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Roller Cam Positioners*

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Abstract

Roller Cam Positioners could support the LCLS undulator sections allowing micron sized alignment adjustment of each undulator in 5 degrees of freedom. The supports are kinematic with the number of degrees of freedom matched to the number of constraints. Ton loads are supported on simple ball bearings. Motion is intrinsically bounded. Positioning mechanisms are based on pure rolling motion with sub-micron hysteresis and micron resolution. This note describes a general purpose positioning mechanism suitable for undulator support.

1 Mechanical Design

A single module design can be used at all 5 roller cam supports. It consists of an aluminum block housing a 0.75mm lift camshaft with roller cam follower bearing supported on 2 deep groove ball bearings. A cross section of the assembly is shown in Figure(1).

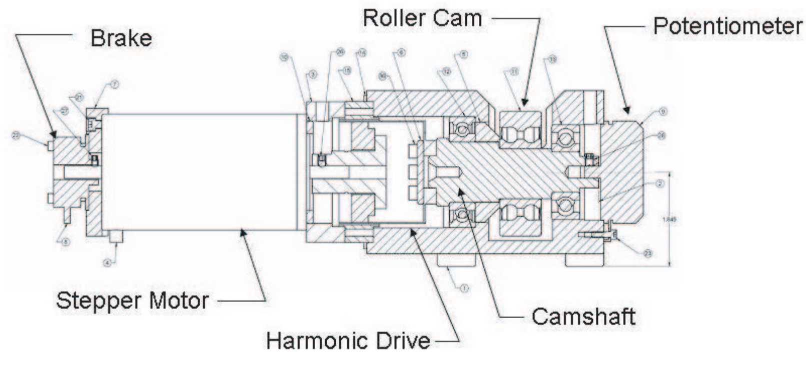

Figure 1: Cross section of roller cam module

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The camshaft is driven by a 1.9 N*m stepper motor through a 120:1 Harmonic Drive reduction unit. Angular step size for standard stepper motors is $(2\pi/200)$. The single step resolution of the positioner is $\Delta y = (\frac{2\pi}{200})(\frac{1}{120})(0.75\text{ mm}) \simeq 0.2$ microns. Ignoring frictional losses, motor lift capacity is $F = (1.9\text{ N} * \text{m})(120\text{ reduction})/(0.75\times10^{-3}\text{ m}) = 3.05\times10^{5}\text{ N} =$ 68000 lbs. Actual lifting force is reduced by friction but more than adequate for any load the bearings are able to support.

1.1 Bearings

Location	SKF part	size	static load (N)	lbs
left ball bearing	SKF 6006	55 OD, 30 ID, 13 W	8.3 kN	1866 lbs
cam roller bearing	SKF 305805 C-2Z	62 OD, 25 ID, 20.6 W	12.5 kN	2810 lbs
right ball bearing	SKF 6204	47 OD, 20 ID, 14 W	6.55 kN	1472 lbs

Table 1: Bearing sizes and loads

Bearing static load ratings are tabulated in Table(1). The cam roller bearing is made with a thickened outer race to support point loads. Its OD is slightly crowned to keep the undulator cradle contact point localized near the center of the bearing. Since motion range for the 3.4 meter long undulator is only $\simeq$ 1 mm, the direction of bearing loads changes little over motion range and self-aligning bearings are unnecessary. The load distribution of the 1800 lb undulator depends on the cam layout but for symmetric support, the cam contact loads are tabulated in Table(2).

Cam 1	Cam 2	Cam 3	Cam 4	Cam 5
$\frac{1800}{4}$	$\frac{\sqrt{2}*1800}{8}$	$\frac{\sqrt{2}*1800}{8}$	$\frac{1800}{4*\cos(37^\circ)}$	$\frac{1800}{4*\cos(37^\circ)}$
450 lbs	318.2 lbs	318.2 lbs	563.5 lbs	563.5 lbs

