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Wakefields in the LCLS Undulator Transitions*

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Abstract

We have studied longitudinal wakefields of very short bunches in non-cylindrically symmetric (3D) vacuum chamber transitions using analytical models and the computer program ECHO. The wake (for pairs of well-separated, non-smooth transitions) invariably is resistive, with its shape proportional to the bunch distribution. For the example of an elliptical collimator in a round beam pipe we have demonstrated that—as in the cylindrically symmetric (2D) case—the wake can be obtained from the static primary field of the beam alone. We have obtained the wakes of the LCLS rectangular-to-round transitions using indirect (numerical) field integration combined with a primary beam field calculation. For the LCLS 1 nC bunch charge configuration we find that the total variation in wake-induced energy change is small (0.03% in the core of the beam, 0.15% in the horns of the distribution) compared to that due to the resistive wall wakes of the undulator beam pipe (0.6%).

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INTRODUCTION

In the LCLS with the nominal, 1 nC of charge configuration the longitudinal wakefields in the undulator region are significant and will affect the laser performance. The dominant contribution to the wakefields is the resistive wall wake (with the ac conductivity included) of the undulator beam pipe [1]. To ameliorate the wake effects it has been proposed to change the pipe inner surface material from copper to aluminum and to change the cross-section from a round to an approximately rectangular shape (while leaving the vertical aperture fixed). Within the undulator, however, the beam pipe will need to be interrupted 33 times by pipes with round cross-sections that house the beam position monitors. Thus there will be 33 pairs of rectangular-to-round transitions. In this note we calculate the (geometric) wakefields generated in these transitions.

To solve this problem by direct (numerical) integration of the fields as the beam moves through the structure is problematic for the short LCLS bunch: to allow the wake to "catch-up" to the beam, integration through a long exit beam pipe is required, a procedure that tends to accumulate numerical errors. An indirect integration algorithm for non-cylindrically symmetric (3D) structures is required. For 3D cavity-like structures with beam pipes, where the minimum aperture of the structure coincides with the beam pipe aperture, an indirect procedure based on the solution of Laplace's equation was derived by T. Weiland and R. Wanzenberg [2]. For cylindrically symmetric (2D), collimator-like structures, an indirect procedure was developed by O. Napoly, *et al*; according to this method, after the beam passes the collimator, the long on-axis integration of fields is replaced by a radial integration to the walls [3]. In this report we make use of an approach to solving this problem that is applicable to the (3D) LCLS rectangular-to-round transitions.

The geometry of an LCLS transition pair is shown on Fig. 1. The rectangular pipe has dimensions 10 mm by 5 mm (horizontal x by vertical y) and the round pipe has radius 4 mm. The length of a rectangular section is on the order of 3 m, and of a round section $\geq$ 470 mm. Because of the large distance between transitions, a pair of transitions has the same wakefield irrespective of which transition comes first. In this report numerical calculations are done with ECHO, a finite-difference, time-domain program that can accurately obtain wakefields of short bunches in long structures (it generates minimal errors due to "mesh dispersion" and mesh-to-boundary mismatch) [4]; the meshing is carried out in Microwave Studio [5]. Beam parameters in the LCLS undulator region are given in Table I.

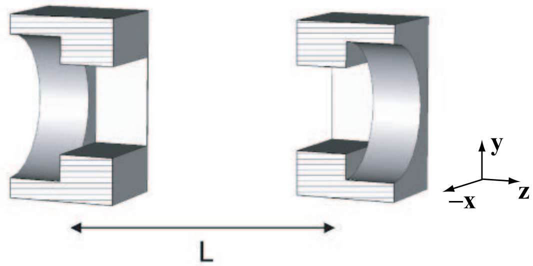

FIG. 1: A pair of LCLS rectangular-to-round transitions shown in a longitudinal cut view.

TABLE I: Selected beam parameters in the LCLS undulator region.

