Requirements For The LCLS Undulator Magnetic Measurement Bench

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Abstract

The magnetic measurements for the LCLS undulators will be done using a measurement bench which transports probes through the undulator under test. Errors in the bench construction cause errors in the probe motion through the undulator. This in turn causes measurement errors. In this note, limits are placed on the bench construction errors such that the measurements remain within tolerance. Secondary requirements for the bench are also discussed.

1 Introduction

The magnetic field in the LCLS undulators will be measured and tuned to be within specifications. The measurements will be done with probes moved through the undulators by a measurement bench. In this note, we derive basic requirements for the measurement bench such that its effect on measurement accuracy is within acceptable limits.

The motion of the probes through the undulator must be a straight line to high accuracy. Previous experience at other laboratories has shown that the best way to accomplish this is to use a large granite block with precision surfaces. A carriage riding on air bearings uses the precision surfaces as a guide to move the probes.

The note begins with an overview of the measurement system. Basic requirements such as the length of the bench are discussed. This is followed by a determination of the basic measurement accuracy requirements for tuning the undulators. Once the measurement accuracy requirements are known, the effects of probe motion errors on the measurements is studied and limits on the probe motion errors are set. Once the probe motion requirements are set, the tolerances on the granite block can be derived. Finally, secondary requirements for the measurement bench, such as auxiliary systems to handle cables and to protect the probes, are discussed.

A number of parameters specific to the LCLS project are used throughout this note. We summarize these parameters in table 1. The parameters come from the LCLS parameter database.[1]

[1] The LCLS undulator parameters are available online at http://www-ssrl.slac.stanford.edu/htbin /rdbweb/LCLS_params_DB_public/.

Parameter	Value	Units
Electron energy	13.640	GeV
Lorentz factor γ	26693	
Frequently occurring factor $-\frac{q}{\gamma mc}$	2.20×10^{-2}	$\frac{1}{\text{Tm}}$
Undulator period	3.0	cm
Undulator gap on axis	6.8	mm
Pole cant angle	4.5	mrad
Nominal undulator field	1.249	T
Undulator length	3.4	m
Regular long segment break length	0.909	m
Nominal radiation wavelength	1.5	$\mathring{A}$

Table 1: Parameters specific to the LCLS project which are used in this note.

2 Overview Of The Measurement System

An overview of the measurement system is shown in figure 1. The measurement system consists of the bench, probes, one or more racks of electronics, a computer, a Hall probe calibration magnet, a zero Gauss chamber, and several smaller reference and calibration devices. The measurement bench has a carriage which holds the probes and moves along the bench. Typically, the carriage uses an air bearing and linear motor for smooth, precise motion. The carriage has a number of stacked stages which hold the probes. This allows different motions of the probes which are useful for undulator alignment to the measurement bench.

The probes are first put in devices at the end of the bench, such as a zero Gauss chamber, reference magnets, reference poles for capacitive sensors, etc. After the probes are properly adjusted and calibrated, the probes are moved through the undulator to perform measurements. Signals are sent to the electronics rack for acquisition. A computer collects the data and performs necessary calculations. Periodic calibrations of Hall probes are performed with a large electromagnet and NMR system.

The cell boundaries shown in the figure indicate the space assigned to the undulator in the tunnel when in use. This includes half the distance from the undulator under test to its neighbor on each end. The measurements should include everything between cell boundaries, if possible.

3 Size Of The Measurement Bench

The measurement bench must be long enough to perform the standard measurements on each undulator using the configuration shown in figure 1. In addition, it should accommodate certain additional measurements which are important to the project.

One very important measurement involves stretching a wire through the undulator and using it to measure the first and second field integrals. This is a check of the Hall probe measurement which uses field sampling and numerical integration. The tolerances on these measurements are very tight and must be checked.

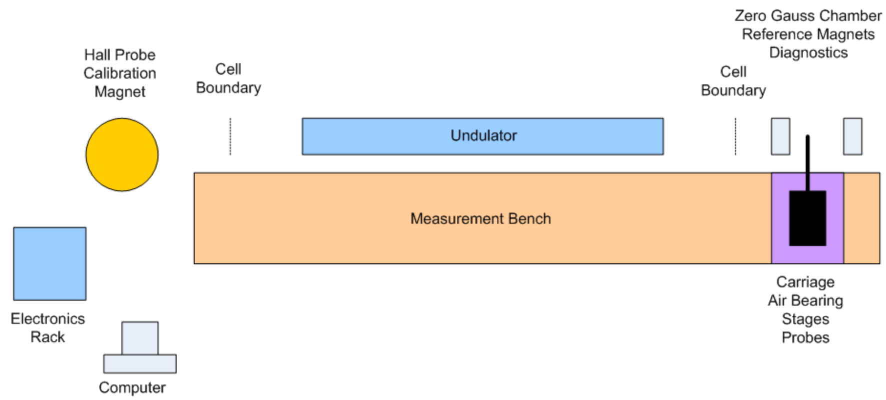

Figure 1: Overview of the LCLS undulator magnetic measurement system.

The configuration for the wire measurements is shown in figure 2. We start at the right side of the figure and discuss the size requirements for different parts of the setup. We assume the carriage is 1.0 m long and leave half of the length, or 0.5 m, inaccessible. The next 1.0 m of the bench is reserved for calibration and diagnostic equipment. This includes a zero Gauss chamber, reference magnets, alignment magnets with vertical and horizontal fields, etc. The next item of length 0.75 m is a base with a stage on top for the wire system. The stage moves the wire horizontally. The following distance of 0.46 m is roughly one half the regular long segment break length of 0.909 m. The undulator length is 3.4 m. This is followed by the other half of the long segment break length. The following 0.75 m is for the other end of the wire system. Finally, we add 0.5 m so the probe can be parked at either end of the bench. This configuration lets us easily bring in the wire system for checks without any major rearranging of the whole measurement system. The total length of the bench with this layout is 7.82 m.

