that the slippage (and phase integral) to scale as the block length to the third power. The length of the phase shifter will increase linearly. The fringe fields will increase with block length due to the increased magnetic charge on the vertical blocks. This effect will be studied below.

3.1 Force Calculations

The force on a phase shifter magnet array is calculated by integrating the stress tensor over a surface enclosing the magnet array. We take the part of the surface outside the phase shifter at infinity where the fields are zero, and the part of the surface inside the phase shifter on the midplane. The i 'th component of the force is given by

$$F_i = \oint dS_j \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} \delta_{ij} B_k B_k \right) \quad (86)$$

where i, j, k indicate the 1, 2, or 3 (x, y , or z) directions. We approximate the surface integral in the gap by using the field along the axis of the phase shifter, the ($x = 0, y = 0$) line, and assuming the field is uniform in the x direction over the width of the magnets W . In the gap the surface element has one component in the y -direction: $dS_2 = W dz$. The force components on the lower magnet array are then

$$F_x = \frac{W}{\mu_0} \int B_x B_y dz \quad (87)$$

$$F_y = \frac{W}{2\mu_0} \int (B_y^2 - B_z^2) dz \quad (88)$$

$$F_z = \frac{W}{\mu_0} \int B_z B_y dz \quad (89)$$

Using these equations, numerical integration of the calculated fields gives the forces.

4 Modeled Results

The pure permanent magnet phase shifter designed in the previous section was simulated by modeling the permanent magnet blocks as equivalent magnetic charges. The details of the model were presented in an earlier note⁴. The magnet block widths were taken to be the LCLS-I block widths, 66.0 mm. The block heights were varied to study the effect on the fringe fields. The block thicknesses were varied to study the effect on the phase integral. The remnant field in the magnet blocks was taken to be 1.2 T. This section discusses the model and how it was used to optimize the design.

4.1 Starting Point Block Dimensions

As a starting point, consider a phase shifter design with 7.5 mm thick blocks for both the vertically and horizontally magnetized blocks. This thickness was chosen to give a 3 cm period in a PPM undulator, a familiar device. Both the block height and gap were varied to study the effect on the phase integral and magnetic length.

Figure 5 shows the phase integral with 7 mm gap as the height of the blocks is varied. The phase integral is below the $8.3 \times 10^{-6} \text{ T}^2 \text{ m}^3$ required value at all block heights, so further design work is required.

Figure 6 shows the magnetic length of the phase shifter as a function of block height. The magnetic length is within tolerance at this minimum gap.

When the gap is opened to 100 mm, the phase integral becomes small, well within the tolerance for maximum gap. This is shown in figure 7.

The fringe fields extend out further when the gap is opened. This is shown in figure 8. Note that a block height below approximately 27 mm is required to stay within the magnetic length tolerance.

⁴Z. Wolf, "Variable Phase PPM Undulator Study", LCLS-TN-11-1, April, 2011.

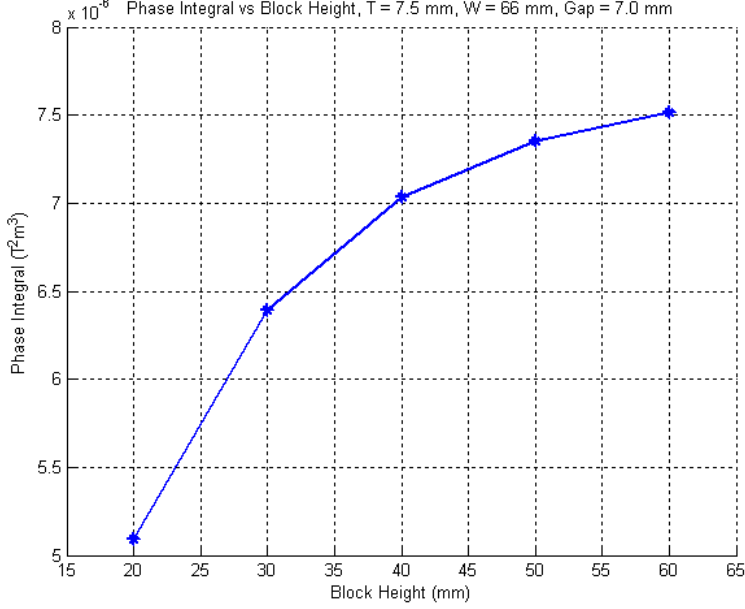


Figure 5: Phase integral as a function of block height for 7 mm gap.

