

Introduction To LCLS Undulator Tuning

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Abstract

This note gives a general introduction to undulator tuning for the LCLS. It starts with a theoretical discussion in which the equations necessary to understand undulator tuning are derived. The trajectory of an electron in an undulator and the relation between the electron motion and a radiation wave are analyzed. Common terms are defined such as slippage, K_{eff} , and B_{eff} . The radiation wavelength is derived. After the theoretical discussion, the results are illustrated with simulations. A program was written which gives simulated undulator field measurements along with an assortment of possible field errors. The simulated measurements are sent to the analysis program we are developing and the effect of various field errors is studied. The shims used to correct the field errors are discussed.

1 Introduction

The LCLS is a free electron laser using 33 undulator segments. After the undulators are built, their magnetic properties must be measured and adjusted until they are within specifications, a process called tuning. This note is intended as an introduction to undulators, the errors in undulators, and the tools used in tuning to correct the errors.

The note starts with the calculation of the trajectory of an electron in an undulator. The relation between the electron motion and a radiation wave is analyzed. Explanations of common terms such as slippage, K_{eff} , and B_{eff} are presented. The radiation wavelength is derived. This theoretical material is required to understand undulator tuning.

To illustrate the equations with practical examples, a number of computer simulations are presented. A program we wrote generates simulated undulator field measurements which can contain an assortment of possible field errors. The simulated measurements are sent to the undulator data analysis program we are developing and the effect of the various field errors is studied. The shims used to correct the field errors are discussed.

The goal of undulator tuning is to keep the electrons adding to the radiation wave so the intensity of the radiation grows to a large value. There are two primary requirements for this to happen. First, since the radiation travels in a straight line, the average electron trajectory must be a straight line along the radiation path. Second, the phase of the radiation an electron emits must be in phase with the radiation wave in the undulator to maximize intensity, otherwise destructive interference will take place. The tuning process can basically be divided along these lines. First shims are added to make both the x and the y trajectories straight along the radiation path, and second, shims are added to keep the oscillatory electron motion synchronized with the radiation wave. We will now begin to study the details of this process.

2 Analysis

In this section we derive the equations necessary to understand undulator tuning. We calculate the motion of an electron in an undulator. We calculate the relation between the electron motion and a radiation wave.

The material in this section concentrates on the requirements of undulator tuning. Several excellent general introductions to undulator physics exist.¹ We were also aided by a note which discusses the analysis used for the APS undulators.²

2.1 Electron Trajectory

A relativistic electron in a magnetic field obeys the Lorentz force equation³

$$\frac{d}{dt}(\gamma m \vec{v}) = q \vec{v} \times \vec{B} \quad (1)$$

In this equation $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$ is the Lorentz factor, m is the electron rest mass, \vec{v} is the electron velocity with magnitude v , q is the electron charge (q is negative), and \vec{B} is the magnetic field strength. Since the energy of the electron is not changed by the magnetic field, γ is constant (we neglect the energy losses due to radiation) and can be taken outside the time derivative. The equation then becomes

$$\frac{d}{dt} \vec{v} = \frac{q}{\gamma m} \vec{v} \times \vec{B} \quad (2)$$

¹See, for instance, K. Wille, "Introduction To Insertion Devices", CERN Accelerator School, CERN 98-04, 1998. Also see R. Walker, "Insertion Devices: Undulators And Wigglers" in the same proceedings.

²R. J. Dejus, I. B. Vasserman, S. Sasaki, and E. R. Moog, "Undulator A Magnetic Properties and Spectral Performance", ANL/APS/TB-45, May 2002.

³SI units are used throughout. One exception is that magnetic fields are mentioned which are measured in Gauss (= 10^{-4} Tesla).

We now take z to be the direction of primary electron motion along the undulator, y to be the upward direction, and x to be the direction such that xyz forms a right handed coordinate system. This is illustrated in figure 1. We take the main

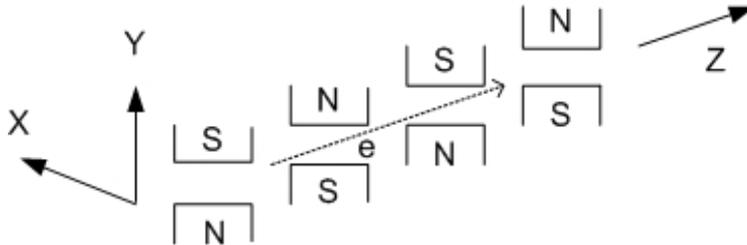


Figure 1: The coordinate system has z along the undulator in the direction of the primary electron motion, y is up, and x forms a right handed system.

undulator field to be in the y direction. We assume there is a small error field in the x direction. We neglect the field in the z direction for calculating the electron trajectory. We will only consider first order effects. Since the main electron motion is in the z direction, B_z has no first order effect. The main velocity component v_z will produce v_x and v_y in the magnetic field of the undulator, but the added dynamics caused by these velocities will not be discussed. With these assumptions, equation 2 can be written in component form as

$$\frac{d}{dt}v_x = -\frac{q}{\gamma m}v_z B_y \quad (3)$$

$$\frac{d}{dt}v_y = \frac{q}{\gamma m}v_z B_x \quad (4)$$

These equations relate the time change of a velocity to the magnetic field at the electron's position in space. It is convenient to change variables from time to the electron's position. The relativistic electron has $v_z \gg v_x$ and $v_z \gg v_y$. It is a very good approximation to use $dt = \frac{1}{v_z}dz$ with v_z constant. This allows us to express the time derivatives as $\frac{d}{dt} = \frac{dz}{dt} \frac{d}{dz} = v_z \frac{d}{dz}$. We denote derivatives with respect to z with a prime. So, $\frac{d}{dt}v_x = v_z v'_x$. Since $v_x = \frac{d}{dt}x = v_z x'$, we have $\frac{d}{dt}v_x = v_z^2 x''$. Using this notation, we arrive at the following equations of motion

$$x'' = -\frac{q}{\gamma m v_z} B_y \quad (5)$$

$$y'' = \frac{q}{\gamma m v_z} B_x \quad (6)$$

The electron's z position is now the independent variable in these equations.