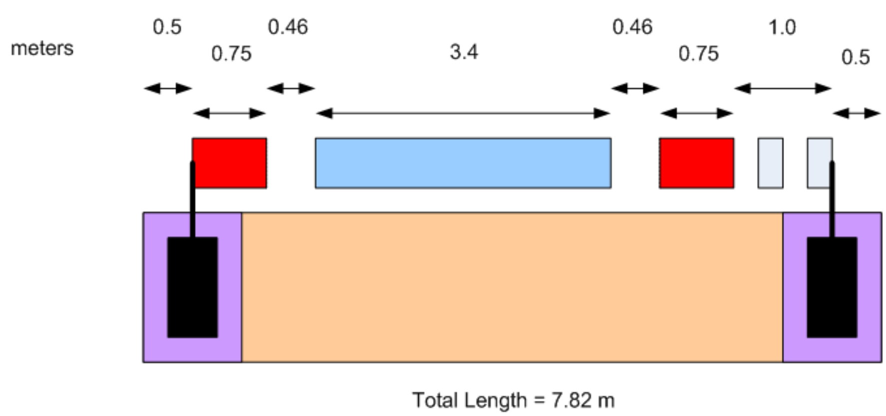

Figure 2: Measurement bench setup for stretched wire measurements.

Another configuration which we must be able to set up is shown in figure 3. We place a second undulator at the end of the first one, separated by a break length. This lets us measure the phasing between undulators. The probe only needs to go a short distance into the second undulator to test the phasing. We chose the bench length to allow a one meter measurement of the second undulator. The total length of the bench for this test must be at least 7.78 m.

In order to perform these tests and have some cushion in our estimates, we require a bench whose total length is 8.0 meters. The height and width of the bench are not critical and do not need to be specified. We will specify motion requirements for the carriage and the manufacturer will choose a height and width in order to meet these requirements.

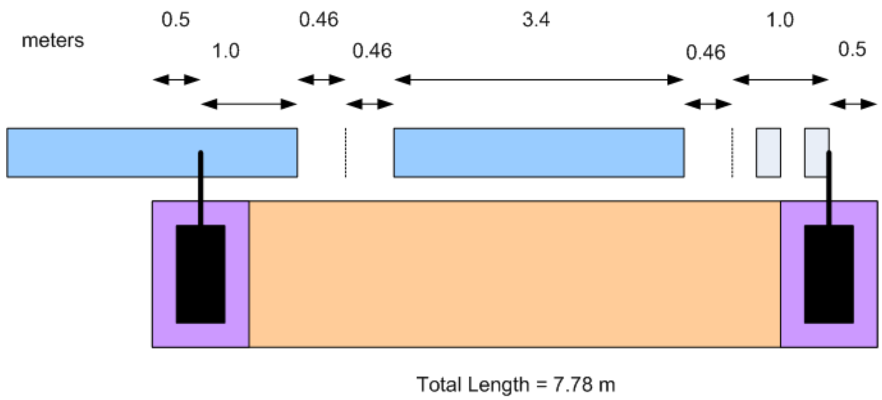

Figure 3: Bench setup for a phase matching measurement.

4 Measurement Accuracy Requirements

In order to specify the requirements for the measurement bench, we must start by determining the measurement accuracy requirements. Then, the effect of probe motion errors on the measurement accuracy can be used to set tolerances on the quality of the probe motion. Once the probe motion requirements are known, specifications for the bench construction can be determined.

In this section, we start by discussing key physics requirements for the undulators. These requirements set limits on how well the undulators must be tuned. The undulator tuning tolerances are then presented.

In the next section, we will study how errors in the probe motion affect the measurements. We use the tuning accuracy tolerances to set limits on probe motion errors. The limits on the probe motion errors are then used to calculate limits on errors in the bench construction.

4.1 Physics Requirements For The Undulators

The physics requirements for the undulators are found in the LCLS parameter database, referenced above. The physics requirements related to undulator tuning and magnetic measurements are given in table 2. Note that some of the requirements are specified in terms of quantities like trajectory walkoff and electron phase, not in terms directly related to magnetic measurements. Formulas are given in a separate note showing how to calculate these quantities from the magnetic measurements.[2]

Parameter	Value	Units
Max trajectory walkoff per 10 m	5	μm
First magnetic field integral suppression	40	μTm
Second magnetic field integral suppression	50	μTm^2
Electron phase deviation per segment	10	degrees
RMS magnet error $\Delta B/B$	0.015	%

Table 2: Undulator physics requirements. The values come from the LCLS parameter database.

The first parameter in table 2 gives the maximum distance the beam can wander from a straight line that is 10 m long. In an undulator, which is 3.4 m long, we limit the beam walkoff to only 2 μm.

The second parameter is related to the slope of the beam when it exits the undulator assuming zero slope at the entrance. The exit slope is given by

$$x' = -\frac{q}{\gamma m v_z} I_1 \tag{1}$$

where q is the electron charge (negative), γ is the electron Lorentz factor, m is the electron mass, v_z is the electron velocity in the forward direction (this can be set to c, the speed of light, with small error), and I_1 is the first field integral which is specified in the table. Using $I_1 = 40 \times 10^{-6}$ Tm and $\gamma = 26693$, we find $x'_{\text{max}} = 8.80 \times 10^{-7}$.

The third parameter in the table is the second field integral I_2. It determines the position at which the beam exits the undulator relative to the entrance position. The formula is

$$x = -\frac{q}{\gamma m v_z} I_2 \tag{2}$$

Using the LCLS parameters and $I_2 = 50 \times 10^{-6}$ Tm^2, the maximum position shift is $x_{\text{max}} = 1.10 \times 10^{-6}$ m.

The fourth parameter gives the phase deviation in the undulator. It is affected by several factors including the strength of the permanent magnet blocks. For our purposes of setting limits on probe motion errors, the slope of the trajectory will be the primary cause of phase errors. The phase is given by

$$P(z) = \frac{2\pi}{\lambda_r} \int_{z_0}^{z} (\frac{1}{2}\frac{1}{\gamma^2} + \frac{1}{2} x'^2)\, dz_1 \tag{3}$$

where λ_r is the radiation wavelength.

Finally, the fifth parameter in the table specifies how well the fields in the undulator must be matched. It is used to ensure that each undulator emits radiation at the same wavelength. It is a limit on the accuracy of the K value.

[2] Z. Wolf, "Introduction to LCLS Undulator Tuning", LCLS-TN-04-07, June, 2004.