From this starting point, we find that the block height must be kept below 27 mm to keep the magnetic length within tolerance at maximum gap. With a 27 mm block height, we must increase the phase integral at 7 mm gap from approximately $6 \times 10^{-6} \text{ T}^2\text{m}^3$ to $8.3 \times 10^{-6} \text{ T}^2\text{m}^3$ to meet the strength requirement.

4.2 Increase Block Length

As noted above, increasing the vertically and horizontally magnetized block thicknesses is an effective way to increase the phase integral. We set the block heights of all blocks to 25 mm to keep the magnetic length within tolerance. We set the gap to 7.0 mm. Figure 9 shows the resulting phase integral as a function of block thickness with $l_v = l_h$. Note that when $l_{block} \gtrsim 8.4 \text{ mm}$, the phase integral exceeds the tolerance of $8.3 \times 10^{-6} \text{ T}^2\text{m}^3$. We set the block length to 9.0 mm.

With the dimensions of the blocks chosen, we must now check the magnetic length. The blocks are 9.0 mm thick, 25 mm high, and 66 mm wide. The magnetic length as a function of gap is shown in figure 10. Note that for gaps up to 70 mm, the magnetic length is below 418 mm. We will limit the gap to 70 mm to stay within the magnetic length tolerance.

The phase integral as a function of gap is shown in figure 11. At a gap of 70 mm, the phase integral is $1.3 \times 10^{-7} \text{ T}^2\text{m}^3$. We take this as close enough to the loosely constrained tolerance of $1.2 \times 10^{-7} \text{ T}^2\text{m}^3$.

4.3 Phase Shifter Design

From the studies above, we take as our design a phase shifter with block thicknesses $l_v = l_h = 9.0 \text{ mm}$, all blocks have the same height of 25 mm, and all blocks have the same width of 66 mm. This is shown in figure 12. The phase shifter has the following properties at its minimum and maximum gap.

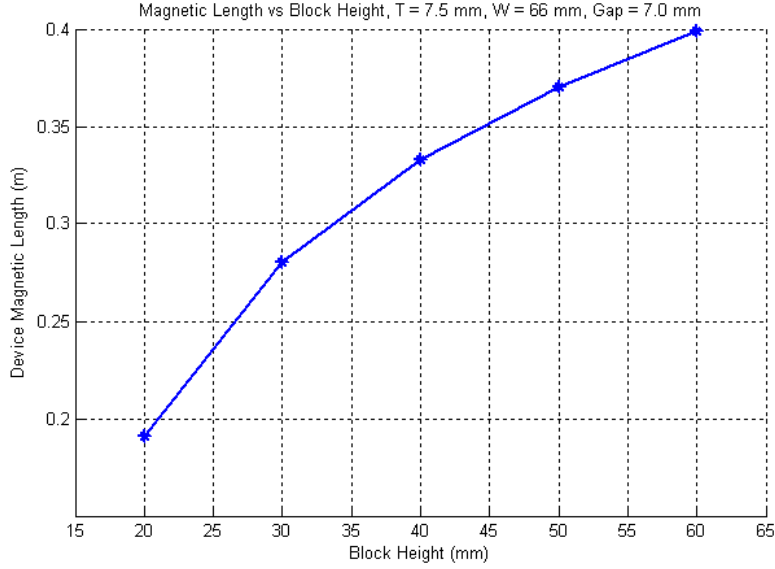


Figure 6: Magnetic length as a function of block height for a 7 mm gap.

Quantity	Value	Units
Minimum gap	7	mm
Phase integral at 7 mm gap	1.004×10^{-5}	T^2m^3
Phase change of 253 eV photon in 400 mm cell, 7 mm gap	504.5	deg
Magnetic length at 7 mm gap	0.226	m
Maximum gap	70	mm
Phase integral at 70 mm gap	1.337×10^{-7}	T^2m^3
Phase change of 253 eV photon in 400 mm cell, 70 mm gap	58.4	deg
Magnetic length at 70 mm gap	0.414	m

Figure 13 shows the magnet blocks with the equivalent magnetic charges. It also shows vectors representing the vertical component of the magnetic field on the phase shifter axis. The gap is 30 mm in this figure.