Table 2: Cam positioner loads

1.2 Motor Brake

Repositioning of undulators will be infrequent; once an hour to once a month. Positioners must hold undulator position static at the micron level without drift for long periods of time. Thermal movements play a major roll on this dimension scale. Holding position without electrical power is a design prerequisite. The positioner must be able to support all loads in the power-off state without the risk of back-driving. The largest back-drive torque at cams 4 & 5 due to undulator weight is $(0.75\text{ mm})(2.5\text{ kN}) = 1.88\text{ kN} * \text{mm}$. Reflected back through a frictionless 120:1 gear reduction, undulator weight applies a torque of 15.66 N*mm to the stepper shaft. The permanent magnets in the Compumotor ES23B-DNR10 provide a 20 N*mm detent torque which is very close to the friction free back-drive torque. Friction in the Harmonic Drive 120:1 gear reducer adds 50 N*mm in front of the the stepper motor detent torque so the drive system should be static in the power-off state. Nevertheless a small 24 volt power-off friction brake (API DELTRAN BRP-15) was fitted to the stepper motor on prototype movers providing an additional 560 N*mm of holding torque. When ever the stepper motor is energized, 24V DC must be applied to the brake to release the stepper armature shaft.

1.3 Gear Reduction

The camshaft is driven through a Harmonic Drive HDC-20-120-2A 120:1 reduction unit. This somewhat unusual mechanism consist of a fixed ring with internal splines which houses a thin flexible steel cup with splines on its external rim. The flex spline cup is stretched to engage the ring by an elliptical ball bearing race pressed into the open end of the cup. Splines engage 2 places where the major axis of the bearing pushes the cup outward. Rotation of the elliptical bearing moves the engagement of splines around the ring as the elliptical bearing rotates inside the cup. One less spline on the cup than on the ring causes a vernier precession of the engaged splines. The input elliptical bearing must rotate 120 times for the cup output to complete one revolution. This mechanism is essentially free of backlash with a hysteresis $<$ 2 arc min or .44 microns at .75 mm lift. (Undulator loads are oriented such that shaft torque never reverses avoiding most of any system hysteresis or back-lash.) The Harmonic Drive torsional stiffness is non-linear. Two torsional constants are published. The initial low torque value for the size -20 drive unit is 1.3×10^4 N $*$ m/radian. The corresponding cam deflection stiffness $k = 1.3 \times 10^4$ N/radian/.00075 m $= 1.733 \times 10^7$ N/m. The force on cams 4 & 5 (563.5 lbs/.2248 lbs/N = 2.5 kN) gives a resonant frequency of

$$f = \sqrt{(1.733 \times 10^7 \text{ N/m})(9.8 \text{ m/sec}^2)/2.5 \times 10^3 \text{ N}}/2\pi = 41.48 \text{ Hz}.$$

It is difficult to build support structures for large accelerator magnets with stiffness above 30Hz. At 40 Hz, cam support stiffness will detract little from the overall mount stiffness.

1.4 Camshaft angle encoder

Because of a cam positioner's intrinsically nolinear kinematics, control requires knowledge of the absolute shaft angle. Incremental digital encoders require return to home on each power-up and high resolution absolute digital encoders are expensive. In this design, camshaft angle is monitored by precision resistance potentiometers (Novotechnik model 6500, 5kΩ) with .05% linearity. This should allow .2° positioning which corresponds to 2-3 microns absolute dead reckoning. In actual use, other more sensitive data from beam operation will be used to compute needed adjustment. Potentiometer signals will only be used inside the positioner control loop to compute the number of steps needed achieve the desired incremental motion The coefficient connecting camshaft degrees turned to microns moved varies with the absolute shaft angle and potentiometers supply this number with sufficient accuracy to make the next move.

2 Kinematics

The connection between camshaft rotations and undulator motion are nonlinear transcendental trigonometric relations. Linear approximations are inadequate if any sizeable portion of the adjustment range is to be used. Derivation of analytic kinematic formulae is straight forward. Since the motions are small (on the order of a millimeter or less) and the undulator is over 3 meters long, the problem can be divided into two separate 2D geometry problems, one for the 3-cam cradle support at the front and another for the 2-cam support at the rear of the undulator. There are several possible formats in which to specify rigid body motion in 3 dimensions. The most common is a set of 3 orthogonal displacement components x, y, z for the center of mass plus 3 rotation angles: pitch, roll, and yaw. Another equivalent representation is to give the x, y position and roll of the undulator in the plane of the front 3-cam support and add to this the x, y position of the undulator in the plane of the rear 2-cam support. The undulator will have its z position restrained axially by a flexure rod which has roll, x and y flexibility.