4.2 Undulator Tuning Tolerances

The process of undulator tuning consists of the following steps. The magnetic field in the undulator is measured. The field's effect on the beam is calculated. The field is adjusted until the calculated physics parameters are within the tolerances given in the previous section.

In order for the tuning process to work, the physics parameters calculated from the field measurements must be at least as accurate as the physics requirements limits. We use this criterion to establish the maximum error on the tuning parameters. Summarizing the values derived above, the maximum errors for the tuning parameters are listed in table 3.

Parameter	**Maximum Error**	**Units**
Calculated trajectory walkoff in undulator	2	μm
Calculated trajectory slope at exit	8.80×10^{-7}	
Calculated beam offset at exit	1.10×10^{-6}	m
Calculated phase deviation	10	degrees
Field measurement accuracy	0.015	%

Table 3: Maximum errors for the undulator tuning parameters.

The field measurement accuracy limit requires some discussion. The requirement means that the measurements must be good enough so that the calculated K value is accurate to 0.015%. In general terms, this means that the measurements must be accurate to 0.015%. We use this as our working requirement, however, larger errors on small sets of field samples can be tolerated.

Errors in the magnetic measurements can not be distinguished from errors in the undulator. Corrections will be made to the undulator until the measurement results are within the undulator physics requirements limits. A measurement error due to an error in the probe motion will be put into the field of the undulator. These errors may not be allowed to exceed the tuning error tolerances given in table 3.

5 Probe Motion Requirements

Ideally, the measurement probe follows a straight line down the center of the undulator. Errors in the bench will cause errors in the probe motion. In this section we derive limits on the probe motion errors such that the tuning accuracy requirements derived in the previous section are still met.

Errors in the probe path come from vertical motion, horizontal motion, axial motion deviations, and probe angle deviations. These errors are illustrated in figure 4. We analyze the effect of these errors in turn. We set limits on the probe path errors such that the undulator tuning parameters are within tolerance. In general, we do not allow a motion error to use the entire allowed tolerance. We choose a suitable fraction of the maximum motion error for the probe path tolerance.

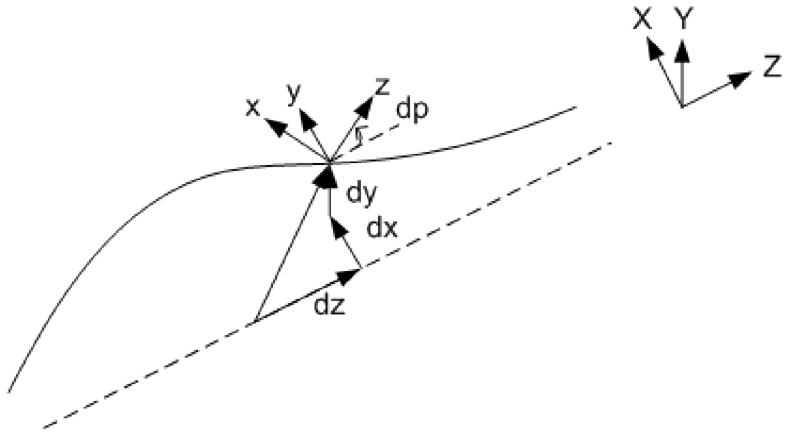

Figure 4: Probe path errors. The dashed line is the ideal probe path. The solid line is the actual path. There is a horizontal error dx, a vertical error dy, an axial error dz, and an error in the angular orientation of the probe dp.

5.1 Vertical Probe Motion

The magnetic field in an undulator can be approximated by the fundamental term in the Fourier expansion

$$B_y(y,z) = B_0 \cosh(\frac{2\pi y}{\lambda_u}) \sin(\frac{2\pi z}{\lambda_u}) \tag{4}$$

In this equation, y is the vertical coordinate, z is in the coordinate along the undulator, and x is the horizontal coordinate. B_y is the main field of interest in the undulator. For now, we assume the field does not vary in the horizontal x direction so B_y depends only on y and z. B_0 is the peak field on the midplane where $y = 0$. λ_u is the undulator period. Note that the field varies as the hyperbolic cosine in y and varies sinusoidally in z.

Consider the effect of vertical probe motion on the measurement of the field. As the probe moves vertically, the hyperbolic cosine term changes the measured value. If we believe the probe is moving on a straight line down the midplane of the undulator, vertical probe motion gives a measurement error. In table 3, we specified that the field measurement error must be below 0.015%. This limits the vertical motion that can be tolerated.

We need to find the maximum deviation from the midplane Δy such that the magnetic field changes by less than 0.015%. This occurs when $\cosh(\frac{2\pi\Delta y}{\lambda_u}) = 1.00015$. For $\lambda_u = 0.03$ m, this occurs when $\Delta y = 8.27 \times 10^{-5}$ m. We do not wish to use our entire error budget on this one error, so we require that the bench move the probes in a straight line with vertical deviations less than 2.0×10^{-5} m, or 20 μm. This guarantees that vertical motions alone will not cause measurement errors larger than 0.015%. The 20 μm tolerance leaves room for fiducialization errors and alignment errors, which will be discussed later.

We have satisfied only one requirement in table 3. We must check the other requirements as well. Suppose we have a bump in the bench which moves the probe vertically so that $\Delta B/B = 0.015\%$. The effect on the first field integral depends on how long the bump is. Since the field varies sinusoidally, a bump over half an undulator period gives the maximum effect. The error in the calculated trajectory slope is

$$\Delta x' = -\frac{q}{\gamma mc}\int_0^{\lambda_u/2}(0.00015)B_0\sin(\frac{2\pi z}{\lambda_u})\,dz \tag{5}$$

$$= -\frac{q}{\gamma mc}(0.00015)B_0\frac{\lambda_u}{\pi} \tag{6}$$

Using $-\frac{q}{\gamma mc} = 2.20\times10^{-2}\ \frac{1}{\text{Tm}}$, $B_0 = 1.249$ T and $\lambda_u = 0.03$ m, we calculate $\Delta x' = 3.94\times10^{-8}$. This is well below the limit of 8.80×10^{-7} specified for the calculated trajectory slope error.