Figure 14 shows the vertical field B_y as a function of z on the phase shifter axis. The positions of the blocks are indicated. The gap is 7 mm for this plot. The horizontal field is zero.

The horizontal trajectory in the phase shifter is shown in figure 15. Note that the phase shifter produces no net slope or offset to the beam. The gap for this figure is 7 mm. The beam energy is 8.57 GeV.

The phase shift through the phase shifter is shown in figure 16. The gap for this figure is 7 mm. The beam energy is 8.57 GeV. The undulator K value is 9.9 and the undulator period is 55 mm. The radiation wavelength is 4.89×10^{-9} m, and the photon energy is 253 eV. This is the longest radiation wavelength which the undulators will produce, making the smallest phase shift in the phase shifter.

The force on each magnet array was calculated. There is only a vertical force. For the calculation, the transverse field distribution was assumed equal to the field on the axis existing over the width of the magnet blocks. This is only an approximation, but it gives an indication of the magnetic forces. With this approximation, the force on each magnet array is estimated to be 1844 N (415 pounds) when the gap is 7 mm.

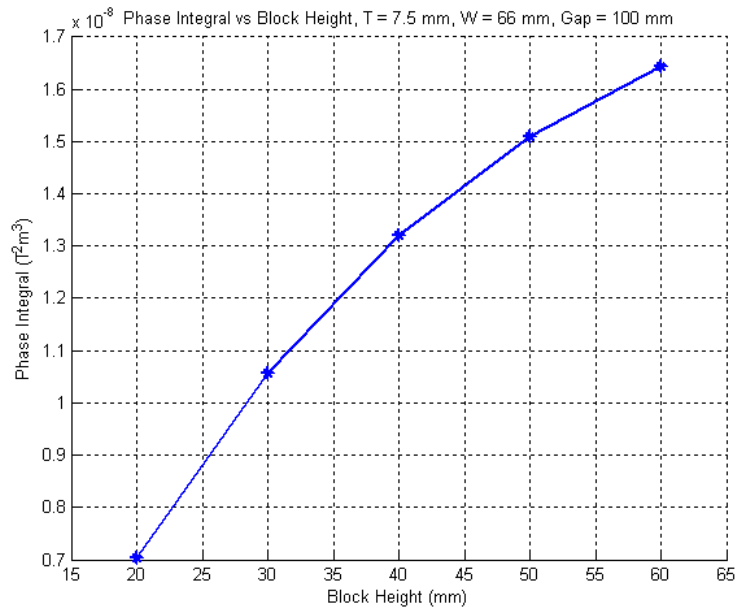


Figure 7: Phase integral vs block height when the gap is 100 mm.

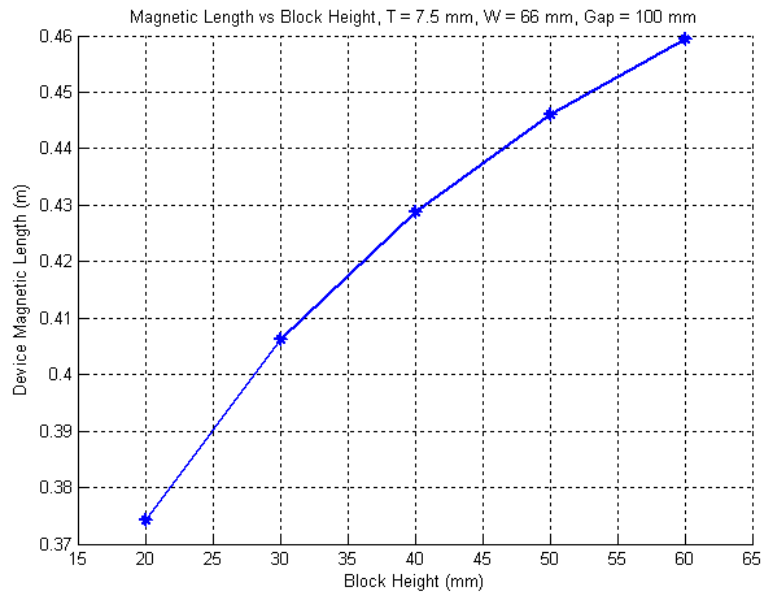


Figure 8: Magnetic length as a function of block height for a 100 mm gap.