To find the horizontal and vertical slopes of the trajectories, we integrate these equations along z . At the initial point $z = z_0$ we take the initial slopes to be zero.

$$x'(z) = \int_{z_0}^z x''(z_1) dz_1 = -\frac{q}{\gamma m v_z} \int_{z_0}^z B_y(z_1) dz_1 \quad (7)$$

$$y'(z) = \int_{z_0}^z y''(z_1) dz_1 = \frac{q}{\gamma m v_z} \int_{z_0}^z B_x(z_1) dz_1 \quad (8)$$

To find the horizontal and vertical trajectories, we integrate again taking the initial positions to be zero.

$$x(z) = \int_{z_0}^z x'(z_2) dz_2 = -\frac{q}{\gamma m v_z} \int_{z_0}^z \int_{z_0}^{z_2} B_y(z_1) dz_1 dz_2 \quad (9)$$

$$y(z) = \int_{z_0}^z y'(z_2) dz_2 = \frac{q}{\gamma m v_z} \int_{z_0}^z \int_{z_0}^{z_2} B_x(z_1) dz_1 dz_2 \quad (10)$$

These important equations allow us to calculate the trajectory of the electron. We measure B_x and B_y as a function of z along the undulator. We then numerically integrate the measured fields to calculate the trajectories.

2.2 Slippage

Suppose we have a radiation wave in the undulator along with the electrons. We now wish to study the relative motion between the radiation wave and an electron in the undulator. The electron can not keep up with the radiation which is travelling at the speed of light. Also, the large, alternating undulator magnetic field B_y causes the x trajectory to oscillate. This oscillation adds path length to the trajectory. The added path length, coupled with the fact that the electron is travelling slower than the speed of light, causes the electron to lag behind the radiation wave. The distance between a point on the radiation wave and the electron is called slippage. In spite of the slippage, the electron motion must keep a proper phase relationship with the radiation if the electron is to add to the radiation field in such a way as to maximize the intensity. The relative motion between the radiation wave and the electron will have important implications for the performance of the undulator. We now analyze the relative motion, or slippage, in detail.

The radiation in the undulator moves with the speed of light c in the z direction and the electron moves with speed v_z . In time dt , the slippage changes by

$$dS = (c - v_z)dt \quad (11)$$

We have defined S to be the slippage. We now wish to change variables, as we did in the trajectory calculation. Instead of using time, we use the z position of a point on the radiation wave as our independent variable. With this change of variables, we have $dz = cdt$. Then,

$$dS = \left(1 - \frac{v_z}{c}\right)dz \quad (12)$$

With z giving the position of a point on the radiation wave, the position of the electron is z minus the slippage distance.

If the electron has velocity v with components v_x and v_z , $v^2 = v_z^2 + v_x^2$. We can write this as $v^2 = v_z^2(1 + \frac{v_x^2}{v_z^2}) = v_z^2(1 + x'^2)$. The velocity can be expressed in terms of γ , $v^2 = c^2(1 - \frac{1}{\gamma^2})$. Combining these formulas, we have $c^2(1 - \frac{1}{\gamma^2}) = v_z^2(1 + x'^2)$. This lets us solve for v_z , $v_z = c\sqrt{(1 - \frac{1}{\gamma^2})} \div \sqrt{(1 + x'^2)}$. It is a very good assumption for relativistic electrons in an undulator that $\frac{1}{\gamma^2} \ll 1$, and $x'^2 \ll 1$. Using these approximations and keeping only first order terms in small quantities, $v_z \simeq c(1 - \frac{1}{2\gamma^2}) \div (1 + \frac{1}{2}x'^2)$, or

$$v_z \simeq c(1 - \frac{1}{2\gamma^2} - \frac{1}{2}x'^2) \quad (13)$$

The slippage differential can now be written as

$$dS = (\frac{1}{2\gamma^2} + \frac{1}{2}x'^2)dz \quad (14)$$

Integrating from initial position z_0 to z , we find the change in slippage

$$\Delta S = \int_{z_0}^z (\frac{1}{2\gamma^2} + \frac{1}{2}x'^2)dz_1 \quad (15)$$

If we define the initial position to have zero slippage, $S(z_0) = 0$, we can define the slippage as

$$S(z) = \int_{z_0}^z (\frac{1}{2\gamma^2} + \frac{1}{2}x'^2)dz_1 \quad (16)$$

Two terms in the integral contribute to the slippage. The first term arises because the electron is moving slower than the speed of light. The second term comes from the extra path length of the trajectory when the electron is moving side to side in the undulator. Outside the undulator, where $x' = 0$, the slippage varies linearly with z as $S(z) = \frac{1}{2\gamma^2}z + const$. Inside the undulator, both terms contribute and the slippage varies more rapidly.

In a more detailed analysis in which error fields are included, we would include the contribution of y' to the slippage. Ideally, the y trajectory is a straight line at $y = 0$ with $y' = 0$. y trajectory errors, however, generate y' and do cause slippage errors. Starting with $v^2 = v_z^2 + v_x^2 + v_y^2$ and going through the above analysis, equation 16 becomes

$$S(z) = \int_{z_0}^z (\frac{1}{2\gamma^2} + \frac{1}{2}x'^2 + \frac{1}{2}y'^2)dz_1 \quad (17)$$

We will assume the y trajectory can be made fairly straight and will only mention the y' contribution briefly in future sections.

2.3 Sinusoidal Field

Consider now the special case of a sinusoidal magnetic field with no field errors. Furthermore, consider only the oscillatory motion of the electron in the central region of the undulator. We neglect end effects and constants of integration because they do not add to our present discussion.