2.1 Front 3-cam support 2D kinematics

At the front there is a support cradle with 3 cam contact surfaces supporting the undulator on roller cams. In this particular design, the 3 contact surfaces are straight lines. This 2D geometry is illustrated in Figure 2. Derivation of the relation between undulator position and camshaft angles $\theta_1, \theta_2, \theta_3$ starts by expressing the 3 cradle cam supports as lines in 2D:

$$y_i = a_i x_i + b_i \tag{1}$$

With the undulator in its home position $x, y, \alpha = 0$, the 3 cam contact lines have slopes a_i and y axis intercepts b_i set by the cradle design geometry.

$$\begin{array}{ll} a_1 = 0 & b_1 = -v_1 + R \\ a_2 = \tan(-\frac{\pi}{4}) & b_2 = h_{23} - \frac{s_{23}}{2} - v_1 + \sqrt{2}R \\ a_3 = \tan\frac{\pi}{4} & b_3 = -h_{23} - \frac{s_{23}}{2} - v_1 + \sqrt{2}R \end{array} \tag{2}$$

Undulator position adjustment can be thought of as a 2 step process: first rotate undulator about beam axis by angle α and then translate the rotated undulator by $\Delta x, \Delta y$. Any point on the undulator originally at (x, y) will move to new coordinates (x', y'). The new position (x', y') is related to the initial position (x, y) by a rigid body rotation through angle α followed by a translation $(\Delta x, \Delta y)$:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \tag{3}$$

After motion, new slopes a'_i and new y intercepts b'_i can be derived by substituting $[x', y'] = [0, b']$ and $y = ax + b$ into Eq(3):

$$\begin{array}{ll} a'_1 = \tan(\alpha - 0) & b'_1 = \Delta y + b_1\cos\alpha + (b_1\sin\alpha - \Delta x)(\sin\alpha + a_1\cos\alpha)/(\cos\alpha - a_1\sin\alpha) \\ a'_2 = \tan(\alpha - \frac{\pi}{4}) & b'_2 = \Delta y + b_2\cos\alpha + (b_2\sin\alpha - \Delta x)(\sin\alpha + a_2\cos\alpha)/(\cos\alpha - a_2\sin\alpha) \\ a'_3 = \tan(\alpha + \frac{\pi}{4}) & b'_3 = \Delta y + b_3\cos\alpha + (b_3\sin\alpha - \Delta x)(\sin\alpha + a_3\cos\alpha)/(\cos\alpha - a_3\sin\alpha) \end{array}$$