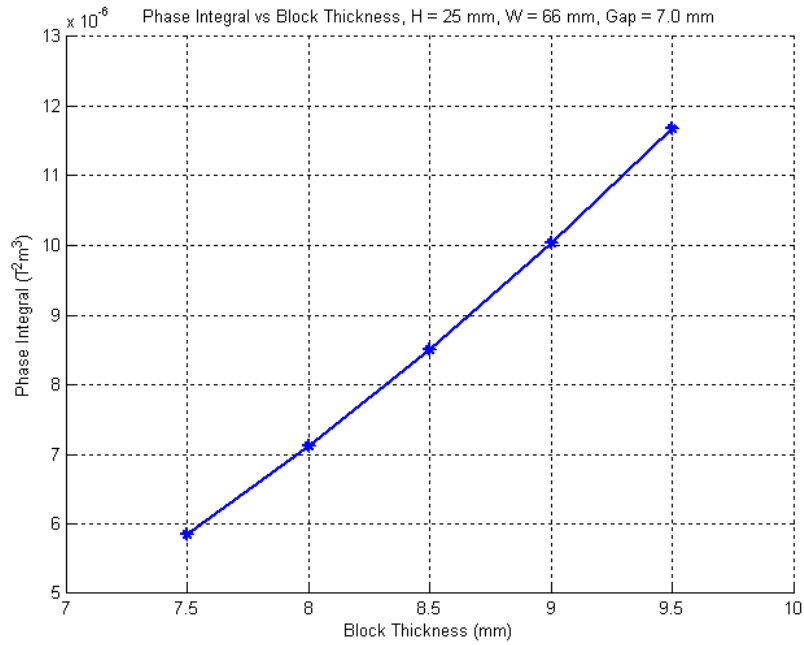


Figure 9: Phase integral as a function of the thickness of the vertically and horizontally magnetized blocks. The gap was 7.0 mm. The block height was 25 mm.

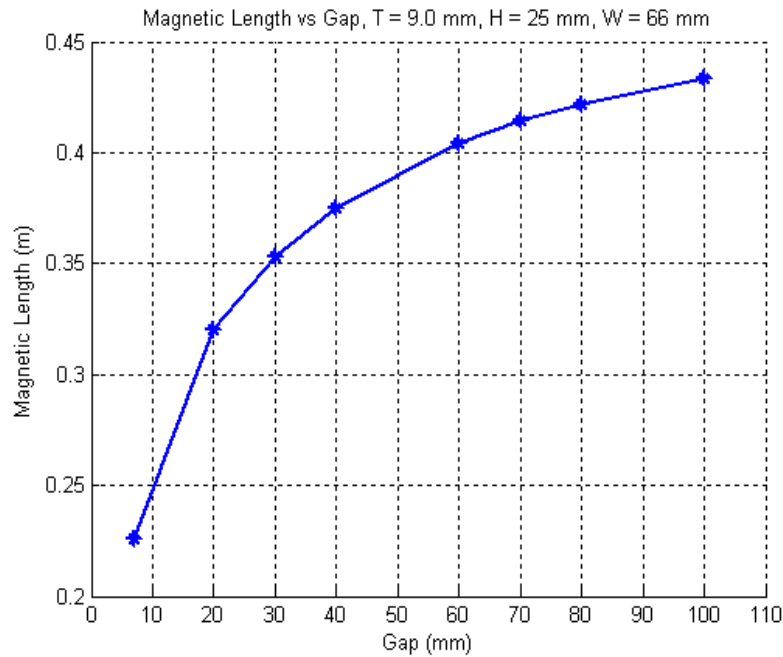


Figure 10: This figure shows the magnetic length as a function of gap. The block thickness is 9.0 mm, the block height is 25 mm, and the block width is 66 mm.