Suppose on the midplane of the undulator that

$$B_y = B_0 \cos\left(\frac{2\pi z}{\lambda_u}\right) \quad (18)$$

Inserting this field into equation 5 and integrating, we find

$$x''(z) = -\frac{q}{\gamma m v_z} B_0 \cos\left(\frac{2\pi z}{\lambda_u}\right) \quad (19)$$

$$x'(z) = -\frac{q}{\gamma m v_z} \left(\frac{\lambda_u}{2\pi}\right) B_0 \sin\left(\frac{2\pi z}{\lambda_u}\right) \quad (20)$$

$$x(z) = \frac{q}{\gamma m v_z} \left(\frac{\lambda_u}{2\pi}\right)^2 B_0 \cos\left(\frac{2\pi z}{\lambda_u}\right) \quad (21)$$

In all integrals, the terms involving the initial position z_0 have been neglected to isolate the oscillatory motion in the body of the undulator.

Consider now the slope x' of the trajectory. In general terms, an accelerating relativistic electron emits radiation in a cone of opening angle $\frac{1}{\gamma}$ relative to its instantaneous direction.⁴ The slope of the trajectory, x' , gives the instantaneous direction of the electron relative to the z axis. If $x' \gg \frac{1}{\gamma}$, a wide path of radiation is created. This is the situation in a wiggler. On the other hand, if $x' \simeq \frac{1}{\gamma}$, a small spot of radiation is created. This is the situation in an undulator. The relation between x' and $\frac{1}{\gamma}$ is expressed in a quantity called the K parameter. We construct K in such a way as to emphasize the relation between x' and $\frac{1}{\gamma}$

$$x' = \frac{K}{\gamma} \sin\left(\frac{2\pi z}{\lambda_u}\right) \quad (22)$$

From this equation, we see that a wiggler has $K \gg 1$ and an undulator has $K \simeq 1$. Using equation 22 and equation 20 for consistency, we define K as follows,

$$K = -\frac{q}{m v_z} \left(\frac{\lambda_u}{2\pi}\right) B_0 \quad (23)$$

In this equation, v_z is often set equal to c with very small error in K .

⁴A. Hofmann, "Characteristics Of Synchrotron Radiation", CERN Accelerator School, CERN 98-04, 1998.

Using equation 22 in equation 15, we evaluate how the slippage changes in a sinusoidal magnetic field.

$$\Delta S = \int_{z_0}^z \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} \frac{K^2}{\gamma^2} \sin^2\left(\frac{2\pi z}{\lambda_u}\right) \right) dz_1 \quad (24)$$

Expanding the \sin^2 term and performing the integral, we get

$$\Delta S = \int_{z_0}^z \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} \frac{K^2}{\gamma^2} \left(\frac{1}{2} (1 - \cos(2\frac{2\pi z}{\lambda_u})) \right) \right) dz_1 \quad (25)$$

$$\Delta S = \frac{1}{2} \frac{1}{\gamma^2} (1 + \frac{1}{2} K^2) \Delta z - \frac{1}{4} \frac{K^2}{\gamma^2} \left(\frac{\lambda_u}{4\pi} \right) \left[\sin\left(\frac{4\pi z}{\lambda_u}\right) - \sin\left(\frac{4\pi z_0}{\lambda_u}\right) \right] \quad (26)$$

The slippage change is made up of a term which varies linearly with Δz and an oscillating term. At the peaks of the magnetic field $(\frac{2\pi z}{\lambda_u}) = 0, \pi, 2\pi, \dots$. If we evaluate the slippage change between the peaks of the magnetic field, the oscillating term is zero. If we define ΔS_p to be the slippage change between field peaks and Δz_p the distance between the field peaks,

$$\Delta S_p = \frac{1}{2} \frac{1}{\gamma^2} (1 + \frac{1}{2} K^2) \Delta z_p \quad (27)$$

Again, the first term comes from the fact that the electron is travelling slower than c . The second term comes from the extra path length due to the oscillating trajectory in the magnetic field.

The pure sinusoidal magnetic field with no errors produces a simple oscillating trajectory and yields a simple expression for the slippage in terms of the undulator parameter K . It is a standard against which actual undulators are compared. For example, we can compare the slippage between peak fields from an actual undulator using equation 15 to the corresponding value from an undulator with a pure sinusoidal field with specified K given in equation 27. We define the difference as the slippage error ε_p ,

$$\varepsilon_p = \int_{z_{p1}}^{z_{p2}} \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2 \right) dz - \frac{1}{2} \frac{1}{\gamma^2} (1 + \frac{1}{2} K^2) (z_{p2} - z_{p1}) \quad (28)$$

As we will see below, the slippage error is a very useful quantity for studying undulator errors.

2.4 K_{eff} , B_{eff} , λ_r

We have just seen how an actual undulator can be compared to an undulator with a pure sinusoidal field in the discussion of the slippage error. This is an example of a direct point by point comparison. It is also possible to compare the average properties of an undulator to the pure sinusoidal case. This is the subject we now

wish to discuss. We seek to describe the average properties of an undulator by an equivalent undulator with a sinusoidal magnetic field. We seek K_{eff} , the effective K value of the actual undulator such that its fundamental properties are the same as an undulator with a sinusoidal magnetic field with $K = K_{eff}$.