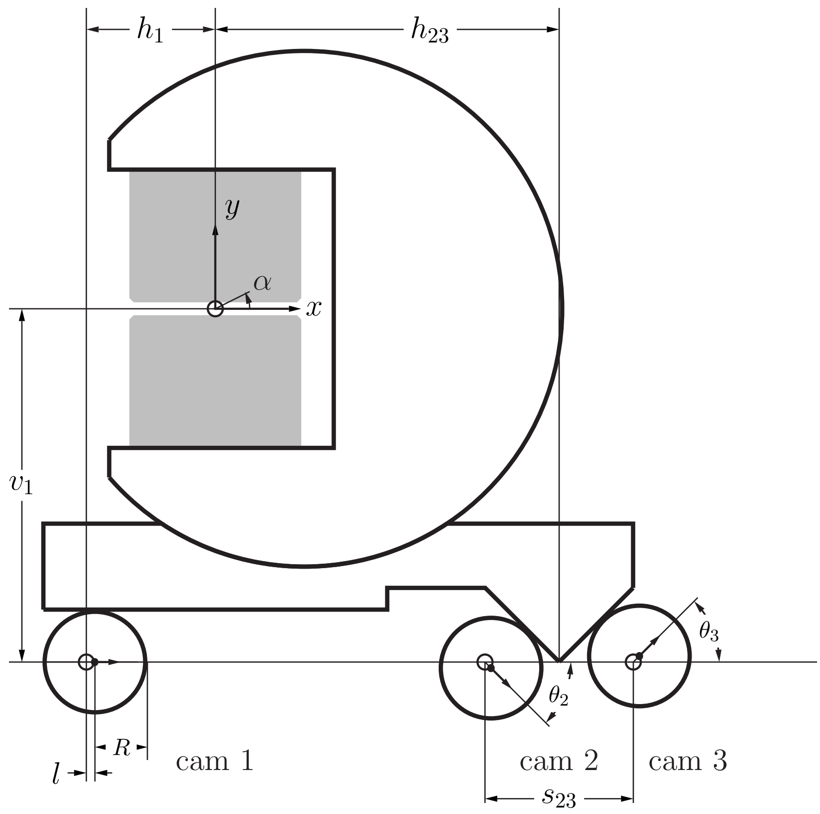


Figure 2: 3-cam undulator cradle in home position with $x, y, \alpha = 0$.

For the cradle geometry set by Eq(2) with cradle contact surfaces at $0°, -45°, +45°$, the slopes a_i and y intercepts b_i simplify to:

$$\begin{array}{ll} a'_1 = \tan(\alpha - 0) & b'_1 = \Delta y + b_1 \cos\alpha + (b_1 \sin\alpha - \Delta x)\tan\alpha \\ a'_2 = \tan(\alpha - \frac{\pi}{4}) & b'_2 = \Delta y + b_2 \cos\alpha + (b_2 \sin\alpha - \Delta x)\tan(\alpha - \frac{\pi}{4}) \\ a'_3 = \tan(\alpha + \frac{\pi}{4}) & b'_3 = \Delta y + b_3 \cos\alpha + (b_3 \sin\alpha - \Delta x)\tan(\alpha + \frac{\pi}{4}) \end{array} \tag{4}$$

Once the displaced cradle contact line parameters a'_i, b'_i, [Eq(4)] are calculated, the 3 camshaft angles $\theta_1, \theta_2, \theta_3$ can be calculated. Each cam contacts its cradle support surface when its cam center is rotated onto line $y = a'x + b' - R/\cos\tan^{-1} a'$ which is parallel to the cradle surface and offset by cam radius R as shown in Figure 3. Note that if the cradle surface is within reach of the cam, there are two camshaft angles which will satisfy the contact condition. Angles $\theta_1, \theta_2, \theta_3$ can be solved for by substituting their cam center coordinates:

$$x0_i = [(-h1),\ \ (h_{23} - s_{23}/2),\ \ (h_{23} + s_{23}/2)] \quad \text{and} \quad y0_i = [-v_1,\ \ -v_1,\ \ -v_1] \tag{5}$$

into equations for the cam centers

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x0_i + l\cos\theta_i \\ y0_i + l\sin\theta_i \end{bmatrix} \tag{6}$$

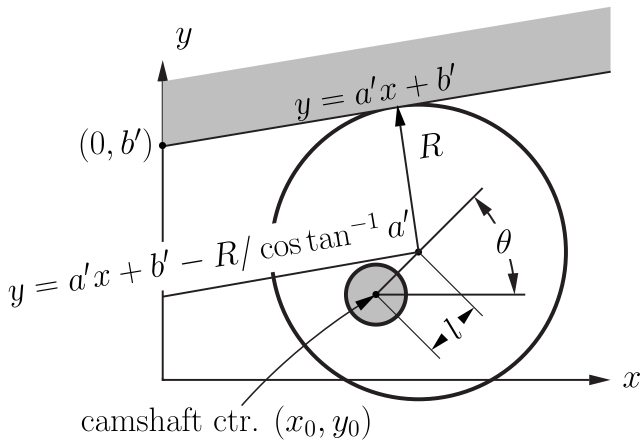


Figure 3: Camshaft angle θ for contact between cam radius R and cradle contact surface specified by line $y = a'x + b'$.