The calculated beam offset error can be determined from the slope error. If the error occurred at the start of the undulator, the beam would exit offset by $\Delta x = \Delta x' L_{und}$, where L_{und} is the length of the undulator. Using the slope of $\Delta x' = 3.94\times10^{-8}$ and $L_{und} = 3.4$ m, we find $\Delta x = 1.34\times10^{-7}$ m. This is below the limit of $x_{\max} = 1.10\times10^{-6}$ m given in table 3.

The maximum trajectory walkoff tolerance of 2 μm is met since the maximum deviation of the beam is $\Delta x = 1.34\times10^{-7}$ m.

The phase change caused by the added trajectory slope is given by

$$\Delta P(z) = \frac{2\pi}{\lambda_r}\frac{1}{2}\Delta x'^2\,z \tag{7}$$

The maximum value is $\Delta P = 6.3\times10^{-3}$ degrees, a negligible amount.

Implicit in this discussion is the assumption that the error $\Delta y(z)$ is a smooth, slowly varying function. Taking $\Delta y(z) = \Delta y_{\max}$ over a half undulator period at the start of the undulator is then a worst case assumption. Clearly, other forms of $\Delta y(z)$, such as a sinusoid in phase with the undulator field, give larger errors. Such errors are not realistic and are excluded from our discussion.

In summary, if the probe has slowly varying vertical deviations less than 20 μm everywhere along the bench, the undulator tuning tolerance limits will be met.

5.2 Horizontal Probe Motion

With wide, parallel undulator poles, the magnetic field is fairly uniform in x, and small motions of the measurement probe in x cause very small errors. It was recently decided, however, to use canted poles in the LCLS undulators. The poles now have an angle with respect to each other so the strength of the magnetic field varies with x. With the canted poles, it is important to restrict the x motion of the probe so that field variations due to probe motion are within acceptable limits.

Figure 5 shows an exaggerated view of the canted poles and the shape of the magnetic field. At present, the total cant angle between poles is 4.5 mrad. This means the gap changes by 45 μm per cm, or $\frac{dg}{dx} = 4.5\times10^{-3}$.

We can estimate the effect of the cant angle on the field strength. We assume that the field at a given gap height is the same as the field in an undulator with parallel poles with that gap height. With this assumption, we need to calculate the field in an undulator with parallel poles. This is done using Ampere's law with the magnetic circuit shown in figure 6.

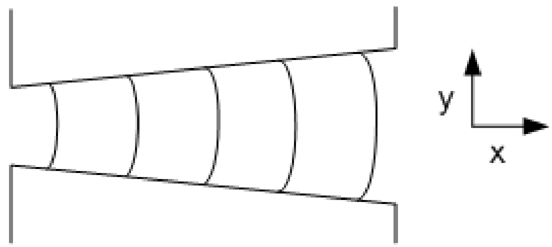

Figure 5: The poles of the undulator are not parellel. The canted poles produce a field which varies with x.

The fields in the undulator are assumed to vary sinusoidally in z with minimal additional harmonic content. This means that the fields vary as the hyperbolic sine or cosine in y with minimal additional terms. Under these conditions, the fields in the undulator are given by

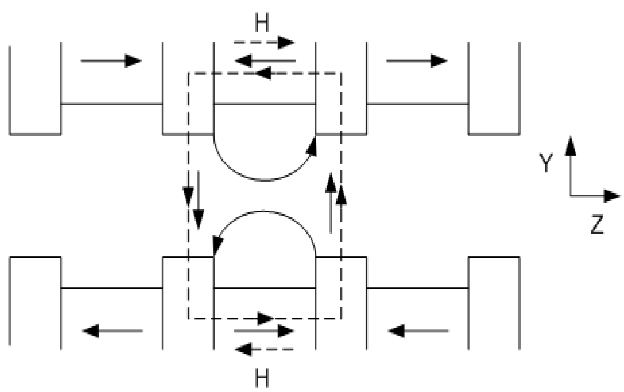

Figure 6: The dashed line shows the path used in the calculation of Ampere's law. The solid arrows show B. The two dashed arrows show H in the permanent magnet blocks.

$$B_y(y,z) = B_0 \cosh(\frac{2\pi y}{\lambda_u}) \sin(\frac{2\pi z}{\lambda_u}) \tag{8}$$

$$B_z(y,z) = B_0 \sinh(\frac{2\pi y}{\lambda_u}) \cos(\frac{2\pi z}{\lambda_u}) \tag{9}$$

Applying Ampere's law around the dashed line in figure 6 gives

$$2\int_{-g/2}^{g/2} \frac{B_0}{\mu_0} \cosh(\frac{2\pi y}{\lambda_u}) dy - 2H_m L_m = 0 \tag{10}$$

In this expression, g is the total gap height of the undulator, B_0 is the strength of the peak vertical field on the midplane, H_m is the magnetic field intensity in the permanent magnet blocks, and L_m is the length of the permanent magnet blocks. The poles are assumed to have infinite permeability. Note that H in the block is in the opposite direction of B, hence the minus sign. Performing the integral, we get

$$2\frac{B_0}{\mu_0}\sinh(\frac{\pi g}{\lambda_u})(\frac{\lambda_u}{2\pi})2 - 2H_m L_m = 0 \tag{11}$$

Simplifying this expression, we find that the field on the midplane is related to the gap height by

$$B_0 \sinh(\frac{\pi g}{\lambda_u}) = \frac{\pi}{\lambda_u}\mu_0 H_m L_m \tag{12}$$

We now let the gap change, but we assume that H in the magnetic material is a constant source and does not change as we change the gap. As g goes to $g + \Delta g$, the field on the midplane B_0 goes to $B_0 + \Delta B_0$. These changes are related by

$$\Delta B_0 \sinh(\frac{\pi g}{\lambda_u}) + B_0 \cosh(\frac{\pi g}{\lambda_u})\frac{\pi}{\lambda_u}\Delta g = 0 \tag{13}$$

Rewriting this expression gives

$$\frac{\Delta B_0}{B_0} = -\coth(\frac{\pi g}{\lambda_u})\frac{\pi}{\lambda_u}\Delta g \tag{14}$$