Consider an arbitrary magnetic field in an undulator. The slippage change between peak fields can be calculated from the measured magnetic field data using equation 15 and equation 7. If we fit the measured slippage at the field peaks with a straight line, we obtain a linear relation between slippage change and z change. This is illustrated in figure 2. We compare to equation 27, which was derived using a sinusoidal magnetic field. The sinusoidal field also had a linear relation between slippage change and z change between peak fields. A sinusoidal field with K_{eff} gives

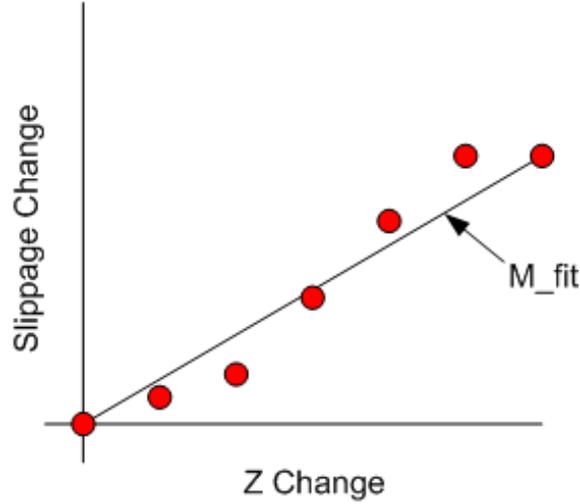


Figure 2: The slippage change at each peak field measured from a reference peak field is plotted as a function of the z change for the actual undulator. A fitted line of slope M_{fit} is used to compare to an undulator with a sinusoidal magnetic field.

the same slope to the ΔS_p vs. Δz_p curve as obtained in the measured data. Let M_{fit} be the fitted slope of the measured ΔS_p vs. Δz_p curve calculated using equation 15 evaluated at the field peaks. Then,

$$M_{fit} = \frac{1}{2} \frac{1}{\gamma^2} \left(1 + \frac{1}{2} K_{eff}^2 \right) \quad (29)$$

$$K_{eff} = \sqrt{2(2\gamma^2 M_{fit} - 1)} \quad (30)$$

Using the relation between K and the amplitude of the sinusoidal magnetic field strength in equation 23, we can define an effective magnetic field in the undulator by

$$B_{eff} = -\frac{mv_z}{q} \left(\frac{2\pi}{\lambda_u} \right) K_{eff} \quad (31)$$

The slippage at the peak fields can be used to calculate the radiation wavelength. Consider an electron at the positive peak of its trajectory corresponding to the peak field of an undulator period as shown in the top of figure 3. The electron adds

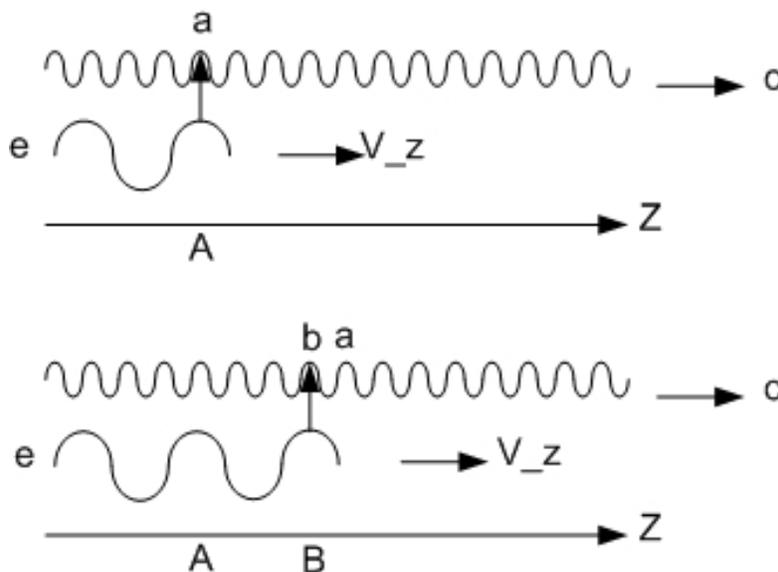


Figure 3: The upper figure shows an electron at position "A" adding photon "a" to the radiation wave. The lower figure shows the situation after the electron has moved one undulator period to position "B". The electron adds photon "b", which is behind "a" because "a" is moving faster than the electron.

radiation to the radiation wave. After the electron has moved one undulator period, it adds more radiation as shown in the lower part of the figure. Suppose this radiation is added in such a way as to maximize the intensity of the radiation wave. The new radiation can not add to the radiation from the previous positive peak because of the forward speed difference between the radiation and the electron. Rather, it adds to the radiation wave one wavelength back. In one undulator wavelength, the slippage increases by one radiation wavelength. Thus, in equation 27, we can set $\Delta S_p = \lambda_r$ when $\Delta z_p = \lambda_u$. Letting $K = K_{eff}$ which we determined from the measured data assuming it represented a sinusoidal magnetic field, we can determine the radiation wavelength,

$$\lambda_r = \frac{1}{2} \frac{1}{\gamma^2} \left(1 + \frac{1}{2} K_{eff}^2\right) \lambda_u \quad (32)$$

The importance of K_{eff} should be noted. It determines the radiation properties of the undulator for given λ_u . In tuning the LCLS undulators, one of the primary jobs will be to make each of the 33 undulators have the same K_{eff} , so each undulator can properly add to the radiation wave at the same wavelength. (In practice, K_{eff} is adjusted to account for the reduction in γ due to energy loss to radiation upstream.)

2.5 Phase

Slippage is a very useful quantity which can be computed directly from the magnetic field measurements. It yields a quantity in meters which, however, is primarily important in its relation to the radiation wavelength. By scaling the slippage by the radiation wavelength, we come up with a quantity, called phase, which indicates the relation between the electron motion and the radiation, but is independent of the radiation wavelength.

We define phase by dividing the slippage by λ_r and multiplying by 2π . The slippage is given by equation 16. We gave a prescription for calculating λ_r in the previous section. Let P be the phase,

$$P(z) = \frac{2\pi S(z)}{\lambda_r} \quad (33)$$

The phase is zero at the initial point where the slippage was defined to be zero. If we let ΔP_p be the phase change between a field peak and some reference field peak in the body of the undulator,

$$\Delta P_p = \frac{2\pi}{\lambda_r} \Delta S_p \quad (34)$$

ΔS_p comes from equation 15 evaluated at the field peaks.