Cam shaft angles are solved for by substitution of $[x, y]$ from Eq(6) into the line equation

$$y = a_i'x + b_i' - R/\cos\tan^{-1} a_i' \tag{7}$$

This substitution generates the trigonometric equations:

$$-a_i'\cos\theta_i + \sin\theta_i = (a_i'x0_i - y0_i + b_i' - R/\cos\tan^{-1} a_i')/l \tag{8}$$

These equations are of the general form

$$\mathbf{a}\cos\theta + \mathbf{b}\sin\theta = \mathbf{c} \tag{9}$$

which has roots

$$\theta = \sin^{-\mathbf{1}} \frac{\mathbf{c}}{\pm\sqrt{\mathbf{a^2} + \mathbf{b^2}}} - \tan^{-\mathbf{1}} \frac{\mathbf{a}}{\mathbf{b}} \tag{10}$$

subject to the constraint that the cradle contact surface is within reach of the cam:

$$\left| \frac{\mathbf{c}}{\sqrt{\mathbf{a^2} + \mathbf{b^2}}} \right| \leq \mathbf{1} \tag{11}$$

For the specific case of Eq(8),

$$\mathbf{a} = -a_i'; \qquad \mathbf{b} = 1; \qquad \mathbf{c} = (a_i'x0_i - y0_i + b_i' - R/\cos\tan^{-1} a_i')/l\ . \tag{12}$$

Substitution of these values for $\mathbf{a}, \mathbf{b}, \mathbf{c}$ into Eq(10) yield the 3 camshaft angles $\theta_1, \theta_2, \theta_3$.

2.2 Rear 2-cam support 2D kinematics

At the undulator rear a circular support cradle is illustrated in Figure 4. The two camshaft angles θ_4, θ_5 are determined by the coordinates of the cradle center (x_b, y_b). After rigid body motion $\Delta x, \Delta y, \alpha$, Eq(3) gives:

$$\begin{bmatrix} x'_b \\ y'_b \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} h_{45} \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \tag{13}$$

The dimension h_{45} can be chosen such that the twist caused by gravity acting on the nonsymmetric undulator strong back can be cancelled [1]. In the case of a circular support cradle, lines of length $(R + R_b)$ connect each cam center to cradle center (x'_b, y'_b). The camshaft angles which meet this distance constraint are determined by equations with the form of Eq(9). For θ_4, θ_5 the values to substitute into Eq(9) for $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are tabulated in Table 2.2 .

Table 2.2

	a	**b**	**c**
θ_4	$2l(x'_b - h_{45} + s_{45}/2)$	$2l(y'_b + v_{45})$	$(x'_b - h_{45} + s_{45}/2)^2 + (y'_b + v_{45})^2 + l^2 - (R + R_b)^2$
θ_5	$2l(x'_b - h_{45} - s_{45}/2)$	$2l(y'_b + v_{45})$	$(x'_b - h_{45} - s_{45}/2)^2 + (y'_b + v_{45})^2 + l^2 - (R + R_b)^2$

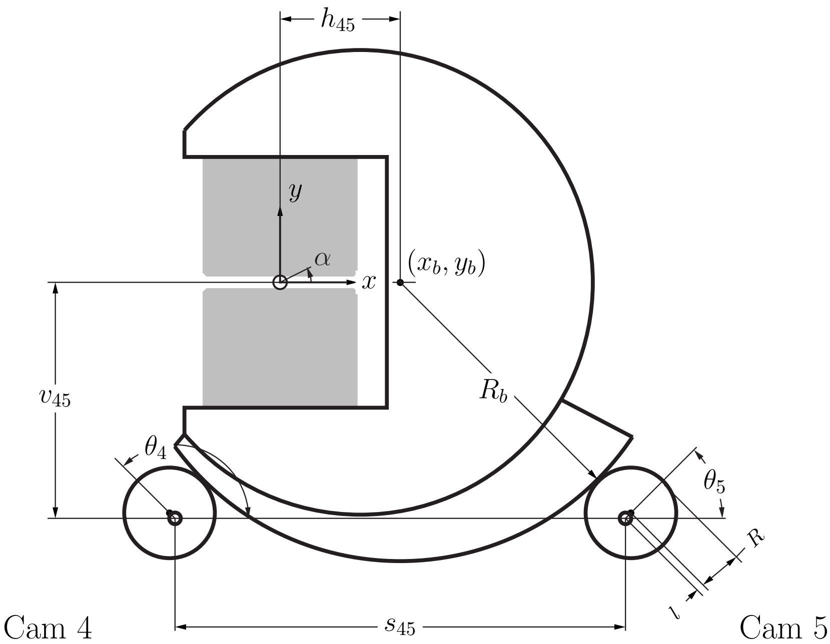


Figure 4: Rear 2-cam cradle support in home position $x, y, \alpha = 0$. Cams are placed symmetrically on each side of cradle center (x_b, y_b).