Parameter	Value
Charge, Q	1 nC
Length (rms), σ_z	20 μm
Energy, E	14 GeV

A 2D MODEL

Analytical Wake

Before proceeding to the 3D calculations let us make an estimate of the wake potential using a 2D (cylindrically symmetric) model. Consider the transition from a round pipe of radius $a = 2.5$ mm to one of radius $b = 4$ mm and back again, or, equivalently, the collimator of radius a with beam pipes of radius b. [Since g is large compared to $2(b-a)^2/\sigma_z$, with $g = 470$ mm the length of the central region and $\sigma_z = 20$ μm the rms bunch length, the impedance is about the same for both configurations.] Note that, since the aperture of the 2D model is everywhere less than or equal to that of the real (3D) transition, we expect the 2D estimate of the wake to be somewhat pessimistic.

For short bunches ($\sigma_z \ll a$) the diffraction model of Heifets and Kheifets (H&K) applies [6], and the wake is resistive (the wake shape is the same as the bunch distribution). For a Gaussian bunch the wake is [6]

$$W(s) = -\frac{Z_0 c \ln(b/a)}{\sqrt{2}\pi^{3/2}\sigma_z} \, e^{-\frac{1}{2}s^2/\sigma_z^2} \ , \tag{1}$$

with s position within the bunch, $Z_0 = 377\ \Omega$, and c the speed of light. (Eq. 1 is valid provided $\sigma_z \gtrsim a/\gamma$, with γ the Lorentz energy factor.) Eq. 1 implies that the impedance at high frequencies (specifically, for $c/\sigma_z \lesssim \omega \lesssim c\gamma/a$) is constant and given by

$$Z_{hi} = \frac{Z_0}{\pi} \ln(b/a) \ . \tag{2}$$

The loss factor (minus the average wake), in terms of Z_{hi}, is given by

$$k_{loss} = \frac{Z_{hi} c}{2\sqrt{\pi}\sigma_z} \ . \tag{3}$$

For our parameters we find that $Z_{hi} = 56.4\ \Omega$ and $k_{loss} = 238$ V/pC. The minimum wake $W_{mn} = -337$ V/pC and the rms of the wake $W_{rms} = 94$ V/pC.

Before leaving this section, let us recall that H&K explained that the wake (as well as the impedance) of one round transition alone can be thought of as composed of a radiation part W^r and a (static) potential energy part W^e. The part W^e is the difference in potential within the primary beam field in the two different pipes. In an in-transition (from big pipe to small) $W^{in} = W^r - W^e$, and in an out-transition (from small pipe to big) $W^{out} = W^r + W^e$. Note that W_r is the same in both cases, since the (radiated) impedance of a transition depends on its shape and not on the direction of traversing it [7]. In their numerical study, H&K further observed that at high frequencies the two parts W^r and W^e were (essentially) equal: for an in-transition, they cancel to give zero wake (one can think of the fields beyond a as being cleanly clipped away, with the beam never aware that a change in the walls occurred); for an out-transition the effect is just twice the potential energy part of the wake. Thus the wake of a transition pair taken together is $W = 2W^e$. We review this here because later, when discussing 3D transitions, we will make use again of some of these concepts.

Numerical Calculations

Numerically obtaining the wake for this problem by direct integration of the electric fields as the bunch passes the transitions is difficult, since it takes a long time for the scattered fields to catch up to the bunch. [The catch-up distance $z_{cu} \gtrsim 2(b-a)^2/\ell$, with ℓ a fraction of σ_z.] A modification of the Napoly indirect method is employed by the computer program ECHO for 2D calculations [8]. An ECHO calculation was performed for a transition pair of the 2D model; for the calculation the pair was modelled as a collimator of length $g = 20$ mm. We find that the numerical result agrees well with the analytical approximation (see Fig. 2). Note that, for this problem, a numerical check on the result finds the catch-up distance to be $z_{cu} = 2.3$ m (or $\ell \approx \sigma_z/10$).