In a perfectly sinusoidal magnetic field, the slippage would change by λ_r as the electron moved a distance λ_u . If we evaluate the phase at the peak field points, it would increase by 2π every undulator period in an ideal undulator, or by π every pole or field peak. In a perfect undulator $\Delta P_p = \pi n_p$, where n_p is the number of the field peak relative to the reference pole.

In an actual undulator, the phase change is not exactly π every pole due to errors in the undulator. We define the phase error ϵ_p at a field peak as the difference between the measured phase change and a multiple of π ,

$$\epsilon_p = \Delta P_p - \pi n_p \quad (35)$$

where n_p is the number of the field peak relative to the reference.

Another useful way to view phase errors is as follows. In figure 2 we fit a line to the slippage at the field peaks vs the z position of the field peak. Now consider the residuals of the fit. They are given by

$$r_p = \Delta S_p - \frac{n_p}{2} \lambda_r \quad (36)$$

In this equation ΔS_p is the measured slippage change between the n_p 'th field peak and the reference field peak. It is represented by a point in the plot. The corresponding point on the line is determined according to the discussion used to determine λ_r . The slippage change along the fitted line is $\Delta S_{line} = \lambda_r$ for every $\Delta z = \lambda_u$, or $\Delta S_{line} = \frac{n_p}{2} \lambda_r$ at the n_p 'th field peak. The difference between the measured slippage

change and the value from the fitted line is the residual. If we now scale the residual by $\frac{2\pi}{\lambda_r}$, we get the phase error. When viewing the phase error, it is often an aid in understanding its shape to think of it in terms of the residuals.

Note that the phase is calculated using the λ_r determined for the undulator, not the ideal λ_r . Phase errors measure how consistently the undulator radiates at λ_r . It is an entirely separate matter to set K_{eff} to the proper value so λ_r has the proper value. In this manner, slippage errors and phase errors are different. Slippage errors measure the difference between the actual slippage and the ideal slippage for specified K . Phase errors measure the residuals between the actual slippage and the linear fit which was used to determine λ_r .

2.6 Phase Matching

In the previous section we noted how the electron motion must stay in phase with the radiation wave in the undulator to maximize output power. We considered small imperfections in the undulator field causing a phase error which we must correct. There is a more global problem, however. We need to adjust each of the 33 individual undulator sections so that the radiation produced in each section is in phase with the overall radiation wave. We assume we have already adjusted K_{eff} to the proper value. This makes the undulator produce radiation with the proper wavelength, but it can still have a possible phase shift. We must now adjust the phase of the electron trajectory motion so that the new radiation is in phase with the existing radiation wave.

Phase matching schemes for the LCLS have been proposed.⁵ One idea is to change the K value over short sections at the ends of the undulator. Another idea is to change the z position of the undulator and use the slippage from $v < c$ outside the undulator to get the phase change. For any scheme, both the phase change to get into the body of the undulator and the phase change at the exit require adjustment.

The LCLS will have spaces between the undulator sections which we divide and assign to undulators. The undulator plus its space at each end make up a cell. We can calculate the phase change in a cell using equations 33 and 16. We know how to calculate λ_r from the measurements. In fact, we will tune the undulators so λ_r has a specified value by setting K_{eff} . We also know how to calculate the slippage from the measurements using equation 16. Following these steps and letting ΔP_{cell} be the phase change in a cell,

$$\Delta P_{cell} = \frac{2\pi}{\lambda_r} \int_{cell} \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2 \right) dz \quad (37)$$

To keep the proper phase relation between cells, we require ΔP_{cell} to be a multiple of 2π .

⁵LCLS Conceptual Design Report, SLAC_R_593.

It is difficult to calculate ΔP_{cell} analytically. The fields at the ends of the undulator are fairly complex and their effect on x' needs to be included in the integral. In general, we will calculate ΔP_{cell} numerically from the magnetic field measurements and perform an adjustment to correct its value.

Making ΔP_{cell} a multiple of 2π is not enough, however. We require the proper phase change from the beginning of the cell into the body of the undulator. Consider figure 4. It shows the x trajectory of an electron in an undulator with an even number of poles, as in the LCLS. The cell boundaries are marked with dotted lines and the reference poles are marked with arrows. The reference poles are the third pole in from each end in this example. Notice that with the reference poles an equal distance in from the ends, there is an extra half oscillation cycle between reference poles. The phase change between reference poles is an integer times 2π plus π for the extra half cycle. We write this as $\Delta P_{und_ctr} = (2n + 1)\pi$, or $\Delta P_{und_ctr} = \pi$ modulo 2π . If the cell is required to have a phase change of an integer times 2π , the phase change at the entrance and exit of the undulator must add to π modulo 2π . We divide this evenly between the ends. Thus we require the phase change from the cell entrance to the first reference pole to be $\Delta P_{enter} = \pm\frac{\pi}{2}$ modulo 2π , and the phase change from the second reference pole to the far cell boundary $\Delta P_{exit} = \pm\frac{\pi}{2}$ modulo 2π . The signs must be the same for entrance and exit.

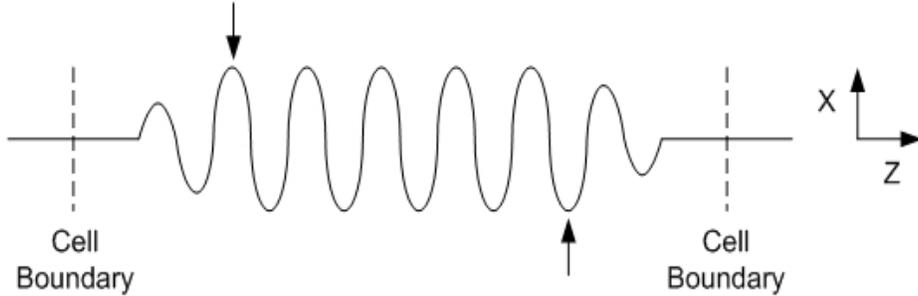


Figure 4: Electron trajectory with the reference poles marked with arrows. The phase from the cell boundary to the first reference pole, the phase between reference poles, and the phase from the second reference pole to the far cell boundary must all be determined.