2.3 Undulator positioning algorithm

The kinematics of roller cam support requires a computer not only for control of stepper motors but also for calculation of the desired camshaft angles. Design dimension inputs to the camshaft algorithm are: $R, l, v_1, h_{23}, s_{23}, v_{45}, h_{45}, R_b, x_b, y_b$. The 5 input position variables are: $\Delta x_{front}, \Delta y_{front}, \alpha, \Delta x_{rear}, \Delta y_{rear}$. Camshaft angles $\theta_{1\rightarrow 5}$ are all roots [Eq(10)] of the same general trigonometric equation $\mathbf{a}\cos\theta + \mathbf{b}\sin\theta = \mathbf{c}$ [Eq(9)] where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ & θ are 5-element arrays. The values of $\mathbf{a}, \mathbf{b}, \mathbf{c}$ for the first 3 cams come from equation (12). The 4th and 5th values come from Table 2.2.

2.4 Cam positioning range

The range of accessible motion provided by cam positioners is set by their lift. The front 3-cam support determines roll α as well as displacement $\Delta x, \Delta y$ which complicates the description of the reachable space. But if roll α is preset to zero, the space reachable by the front 3-cam support is bounded top and bottom by the lift l of cam 1. For a 45° **V** cradle on cams 2 and 3, the space is also bounded at 45° lines at $\pm l$ of cams 2 and 3 as illustrated in Figure(5). The plotted circle of radius l in $\Delta x, \Delta y$ is equivalent to a square diamond in θ_2, θ_3 centered at $135°, 45°$.

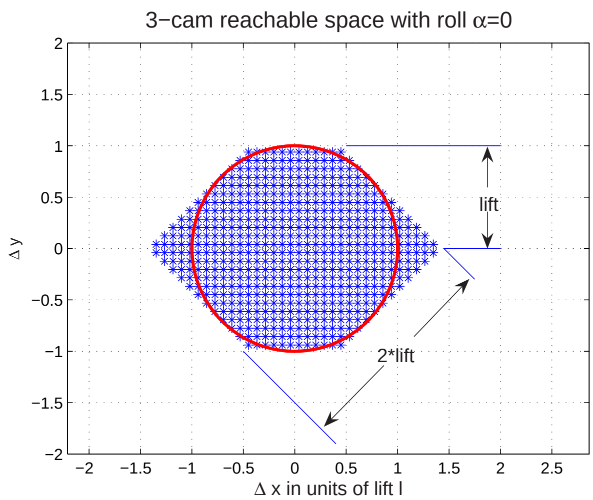
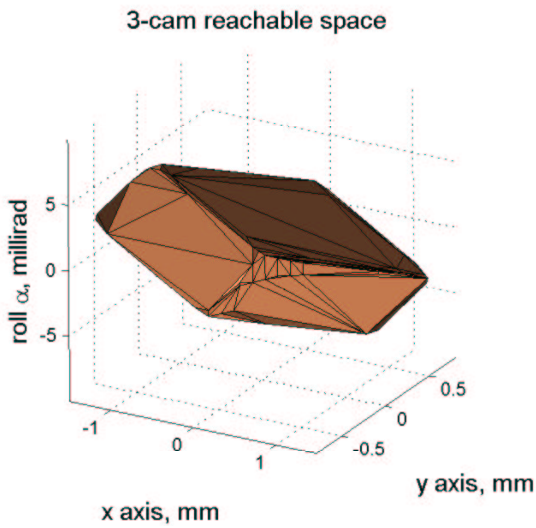


Figure 5: $\Delta x, \Delta y$ positions reachable by front 3-cam support with roll α held constant by cam 1. Units are in cam lift l.