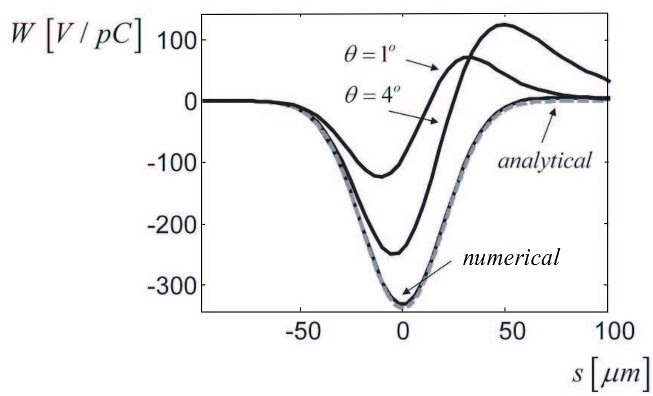

FIG. 2: Wake potential for the 2D model of a pair of LCLS transitions. The analytical result, Eq. 1, is given by dashes. Also shown are 2 results for tapered transitions (discussed below). The bunch is Gaussian with $\sigma_z = 20$ μm (the head is to the left).

Effect of Tapering

To investigate the effect of tapering, we performed calculations for pairs of 2D transitions with taper angles, θ (see Fig. 3). Results, for $\theta = 1°$ and $4°$, are plotted In Fig. 2; k_{loss}, W_{rms}, and W_{mn}, for several values of θ, are given in Table II (the earlier results are labelled $\theta = 90°$). We note that, for the short LCLS bunch, tapering is of little help in reducing the wake, unless the taper angle is very shallow. This is because diffraction radiation is projected at a shallow angle. So, for example, when a short bunch passes by an out-transition, a significant reduction in the wake will not happen until the tapered walls cut into this cone of radiation, *i.e.* not until $\tan\theta \sim \sigma_z/a$, which here implies $\theta \sim 0.5°$. In Table II we see that *e.g.* the rms moment of the wake, W_{rms}, does not begin to significantly decrease until $\theta \lesssim 1°$, in reasonable agreement with this estimate.

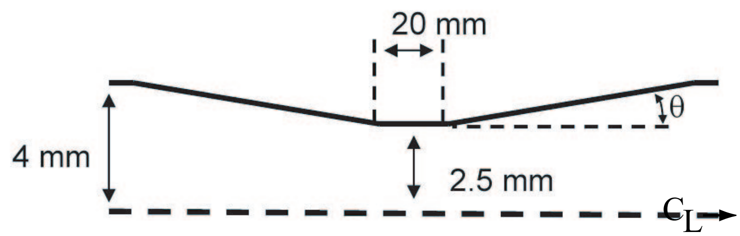

FIG. 3: A tapered, cylindrically symmetric collimator used in 2D simulations.

TABLE II: Wake parameters for the round collimator of Fig. 3; $\sigma_z = 20$ μm.

θ [deg]	k_{loss} [V/pC]	W_{rms} [V/pC]	$-W_{mn}$ [V/pC]
90. (analytical)	238.	94.	337.
90.	235.	93.	331.
8.5	191.	96.	290.
4.	148.	101.	249.
2.	88.	91.	182.
1.	50.	65.	124.
0.5	33.	41.	80.

3D TRANSITIONS

Collimator in Round Beam Pipe

The indirect method has been extended in ECHO to 3D structures that end in a round beam pipe (to be presented in a future report). For calculating the longitudinal wake on axis, as we want to do here, the procedure is relatively simple. For short bunches ECHO employs a moving mesh that encloses the beam longitudinally and reaches transversely to the structure walls. The first part of the calculation is direct integration of the wake forces as the beam passes through the beginning of the structure. When the moving mesh finally is entirely within the round exit pipe region the monopole moment of the transverse field (over the mesh) is extracted; this field is then integrated radially to the wall, following the procedure of the 2D indirect method.