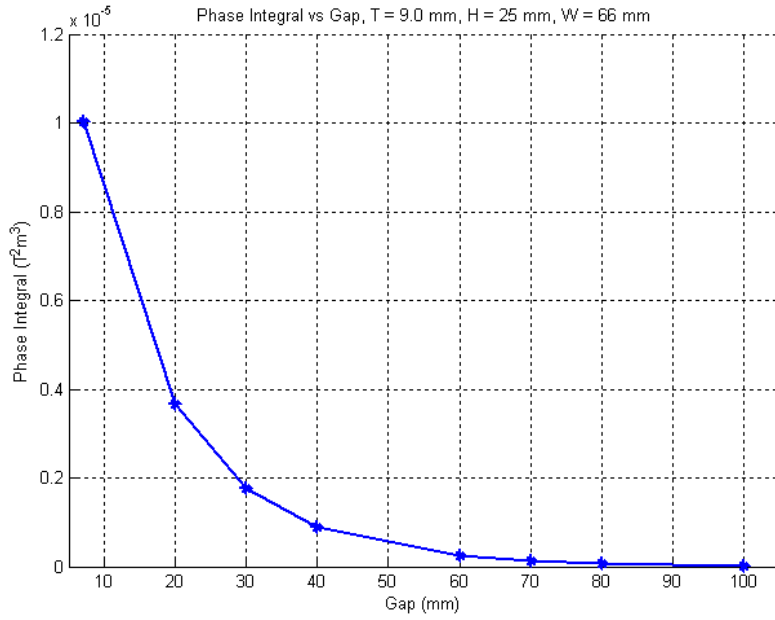


Figure 11: Phase integral as a function of gap. The block thickness is 9.0 mm, the block height is 25 mm, and the block width is 66 mm.

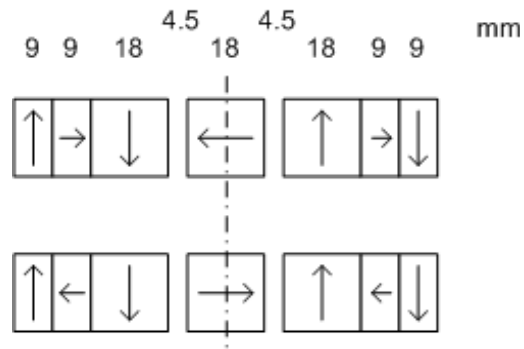


Figure 12: Design of the phase shifter.

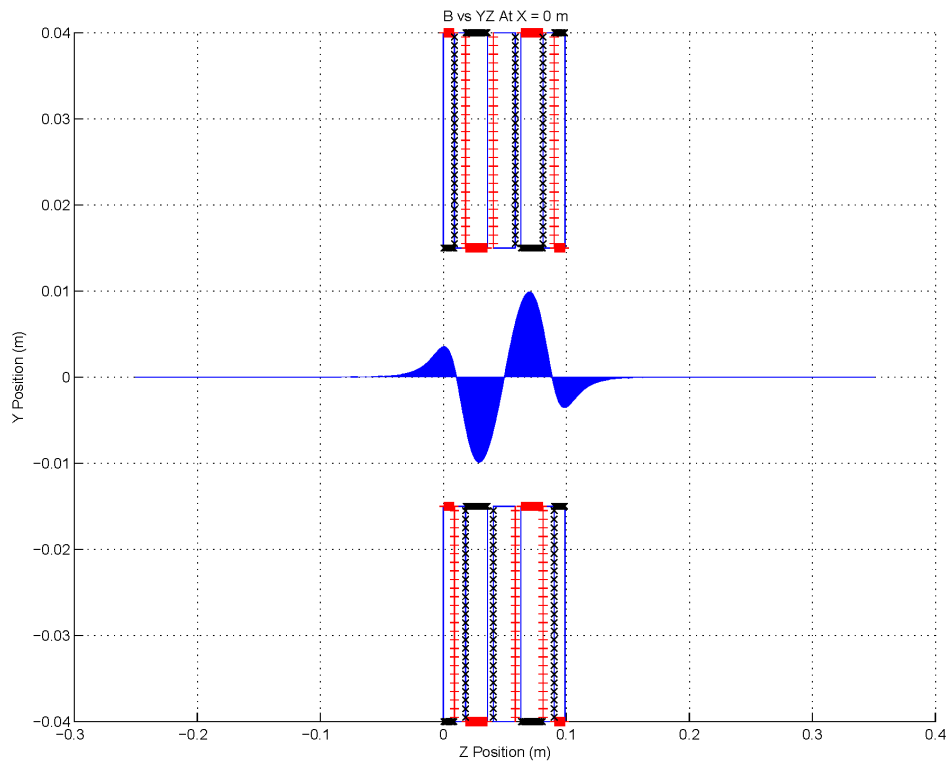


Figure 13: This figure shows the outline of the magnet blocks and the equivalent magnetic charges. The vertical field B_y is represented on the phase shifter axis.