Let ΔP_{enter} be the phase change from the entrance cell boundary at z_{c1} to a given reference pole at z_{r1} .

$$\Delta P_{enter} = \frac{2\pi}{\lambda_r} \int_{z_{c1}}^{z_{r1}} \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2 \right) dz \quad (38)$$

We determine ΔP_{enter} from our measurements and perform adjustments until $\Delta P_{enter} = 2\pi n \pm \frac{\pi}{2}$, where n is an integer.

Similarly, let ΔP_{exit} be the phase change from the second reference pole near the end of the undulator at z_{r2} to the cell boundary at the exit at z_{c2} .

$$\Delta P_{exit} = \frac{2\pi}{\lambda_r} \int_{z_{r2}}^{z_{c2}} \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2 \right) dz \quad (39)$$

We determine ΔP_{exit} from our measurements and perform adjustments until $\Delta P_{exit} = 2\pi m \pm \frac{\pi}{2}$, where m is an integer. The sign of the $\frac{\pi}{2}$ must be the same as in ΔP_{enter} .

Using this scheme for all undulators, the electron motion will be in phase with the radiation in the body of all the undulators. The cells independently preserve the phase relationship.

3 Simulations

The LCLS undulator analysis program we are developing uses the theoretical tools presented above. A program was also written which generates simulated measurement data including an assortment of typical field errors.⁶ We now use these programs to study the effect of various field errors. Recognizing the signatures of the field errors will allow us to make the proper corrections in the tuning process.

The parameters used to generate the simulated data are shown in table 1. The values correspond to current LCLS short wavelength parameters. The sample spacing is a typical value we expect to use for the magnetic field measurements.

Parameter	Value	Units
Electron Energy	14.08	GeV
Lorentz Factor, γ	27549	
Nominal Field, B_0	1.296	T
Undulator Period, λ_u	0.03	m
Undulator Parameter, K	3.6300	
Number of Poles	226	
Sample Spacing	1.5×10^{-4}	m
Number of Samples	26803	

Table 1: Parameters used in generating the simulated magnetic measurement data for all data sets in this note.

An important parameter in the analysis program is the number of end poles. All examples in this note use 5 end poles. The fit to find K_{eff} uses poles 6 to 221. The reference pole from which slippage changes and phase changes are summed is pole 6. The phase change into the undulator is measured from the cell boundary to the peak field position at pole 6. The phase change out of the undulator is measured from the peak field at pole 221 to the far cell boundary. The phase change in the center of the undulator is measured from the field peak at pole 6 to the field peak at pole 221.

⁶The programs are written in Matlab and are available from the author.

3.1 No Field Errors

Figure 5 shows a perfect sinusoidal field. It also shows the first and second integrals of the field calculated numerically. There is a heavy line in each plot which is the average value taken over one undulator period. The end poles have strengths in the ratio of 1/4, 3/4, 1. This gives a trajectory with no offset and no slope. The fields from the end poles in the model are half sinusoids with peak fields in the ratios given. In an actual undulator, the field changes more gradually at the ends, however, the model is adequate for our purposes.

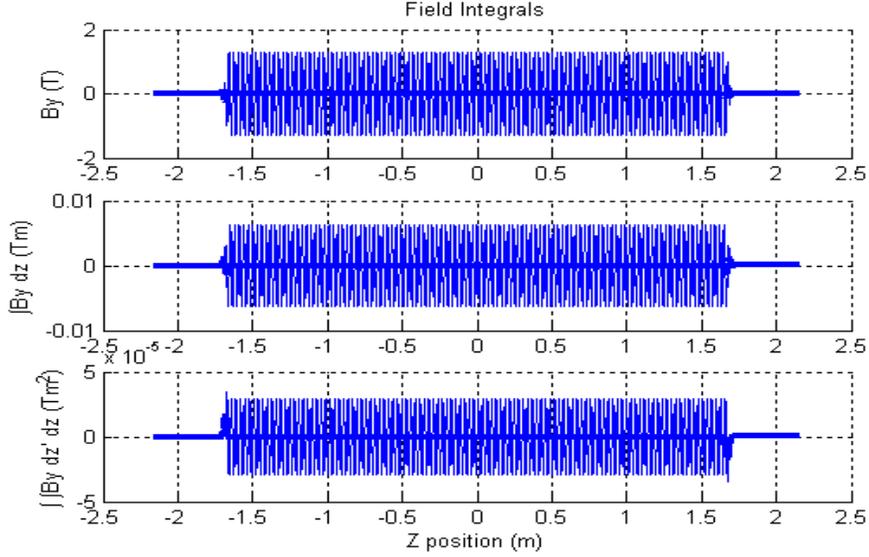


Figure 5: Sinusoidal field and its first and second integrals. The heavy line shows the average over one period.

Using the generated magnetic field, we calculate the x trajectory of an electron using equation 9. The trajectory is shown in figure 6, along with x' and x'' . Note that the electron follows a sinusoidal path with an amplitude of roughly $0.63 \mu\text{m}$.