We now incorporate the fact that the gap change is due to the canted pole and that the gap changes with x according to $\Delta g = \frac{dg}{dx}\Delta x$,

$$\frac{\Delta B_0}{B_0} = -\coth(\frac{\pi g}{\lambda_u})\frac{\pi}{\lambda_u}\frac{dg}{dx}\Delta x \tag{15}$$

For the LCLS, $g = 6.8 \times 10^{-3}$ m, $\lambda_u = 3.0 \times 10^{-2}$ m, $\coth(\frac{\pi g}{\lambda_u}) = 1.634$, and $\frac{dg}{dx} = 4.5 \times 10^{-3}$. Inserting these values, we find

$$\frac{\Delta B_0}{B_0} = -(0.77\ \frac{1}{\mathrm{m}})\Delta x \tag{16}$$

For a field change of $\frac{\Delta B_0}{B_0} = 1.5 \times 10^{-4}$, a probe motion of $\Delta x = 1.95 \times 10^{-4}$ m is required.

The analysis presented above assumed a sinusoidal field in the undulator with the fundamental undulator period. This allowed a simple derivation of the field on the midplane as a function of gap height. Other authors have performed more detailed analyses. Walker added another term in the Fourier expansion of the field and found the field on the undulator midplane to be[3]

$$B_0 = \frac{4B_r}{\sqrt{2}\pi}\left[\frac{1}{\sinh(\frac{\pi g}{\lambda_u})} - \frac{1}{3\sinh(3\frac{\pi g}{\lambda_u})}\right] \tag{17}$$

where B_r $(= \mu_0 H_m)$ is the field in the magnet and the magnet length is set equal to $\frac{\lambda_u}{4}$. The field on the midplane changes with gap according to

$$\frac{\Delta B_0}{B_0} = \left[\frac{1}{\sinh(\frac{\pi g}{\lambda_u})} - \frac{1}{3\sinh(3\frac{\pi g}{\lambda_u})}\right]^{-1}\left[-\frac{\cosh(\frac{\pi g}{\lambda_u})}{\sinh^2(\frac{\pi g}{\lambda_u})} + \frac{\cosh(3\frac{\pi g}{\lambda_u})}{\sinh^2(3\frac{\pi g}{\lambda_u})}\right]\frac{\pi}{\lambda_u}\Delta g \tag{18}$$

[3] R. P. Walker, "Periodic Magnets For Free Electron Lasers", NIM A237 (1985) 366.

Using $\Delta g = \frac{dg}{dx}\Delta x$ and inserting the LCLS parameters, we find

$$\frac{\Delta B_0}{B_0} = -(0.73\ \frac{1}{\mathrm{m}})\Delta x \tag{19}$$

For a field change of $\frac{\Delta B_0}{B_0} = 1.5 \times 10^{-4}$, a probe motion of $\Delta x = 2.07 \times 10^{-4}$ m is required.

Halbach has also given a widely used formula for the field on the midplane of an undulator.[4]

$$B_0 = 3.33 \times \exp[-\frac{g}{\lambda_u}(5.47 - 1.8\frac{g}{\lambda_u})] \tag{20}$$

This formula is a parametrization of data from a computer design study of optimized undulators. The LCLS value of $g/\lambda_u = 0.23$ is within the range of validity $0.07 < g/\lambda_u < 0.7$ of the formula. Using Halbach's formula, we find

$$\frac{\Delta B_0}{B_0} = -(5.47 - 3.6\frac{g}{\lambda_u})\frac{1}{\lambda_u}\Delta g \tag{21}$$

Again using $\Delta g = \frac{dg}{dx}\Delta x$ and inserting the LCLS parameters, we find

$$\frac{\Delta B_0}{B_0} = -(0.70\ \frac{1}{\mathrm{m}})\Delta x \tag{22}$$

For a field change of $\frac{\Delta B_0}{B_0} = 1.5 \times 10^{-4}$, a probe motion of $\Delta x = 2.14 \times 10^{-4}$ m is required.

We have come up with three similar values for the horizontal motion which give $\frac{\Delta B_0}{B_0} = 1.5 \times 10^{-4}$. They are $\Delta x = 1.95 \times 10^{-4}$ m, $\Delta x = 2.07 \times 10^{-4}$ m, and $\Delta x = 2.14 \times 10^{-4}$ m. We wish to stay well below these limits for the allowed horizontal probe motion. We set the maximum allowed horizontal probe motion to be $\Delta x = 0.4 \times 10^{-4}$ m, or 40 μm.

The discussion of the other undulator tuning tolerances given above for the vertical probe motion also applies here. The relative field error $\frac{\Delta B_0}{B_0} < 1.5 \times 10^{-4}$ requirement sets the limit for the Δx motion error.

5.3 Axial Probe Motion

The magnetic measurements for the LCLS will be done "on the fly". The carriage on the bench will move the probes through the undulator at constant speed while the measurements are being performed. An encoder on the bench generates triggers to perform the measurements. This approach allows the measurements to be performed very quickly with many field samples and is expected to give smaller errors than could be obtained using a starting and stopping motion.

A schematic of the probe and the encoder is shown in figure 7. Notice that if the carriage yaws, for instance, the probe will be at a different z position than the trigger point on the encoder. A constant yaw due to the probe mounting is not a problem because the measurements will be used to establish the z coordinate relative to the undulator. Errors in the bench, however, which cause the carriage yaw to change, will cause errors in the data.

On the midplane of the undulator, the field goes as

$$B_y(z) = B_0 \sin(\frac{2\pi z}{\lambda_u}) \tag{23}$$

[4] K. Halbach, "Permanent Magnet Undulators", Journal De Physique C44, (1983) 211.