If roll α is allowed to vary, the reachable space is a 3 dimensional volume with dimensions $\Delta x, \Delta y, \alpha$. The shape of this volume is influenced by the dimensions of the cradle layout as well as cam lift. To a good approximation, the motion boundary is a rectangular block volume in $\Delta x, \Delta y, \alpha$ space as illustrated in Figure(6).



support dimensions		
R	31.0	mm
l	0.75	mm
h_1	76.2	mm
h_{23}	203.2	mm
s_{23}	87.38	mm
v_1	208.79	mm

Figure 6: Reachable space for 3-cam support

When roller cam support is located below the undulator, the Δx position is coupled to roll α. If α is allowed to vary, this rocking significantly increases the bounds on Δx motion. Figure(7) plots the 3-cam reachable space projected onto the $\Delta x, \Delta y$ plane. The inscribed boundary of motion with roll $\alpha = 0$ is plotted for comparison.

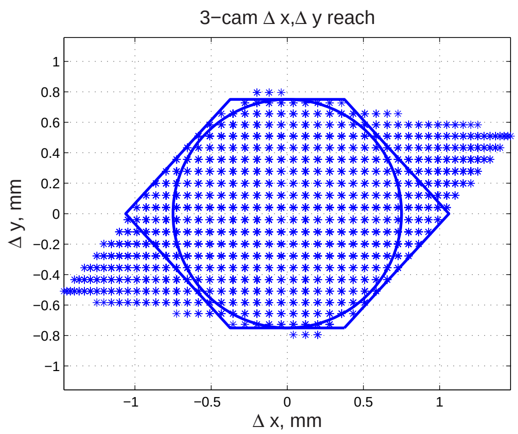


Figure 7: $\Delta x, \Delta y$ positions reachable by front 3-cam support for all values of roll α. Cradle support dimensions are same as Figure(6).

To reduce coupling between roll and Δx displacement it is necessary to move the cam cradle up to beamline height. Making dimension $v_1 = R$ diminishes the interaction between Δx and α. Such a layout is illustrated in Figure(8).
A small range of roll adjustment is useful for maintaining alignment but beam pipe bellows do not have the torsional compliance needed to absorb significant roll motion. Roll motion can be limited by design geometry or held to zero by control software but design of a mechanical linkage to constrain roll to zero is probably impractical for roller cam support.

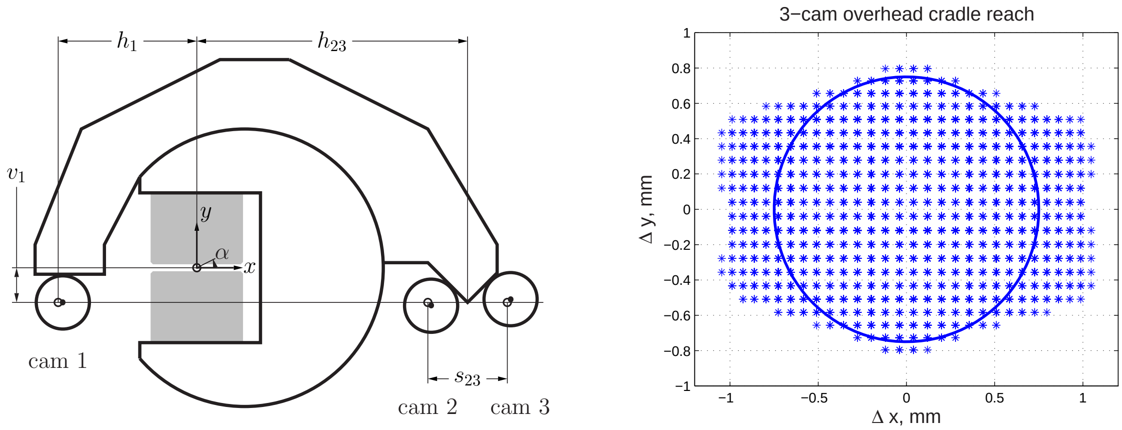


Figure 8: 3-cam overhead cradle decouples roll and Δx motion

References

[1] SLAC PUB 10958, LCLS-TN-02-6, *Undulator Gravitational Deflection*, G.Bowden, Oct. 2002.

[2] *Theoretical Kinematics*, Bottome and Roth, Dover, 1979, ISBN 0-486-66346-9

[3] *Ebene Kinematik*, W. Blaschke und H.R.Müller, 1956