Figure 7 shows the slippage as a function of z both outside and inside the undulator. It was calculated using equation 16. Note that outside the undulator, the path of the electron has $x' = 0$ and the slippage comes only from the speed of the electron being less than c . Using $\gamma = 27549$, in the drift region outside the undulator we find $\Delta S_{drift} = \frac{1}{2} \frac{1}{\gamma^2} \Delta z = 6.59 \times 10^{-10} \Delta z$. We can convert this to phase, $\Delta P_{drift} = \frac{2\pi}{\lambda_r} \Delta S_{drift}$. Using $\lambda_r = \frac{1}{2} \frac{1}{\gamma^2} (1 + \frac{1}{2} K^2) \lambda_u$ given by equation 32, we find

$$\Delta P_{drift} = 2\pi \frac{\Delta z}{(1 + \frac{1}{2} K^2) \lambda_u} \quad (40)$$

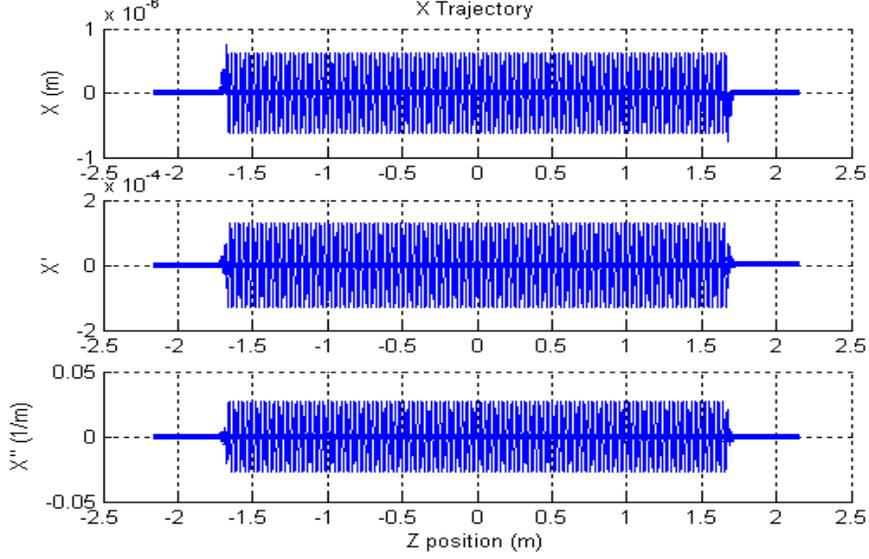


Figure 6: Electron trajectory x , along with x' and x'' .

Inserting $K = 3.630$ and $\lambda_u = 0.03$ m and converting to degrees, we find $\Delta P_{drift} = (1.58 \times 10^3 \text{ degrees per meter})\Delta z$. Every centimeter of drift gives a phase shift of 15.8 degrees. A drift length of 0.2279 m gives a phase change of 360° . These numbers are useful for phase matching. Also note that the phase change in a drift region is independent of electron energy.

Inside the undulator, the oscillating path of the electron causes the slippage to increase much more rapidly. From equation 27, the slippage at the field peaks is greater than the free space slippage by a factor of $(1 + \frac{1}{2}K^2)$. Using $K = 3.630$, this factor is 7.59. Inside the undulator, $\Delta S = 5.00 \times 10^{-9}\Delta z$, and $\Delta P = (1.20 \times 10^4 \text{ degrees per meter})\Delta z$. The phase change corresponds to 360° every $\lambda_u = 0.03$ m, as expected.

The program performs numerical calculations to determine K_{eff} , B_{eff} , λ_r , etc. The values of important parameters calculated by the program are given in table 2. Note that K_{eff} calculated by the analysis program agrees with the input value of $K = 3.63000$. This is an important check of the numerical methods.

The phase matching results given in table 2 were obtained after choosing the proper cell length. This was done by first making an initial determination of the phase matching and then calculating the change in the drift length required to give the correct phase matching. The estimate given above of $\Delta P_{drift} = 15.8^\circ$ per centimeter of drift length was used in calculating the change. The resulting cell length is 3.840 m. It is made of a drift length of 0.225 m added to each end of the 3.390 m long magnet. The total phase change in the cell is $114 \times 360^\circ$.

The cell length for the short segments in the LCLS is presently specified to be 3.896

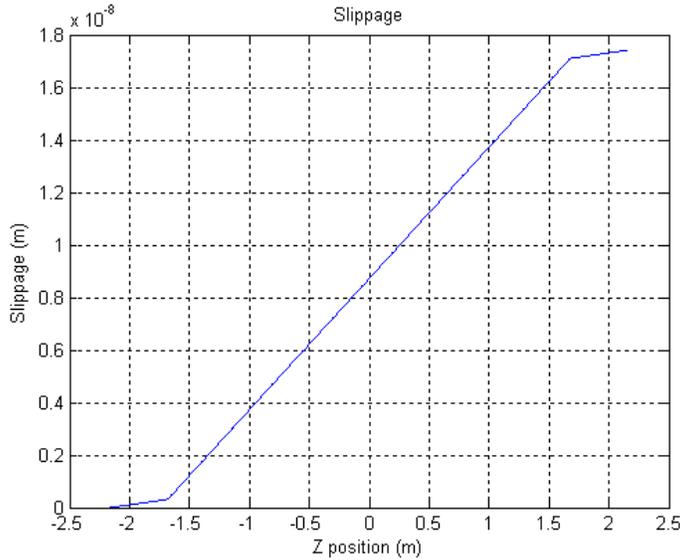


Figure 7: Slippage both outside and inside the undulator.

Parameter	Value	Units
K_{eff}	3.62998	
B_{eff}	1.29587	T
λ_r	1.4998×10^{-10}	m
$\Delta P_{enter, \text{ mod } 360^\circ}$	89.99	deg
$\Delta P_{und_ctr, \text{ mod } 360^\circ}$	180.00	deg
$\Delta P_{exit, \text{ mod } 360^\circ}$	89.99	deg
ΔP_{cell}	114.00	360 deg

Table 2: Undulator parameter values numerically calculated by the analysis program.

m. This differs from the value of 3.840 m calculated numerically in the sinusoidal field model by 5.6 centimeters. The difference is probably due to the sinusoidal form of the end fields in the model, which are undoubtedly different than the actual end fields.