As a concrete example consider an elliptical collimator in a round beam pipe (see Fig. 4), and a Gaussian bunch with $\sigma_z = 25$ μm. The wake as obtained by the just-described indirect method is shown in Fig. 5 (the solid curve). The loss factor $k_{loss} = 116.6$ V/pC. To obtain an analytical result, we calculate the potential difference in the primary beam field when it is in the elliptical *vs.* in the round pipe, giving us W^e (more details of such a calculation are given in the next section); then, as in the 2D case, we let $W = 2W^e$. The result is shown by dashes in Fig. 5, and we see good agreement with the numerical result ($k_{loss} = 118.5$ V/pC). This good agreement suggests that this analytical method can be used to find the short-bunch wakes of a large class of 3D collimators (specifically, those that are abrupt-edged and translationally symmetric).

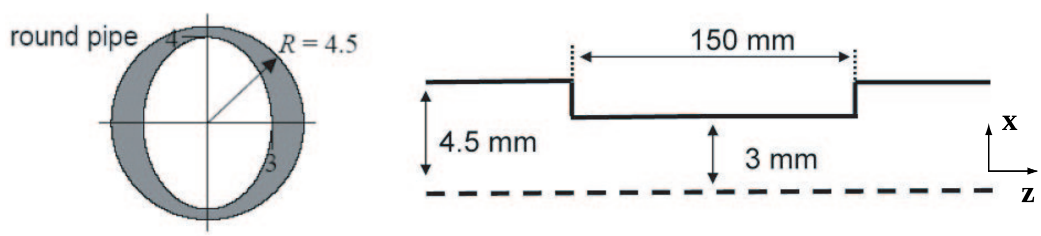

FIG. 4: Geometry of 3D collimator in a round beam pipe.

Before leaving this section, recall that in the 2D (round) collimator case W^e could be found either by the change in potential in the primary field in the two pipes, or by the potential in the field "clipped" away by the collimator: the two calculations gave the same answer. For the 3D collimator this is no longer true. This is because the steady-state field pattern within the collimator aperture (here elliptical) differs from that in the beam pipe (here round). Thus, we find here, according to a clipping calculation, $k_{loss} = 110.3$ V/pC; this result is different from before, but it agrees well with a direct calculation for a short

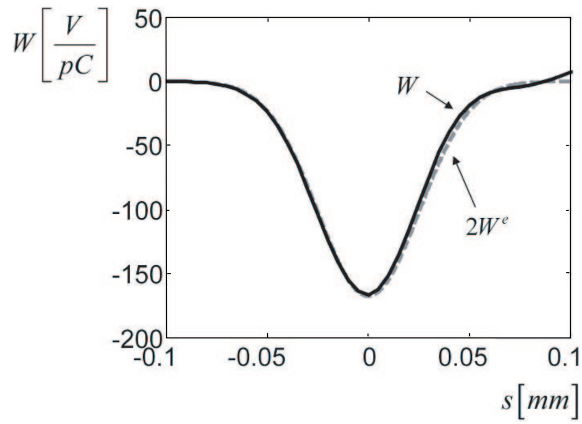

FIG. 5: Wake of a $\sigma_z = 25$ μm Gaussian bunch in the elliptical collimator (head is to the left): comparison of the indirect numerical result (solid line) with the analytical potential energy calculation (dashes).

(3 mm-long) elliptical collimator, where $k_{loss} = 109.9$ V/pC. The implication is that, when a beam encounters a collimator it takes time for its fields to adjust themselves to a new steady-state configuration. Thus, for a short collimator, a clipping calculation is correct; for a long collimator the change in potential in the two pipes gives the correct answer.