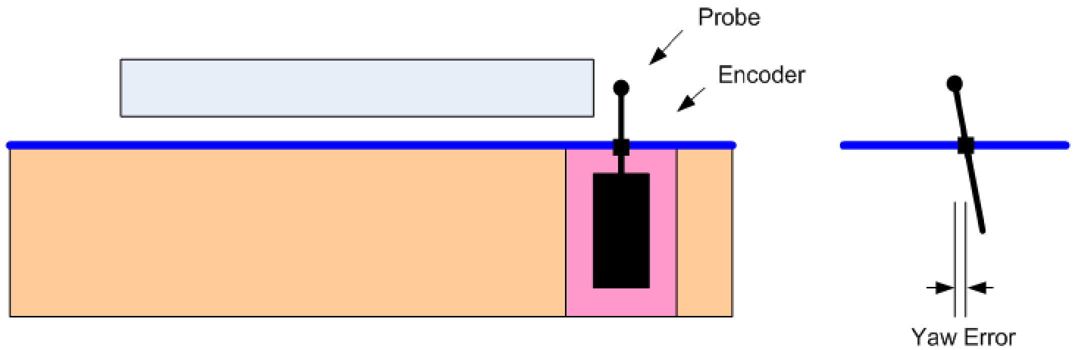

Figure 7: The measurements are triggered by an encoder on the bench. Yaw (and pitch for elevated probes) in the carriage causes the probe to be at a different z position than the trigger position.

If the trigger occurs at position z and the probe is at position $z + \Delta z$, there will be an error in the field reading of

$$\Delta B_y(z) = B_0 \cos(\frac{2\pi z}{\lambda_u})\frac{2\pi}{\lambda_u}\Delta z \tag{24}$$

Notice that at the field peaks, the field profile is flat and the error in B_y is zero. At the zeros in the field, where the field is changing rapidly, the errors due to axial motion deviations are the largest.

To derive a limit on the axial motion deviations, consider the criterion that the bench must not cause measurement errors larger than 1.5×10^{-4} of the peak field. As noted previously, this criterion refers to the K values of the undulators which are calculated from all the measurements. It does not refer to each measurement point. It is interesting, however, to see what the limit is. From equation 24, we conclude that

$$\frac{2\pi}{\lambda_u}\Delta z_{\mathrm{max}} = 1.5 \times 10^{-4} \tag{25}$$

Using $\lambda_u = 0.03$ m, we find the limit $\Delta z_{\mathrm{max}} = 7.16 \times 10^{-7}$ m. This would be a very difficult limit to achieve. Fortunately, it is not required and we do not pursue this limit further.

Consider now the tolerance on the calculated trajectory slope. The slope of the trajectory is given by

$$x'(z) = -\frac{q}{\gamma mc}\int_{z_0}^{z} B_y(z_1)\, dz_1 \tag{26}$$

The change in the calculated slope of the trajectory due to an axial error in the probe position is given by

$$\Delta x' = -\frac{q}{\gamma mc}\int_{z_1}^{z_2} \Delta B_y(z)\, dz \tag{27}$$

where z_1 and z_2 are the starting and ending positions of the error, respectively. Using equation 24 for the error in the measured field, we find

$$\Delta x' = -\frac{q}{\gamma mc}\int_{z_1}^{z_2} B_0 \cos(\frac{2\pi z}{\lambda_u})\frac{2\pi}{\lambda_u}\Delta z(z)\, dz \tag{28}$$

Note that for $\Delta z(z)$ a smooth function of z over some range of z, the range that maximizes $\Delta x'$ is half an undulator period, or an odd multiple. Otherwise, cancellations occur with the oscillating cosine term. As a worst case estimate for smooth, non-oscillating $\Delta z(z)$, we take $\Delta z(z) = \Delta z_{\mathrm{max}}$, constant over half an undulator period. The slope error is then

$$\Delta x' = -\frac{q}{\gamma mc} \int_{-\lambda_u/4}^{\lambda_u/4} B_0 \cos(\frac{2\pi z}{\lambda_u}) \frac{2\pi}{\lambda_u} \Delta z_{\mathrm{max}} \, dz \tag{29}$$

Evaluating the integral gives

$$\Delta x' = -\frac{q}{\gamma mc} 2 B_0 \Delta z_{\mathrm{max}} \tag{30}$$

We require this error to be less than the limit on the calculated trajectory slope error $x'_{\mathrm{max}} = 8.80 \times 10^{-7}$, given above. Inserting $-\frac{q}{\gamma mc} = 2.20 \times 10^{-2}$ 1/(Tm) and $B_0 = 1.249$ T, we find $\Delta z_{\mathrm{max}} = 1.60 \times 10^{-5}$ m.

The tolerance on the calculated beam offset provides another limit. The limit is $x_{\mathrm{max}} = 1.10 \times 10^{-6}$ m. Since the undulator is 3.4 m long, this limits the maximum slope of the trajectory to $x'_{\mathrm{max}} = 3.24 \times 10^{-7}$. Using the above analysis, we find $\Delta z_{\mathrm{max}} = 5.89 \times 10^{-6}$ m.

Finally, the small slopes from the limits already established do not contribute significant phase errors, as already seen when discussing the probe vertical motion. Thus, the phase error tolerance does not provide the limit for Δz_{max}.

We have arrived at the following limits. The calculated trajectory slope tolerance requires $\Delta z < 1.60 \times 10^{-5}$ m. The calculated beam offset tolerance requires $\Delta z < 5.89 \times 10^{-6}$ m. We use the tighter limit and set $\Delta z_{\mathrm{max}} = 5.89 \times 10^{-6}$ m. Again, we do not want to use our entire error budge on one error, so we set our tolerance to be $\Delta z < 3 \times 10^{-6}$ m.

5.4 Probe Angle Deviations

Hall probes and coils, both of which will be used on the carriage, are direction sensitive. The sensitivity of the probe goes as the cosine of the angle between the field and the normal to the probe.

To set a limit on the angle error of the probe, we find the angle of probe rotation which causes a 0.015% error in the measurement. If ϕ is the rotation angle, we require $\cos(\phi) > 1 - 0.00015$. This leads to $\phi < 0.017$ rad. This is a very large angle to come from errors in the bench. Once the probes are aligned, imperfections in the bench will not lead to deviations of the probe angle by this large a value.

Even if errors in the bench construction will not lead to probe angle problems, the probes must be aligned to better than 17 mrad. This means that adjustments must be built into the system for accurate probe alignment. Also, both a vertical and horizontal field standard should be provided for probe alignment. We take 8 mrad as our tolerance for probe alignment.

5.5 Summary Of Probe Motion Requirements

A summary of the probe motion requirements derived in this section is presented in the following table.