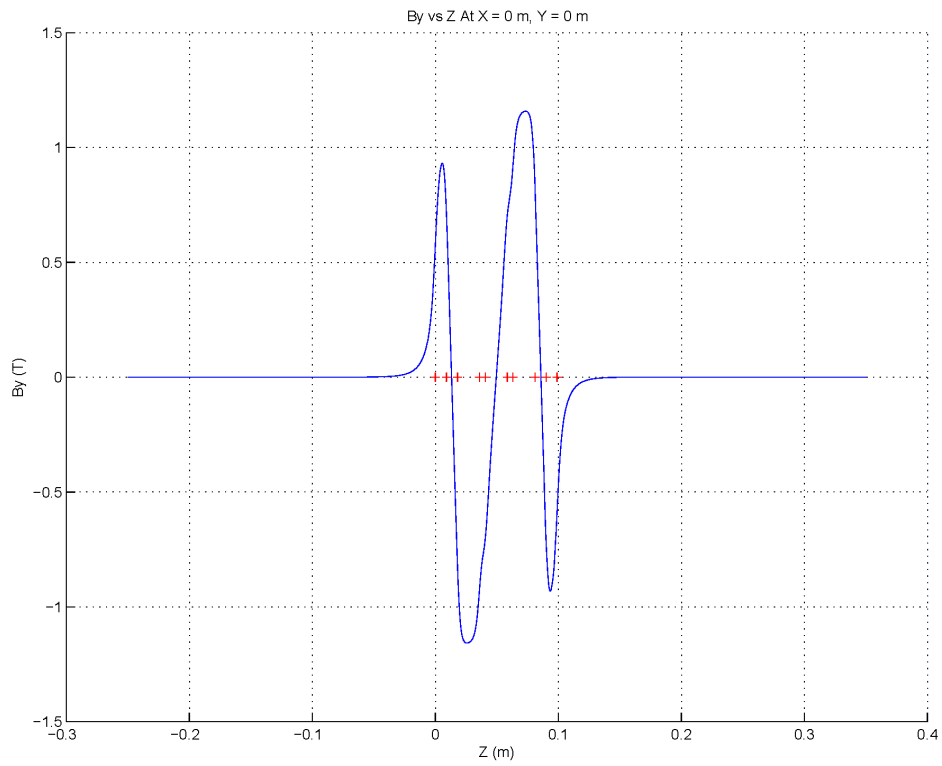


Figure 14: B_y as a function of z on the phase shifter axis. The positions of the magnet blocks are indicated.