LCLS Rectangular-to-Round Transitions

Consider now the pair of LCLS rectangular-to-round transitions shown in Fig. 1. Both the rectangular and the round regions are long and we can consider them independently. First note that the analytical methods used above for the elliptical collimator—obtaining the wake from the primary field potential alone—cannot be used here, since neither cross-section is inscribed entirely within the other. Also note that the indirect numerical method, described above, can only be applied to the rectangular-to-round half of the pair since it ends in a round pipe (and not to the converse, round-to-rectangular one). The wake of the pair of transitions is given by

$$W = W^{in} + W^{out} = 2(W^{out} - W^e) \ , \tag{4}$$

where W^{out} represents the numerically-obtained, rectangular-to-round wake and W^e is the potential difference in the primary beam fields of the two pipe cross-sections.

To obtain W^e we first use a Matlab Poisson equation solver to find the field energy for the analogous 2D (x-y) problem with our cross-section geometries. Fig. 6 shows the field energy density for the two cases. For each geometry the total field energy diverges at the origin, but the difference between the two totals does not (near the origin the fields are the same). In the numerical calculation of field energy, to avoid the singularity at the origin, the integration was stopped at a small circle of radius ϵ. In Table II we give several values of $w(\epsilon) = u/Q^2$, with u the energy in the (2D) fields, and the difference in w between the two regions. [For the round case w_{round} just equals $Z_0 c \ln(b/\epsilon)/(2\pi)$.] Note that the difference is unchanged for $\epsilon \lesssim 1$ mm. Finally, $W^e = \lambda_z(w_{round} - w_{rect})$ with $\lambda_z = e^{-\frac{1}{2}s^2/\sigma_z^2}/(\sqrt{2\pi}\sigma_z)$.

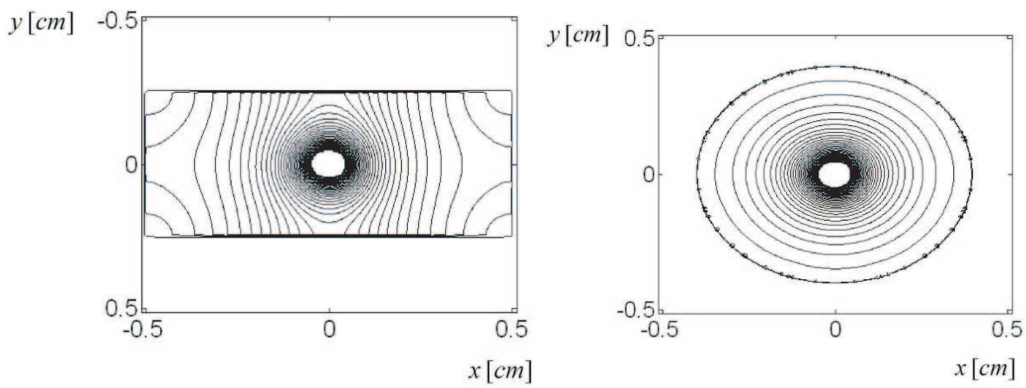

FIG. 6: Energy density in the fields for the rectangular and round beam pipes.

TABLE III: Field energy of the beam beyond radius ϵ in the rectangular and round beam pipes.

$\frac{2\pi}{Z_0 c}\, w(\epsilon)$			
ϵ [mm]	Rectangular	Round	Difference
1.50	0.73701	0.98350	0.2465
1.00	1.14727	1.38892	0.2417
0.50	1.85149	2.09222	0.2407
0.25	2.54466	2.78534	0.2407

To test the indirect method on the LCLS rectangular-to-round transitions we first perform calculations for a longer bunch ($\sigma_z = 200$ μm). The calculations are done twice: first, using direct integration on a pair of transitions with separation $L = 50$ mm, and then using the earlier-described indirect method. The results are shown in Fig. 7; we see good agreement.