Parameter	Value	Units
Maximum vertical deviation Δy	20	μm
Maximum horizontal deviation Δx	40	μm
Maximum deviation in distance between probe and encoder	3	μm
Maximum probe angle error	8	mrad

Table 4: Summary of the probe motion requirements derived in this section.

6 Accuracy Requirements For The Bench Construction

Up to this point, we have discussed limits on imperfections in the probe motion. We now discuss limits on imperfections in the measurement bench such that the probe motion limits are met.

The carriage on the bench is supposed to have only one degree of freedom, namely motion along the z axis. All other motions are errors and we must establish limits on these error motions. These error motions of the carriage are changes in x, y, roll, pitch, and yaw. In this section, we will discuss these errors in turn.

6.1 Carriage X Motion

Motion of the carriage in the x direction is illustrated in figure 8. X motion of the carriage translates directly into x motion of the probe. Using our maximum allowed x motion of the probe of 40 μm, we limit the maximum allowed deviation of the carriage in the x direction to 40 μm.

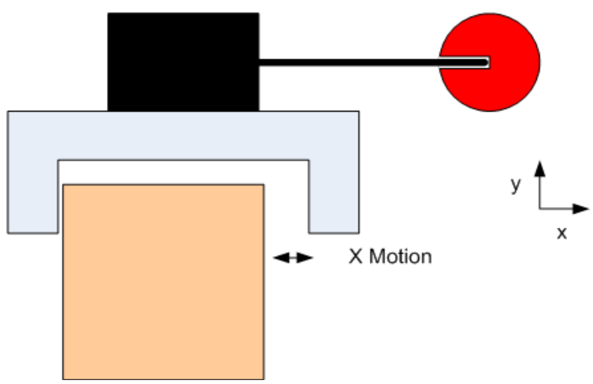

Figure 8: X motion of the carriage translates directly into x motion of the probe.

6.2 Carriage Y Motion

Motion of the carriage in the y direction is illustrated in figure 9. Y motion of the carriage translates directly into y motion of the probe. Using our maximum allowed y motion of the probe of 20 μm, we limit the maximum allowed deviation of the carriage in the y direction to 20 μm.

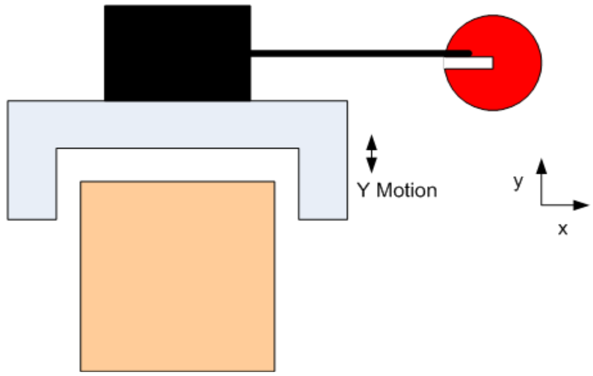

Figure 9: Y motion of the carriage translates directly into y motion of the probe.

6.3 Carriage Roll

Roll of the carriage causes y motion of the probe. The x motion is second order. This is illustrated in figure 10. We assume a 1 meter arm from the center of the carriage to the probe. Limiting the y motion to 20 μm places a limit on the carriage roll of 20 μrad.

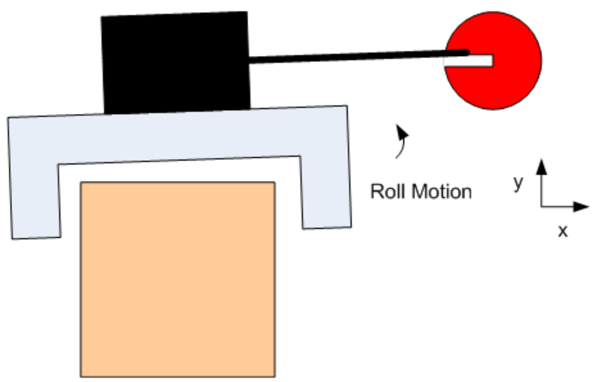

Figure 10: Roll motion of the carriage primarily causes y motion of the probe.

6.4 Carriage Pitch

As long as the probe arm is centered on the carriage, pitch of the carriage does not affect the x, y, or z position of a probe. This is because in this case, pitch is a rotation along the probe arm. If a probe is placed away from the center of the carriage, however, carriage pitch will generate errors.

Assume we have a 1 meter long carriage. The maximum distance a probe can be placed from the center of the carriage is 0.5 m. Pitch of the carriage then causes a y motion of the probe. This is illustrated in figure 11 on the left side of the figure. The maximum allowed deviation of the probe in y is 20 μm. This means the maximum allowed pitch of the carriage is 20 μm/0.5 m, or 40 μrad.

Pitch also causes z motion of the probe for probes elevated above the carriage. Suppose the maximum height of a probe above the carriage is 0.5 m, again a typical value. The maximum allowed z motion between the probe and the encoder is 3 μm. The encoder is very close to the carriage, so the carriage position is taken to be the encoder position. In this case, the maximum allowed pitch is 3 μm/0.5 m, or 6 μrad.

The smaller of these two limits, 6 μrad, is the maximum allowed pitch of the carriage.

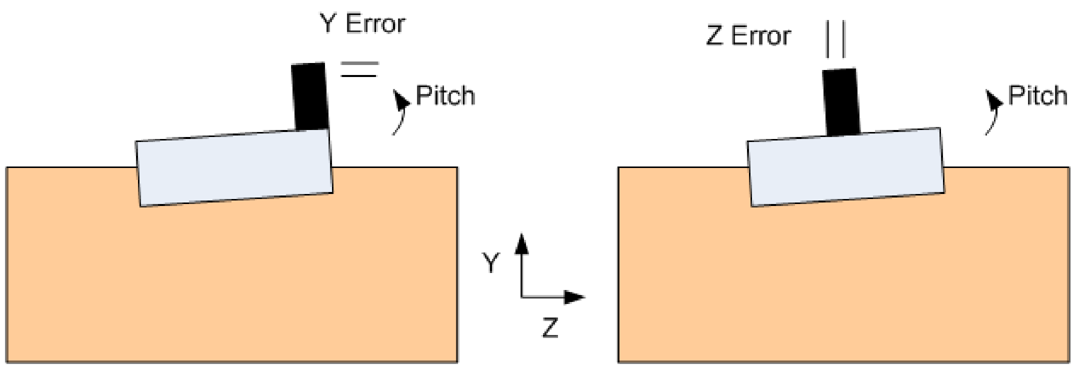

Figure 11: Pitch of the carriage causes y errors for offset probes, left, and z errors for elevated probes, right.