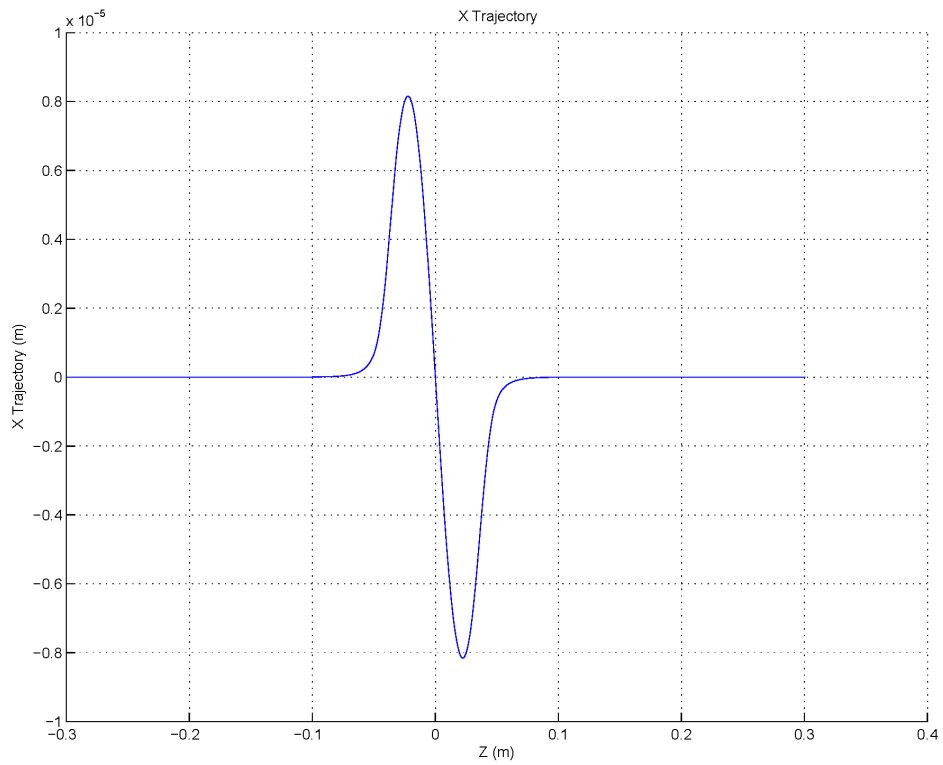


Figure 15: Horizontal trajectory of a 8.57 GeV beam in the phase shifter. The gap is 7 mm.

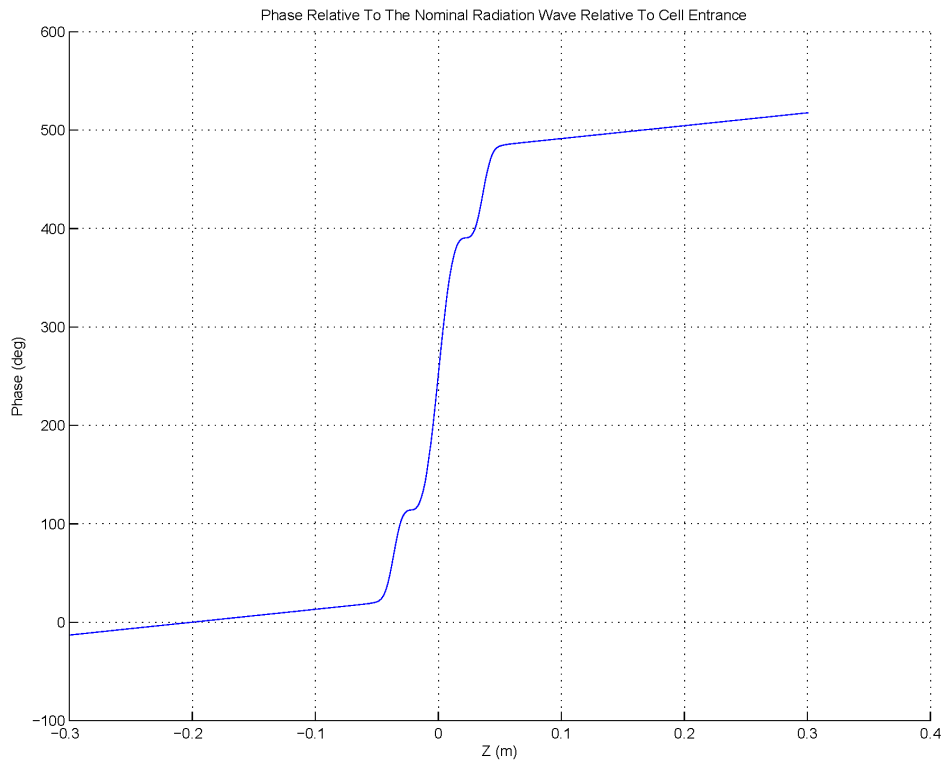


Figure 16: Phase shift through the phase shifter. The electron energy is 8.57 GeV and the photon energy is 253 eV, produced when the undulator K value is 9.9 and the undulator period is 55 mm.

5 Conclusion

A PPM phase shifter was designed which met the requirements appropriate for LCLS-II listed in this note. The advantage of this phase shifter is that it can be placed in the fringe fields of the undulators without making a slope change to the beam trajectories, and without accounting for the phase shifter during undulator tuning. At a gap of 7 mm, the phase integral is $1.004 \times 10^{-5} \text{ T}^2\text{m}^3$. The magnetic length of the phase shifter at a maximum gap of 70 mm is 414 mm.

Acknowledgements

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