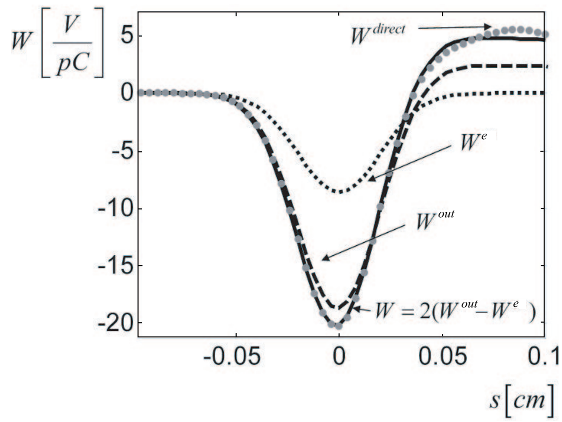

FIG. 7: Wake of a pair of LCLS rectangular-to-round transitions assuming a Gaussian bunch with $\sigma_z = 200$ μm.

For the nominal 20 μm long bunch it is difficult to obtain an accurate wake using the direct method. Thus, for this case we perform the indirect calculation only. The results are given in Fig. 8. We obtain $W_{mn} = -216$ V/pC, $k_{loss} = 153$ V/pC, and $W_{rms} = 61$ V/pC; or, using Eq. 3, we find that the high frequency impedance $Z_{hi} = 36.2$ Ω. The 3D results are $\sim 2/3$ the 2D estimates (see Table II with $\theta = 90°$).

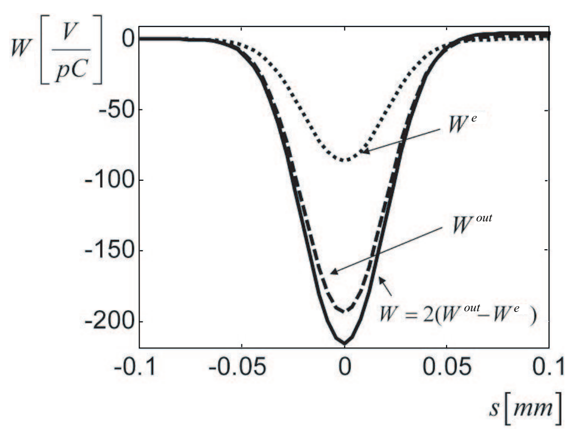

FIG. 8: Wake of a pair of LCLS rectangular-to-round transitions assuming a Gaussian bunch with $\sigma_z = 20$ μm.

When this report was nearly completed a 3D indirect wakefield calculation method was developed that no longer is limited to structures with round exit pipes (to be presented in a future report). The method was applied independently to each of two transitions of a pair and the results were added for: the elliptical collimator example and LCLS rectangular-to-round transitions presented in this report; in both cases the new results agree closely with and confirm the earlier results.

Wake Induced Energy Change

The LCLS bunch shape is not Gaussian; in fact it can be described as flat with leading and trailing spikes or horns, and with an rms length of 20 μm. Since for short bunches the wake is resistive, the wake will still be proportional to the bunch shape, with $W = -Z_{hi}c\lambda_z$. Taking $Z_{hi} = 36.2\ \Omega$ (the effect of all 33 pairs of rectangular-to-round transitions), the resulting induced energy change is given in Fig. 9. Results are shown for the actual bunch shape (the solid curve) and for the Gaussian model (dashes). Note that, even though the rms lengths of the horns are very small ($\sim 2\ \mu$m), they are still large compared to a/γ ($\sim 0.1\ \mu$m); thus the resistive model of wakes/impedances still applies.