6.5 Carriage Yaw

Carriage yaw moves the probe in the z direction. This is illustrated in figure 12. There is a small second order x motion which we neglect. We assume a distance of 0.5 m between the encoder and the probe. The maximum allowed z motion between the probe and the encoder is 3 μm. In this case, the maximum allowed yaw is 3 μm/0.5 m, or 6 μrad.

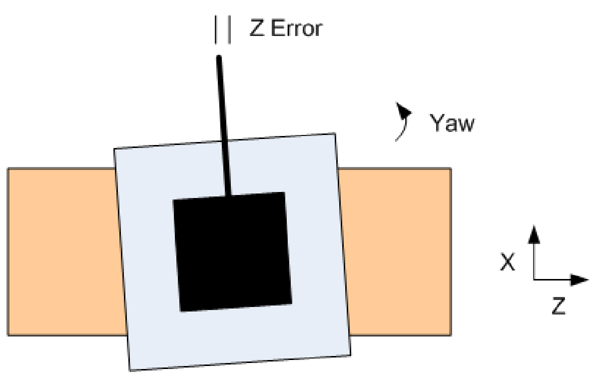

Figure 12: Yaw motion of the carriage primarily causes z motion of the probe.

6.6 Summary Of The Accuracy Requirements For Bench Construction

The following table summarizes the accuracy requirements for the bench construction. Constant offsets of these parameters are allowed. The parameters are the maximum allowed deviations from the constant offsets.

Parameter	**Probe Motion**	**Single Error**	**Quad Limit**	**Units**
Carriage x deviation	Δx	40	40	μm
Carriage y deviation	Δy	20	14	μm
Carriage roll deviation	Δy	20	14	μrad
Carriage pitch deviation	Δy, Δz	6	4.2	μrad
Carriage yaw deviation	Δz	6	4.2	μrad

Table 5: Summary of the derived accuracy requirements for the bench construction. The effect of each error on the probe motion is noted. The column labeled "quad limit" adjusts the errors so that the quadrature sum gives the maximum allowed probe motion.

In table 5, note that several errors in the bench construction give probe motion in the y and z directions. The column "single error" summarizes the bench errors discussed so far. These numbers use the entire allowed probe motion error to compute the bench error. They must be corrected so that the quadrature sum of all errors giving y motion is the maximum allowed probe y motion error, for instance. The corrected limits are given in the column "quad limit" where the errors are added in quadrature. These limits are our result.

6.7 Note About Undulator Fiducialization

It should be noted that the magnetic measurement bench will be used to fiducialize the undulators. The alignment tolerances given in the LCLS parameter database are given in table 6. The fiducialization errors must be a small fraction of the total alignment errors given in the table. The limits we derived above for the bench construction will satisfy the fiducialization requirements, but not by a large factor. This is especially true for the carriage y deviation. The magnets used in the fiducialization will be at the ends of the undulator. We must be especially careful that the bench does not have "bumps" in the region of the fiducialization magnets, even if they are within specification. Otherwise, a large part of the alignment error will be used up.

Parameter	**Value**	**Units**
Horizontal segment location tolerance	250	μm
Vertical segment location tolerance	50	μm
Longitudinal segment location tolerance	500	μm
Segment roll tolerance	1000	μrad
Segment yaw tolerance	50	μrad
Segment pitch tolerance	30	μrad

Table 6: Alignment parameters taken from the LCLS parameter database.

7 Secondary Bench Requirements

The primary requirements of the bench have been listed. We now list several secondary requirements.

1. The bench must have a cable handling system. The bench manufacturer will build the bench to our specifications with no torques on the carriage. A secondary cable handling system, such as used at DESY, must be included to handle the unknown torques on the carriage from the cables.

2. Both x and y stages are required on the carriage. Their purpose is to perform scans of the probes in x and y to magnetically align the undulator to the bench. They then position the probe on the undulator axis.

3. Linear encoders are required on all three axes, x, y, and z.

4. Since the probes are direction sensitive, a mounting method must be provided so the probes can be set to the correct angle. Two angles must be provided. Using the same convention as for the carriage, they are roll and pitch. The pitch angle can be provided by a rotary stage. The roll angle must be provided by a goniometer. This keeps the y position of the probe fixed as the angle is varied.

5. All stages must have brakes. The probes are very delicate. In case of power loss or any motion error, all motion must stop. As a secondary note, the undulator support must also have a brake so the undulator does not move in case of power loss.

6. All stages must have limit switches and soft mechanical limits.

7. The test bench must be nonmagnetic. It must not distort the Earth's magnetic field. The same is true of the undulator stand.

8. It must be possible to level the bench. This is an aid for alignment.

9. The carriage must be constrained so that its air bearing can not accidently move it away from its guide surfaces.

10. The center of gravity of the carriage and probe system will change during probe scans in the x direction. The carriage must meet its motion accuracy requirements for loads up to approximately 50 kg placed anywhere on the carriage.

8 Summary

The requirements of the LCLS undulator magnetic measurement bench have been specified. The bench must be at least 8.0 m long. The carriage must travel down the bench with tolerances given in table 7. Precautions must be taken so that no torques from cables are applied to the carriage. Motion in the x and y directions must also be provided. Linear encoders must be on each axis. Provisions must be made to orient the probes properly. Protection systems for the probes and the measurement bench must be in place.

Parameter	**Value**	**Units**
Carriage x deviation tolerance	40	μm
Carriage y deviation tolerance	14	μm
Carriage roll deviation tolerance	14	μrad
Carriage pitch deviation tolerance	4.2	μrad
Carriage yaw deviation tolerance	4.2	μrad

Table 7: Accuracy requirements for the bench construction.

Acknowledgements

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