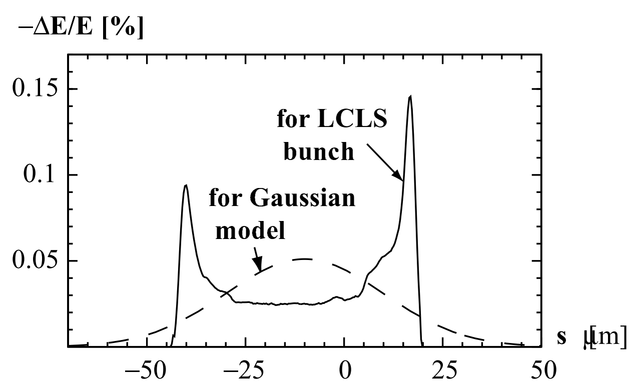

FIG. 9: Total energy change induced in the 33 pairs of rectangular-to-round transitions in the LCLS undulator region when in the 1 nC bunch charge configuration. Results are given for the actual bunch shape (solid) and for the Gaussian approximation (dashes). The head of the bunch is to the left; negative $\Delta E/E$ indicates energy loss.

It is the variation in induced energy change that is important for the LCLS, since an average change can always be compensated by tapering the undulator strength. We see that over the core of the beam (that part that excludes the horns) the total variation in induced energy is $\sim 0.03\%$. In the horn regions it grows to $\sim 0.15\%$ (however, it is questionable how effective in lasing horn particles will actually be). These numbers, however, are still small compared to the variation due to the resistive wall wake of the undulator beam pipe (which, for the flat, aluminum chamber is 0.6%) [1]. Thus we conclude that the wake effect of the LCLS rectangular-to-round transitions is relatively small.

I. CONCLUSION

We have studied the longitudinal wakefields of very short bunches ($\sigma_z/a \ll 1$ with σ_z rms bunch length and a beam pipe radius) in 3D vacuum chamber transitions using analytical models and the computer program ECHO. Here the diffraction model of Heifets and Kheifets applies, with the wake proportional to the bunch distribution. For this problem, direct numerical integration of the fields to obtain the wake (*i.e.* the constant of proportionality) is particularly difficult due to the long "catch-up" distance, and an indirect method of calculation is needed.

We find that for LCLS parameters—because of the short bunch length—tapering will not significantly reduce the wake unless the taper angle is very shallow ($\lesssim 1^\circ$). For a short bunch and a long, elliptical collimator in a round beam pipe we have demonstrated that—as is the case with a purely 2D (round) collimator—the wake can be obtained from the static primary fields of the beam alone. Finally, we have obtained the wakes of the LCLS bunch in the rectangular-to-round transitions using a hybrid method that includes indirect numerical (field) integration and a potential calculation using the beam's primary field.

For the LCLS 1 nC bunch charge configuration, we find that the wake-induced energy change of the transitions is proportional to the (double-horned) bunch distribution; in addition, the total variation in energy change is small (0.03% in the core of the beam, 0.15% in the horns) compared to that due to the resistive wall wakes of the undulator beam pipe (0.6%).

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REFERENCES

[1] K.L.F. Bane and G. Stupakov, "Resistive wall wakefield in the LCLS undulator beam pipe," SLAC-PUB-10707, Sep. 2004.

[2] T. Weiland and R. Wanzenberg, "Wake Fields and Impedances," CCAS-CAT-CERN Accelerator School, 1993, Vol. 1, p. 140.

[3] O. Napoly, Y. Chin and B. Zotter, "A Generalized Method for Calculating Wake Potentials," Num. Instrum. Meth., **344**, 255 (1993).

[4] I. Zagorodnov and T. Weiland, "TE/TM Field Solver for Particle Beam Simulations without Numerical Cherenkov Radiation," Physical Review - STAB, **8**, 042001 (2005).

[5] CST Microwave Studio is a trademark of CST GmbH, Buedinger Strasse 2a, 64289 Darmstadt, Germany (www.cst.de).

[6] S. Heifets and S. Kheifets, Review of Modern Physics **63**, 631 (1991).

[7] B.W. Zotter and S.A. Kheifets, *Impedances and Wakes in High-Energy Particle Accelerators*, (World Scientific, London, 1998).

[8] I. Zagorodnov, R. Schuhmann, and T.Weiland, "Long-Time Numerical Computation of Electromagnetic Fields in the Vicinity of a Relativistic Source," Journal of Computational Physics, **191**, 525-541 (2003).