3.1.1 Analytical Calculation Of The Cell Length

It is instructive to perform an analytical calculation of the cell length in order to get a deeper understanding of the results just presented. To do this, we further simplify our model of the undulator field. We approximate the field from each pole by a delta function with the same integrated field strength as the pole. This model will give a clear explanation of why the end poles have strengths in the ratio 1/4, 3/4, 1. It will also lead to simple expressions for the phase change through the undulator

and through the cell. The derived expressions give fairly good agreement with the numerical calculations of the sinusoidal field. The model will explain why the total phase change in the cell is $114 \times 360^\circ$, as given above, when there are 113 undulator periods and nearly 360° of phase change in each of the drift regions both before and after the undulator.

Consider the effect of an electron going through the field of a single pole. From equation 7, the slope of the trajectory will change. If the trajectory slope change is $\Delta x' = m$, we replace the pole in our model by a very narrow pole with integrated field strength also giving $\Delta x' = m$. This is illustrated in the upper part of figure 8. Replacing all the poles in the undulator by the very narrow poles gives the situation shown in the lower part of figure 8. The integrated field strength of each pole is represented by a vertical line. The trajectory is a series of straight lines between the narrow poles. The poles cause the slope of the trajectory to change. From the cell boundary to the undulator magnet end, the trajectory has $x' = 0$. From the start of the undulator to the first pole, a distance of $\frac{\lambda_u}{4}$, the slope is also $x' = 0$. The first pole has one quarter strength and gives the trajectory a slope of $x' = \frac{1}{4}m$. The amplitude of the trajectory at the second pole is $x = x' \Delta z = \frac{m}{4} \frac{\lambda_u}{2}$. The second pole is $-3/4$ strength and changes the slope by $\Delta x' = -\frac{3}{4}m$. The slope after the second pole is $x' = \frac{1}{4}m - \frac{3}{4}m = -\frac{1}{2}m$. The amplitude at the third pole is $x = x_{init} + x' \Delta z = \frac{m}{4} \frac{\lambda_u}{2} - \frac{m}{2} \frac{\lambda_u}{2} = -\frac{m}{4} \frac{\lambda_u}{2}$. This is the negative of the amplitude at the second pole. The third pole is full strength and changes the slope to $x' = -\frac{1}{2}m + m = \frac{1}{2}m$. At the fourth pole, the amplitude of the trajectory is back to $x = \frac{m}{4} \frac{\lambda_u}{2}$. This pattern continues through the undulator. Notice how the 1/4, 3/4, 1 pole ratios results in a trajectory with no net slope and no net offset. At the far end of the undulator, this process is reversed as shown in figure 8.

The undulator model with impulsive forces results in a trajectory consisting of straight line segments. This makes it very easy to calculate the slippage in the cell. Each straight line segment adds a slippage change given by equation 15, which for constant slope becomes $\Delta S = (\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2) \Delta z$.

We start by calculating the slippage change in the magnet. From the magnet end to the first pole, the slope is zero and the distance is $\frac{\lambda_u}{4}$, so $\Delta S_0 = (\frac{1}{2} \frac{1}{\gamma^2}) \frac{\lambda_u}{4}$. From the first pole to the second pole the slope is $\frac{m}{4}$ and the distance is $\frac{\lambda_u}{2}$, giving $\Delta S_1 = (\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} (\frac{m}{4})^2) \frac{\lambda_u}{2}$. After this, there are $(N_p - 3)$ segments with slope $\frac{m}{2}$, giving a slippage change of $\Delta S_{center} = (N_p - 3) (\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} (\frac{m}{2})^2) \frac{\lambda_u}{2}$, where N_p is the number of poles in the undulator. Finally, we add another ΔS_1 and ΔS_0 for the far end of the

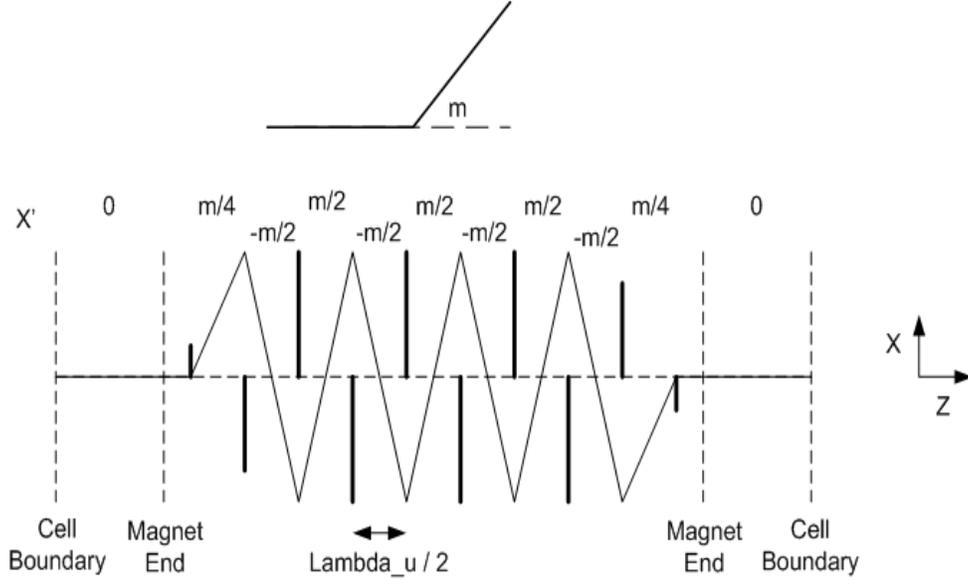


Figure 8: The undulator field is replaced by impulsive forces. The upper part of the figure shows the effect of a short pole giving the trajectory a slope change equal to that of a full strength pole. The lower part of the figure shows the model of the undulator field replaced by the impulsive forces. The trajectory is shown and the trajectory slopes are given.

undulator. The total slippage in the magnet is then

$$\begin{aligned}
\Delta S_{mag} &= \left(\frac{1}{2} \frac{1}{\gamma^2}\right) \frac{\lambda_u}{4} + \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} \left(\frac{m}{4}\right)^2\right) \frac{\lambda_u}{2} \\
&+ (N_p - 3) \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} \left(\frac{m}{2}\right)^2\right) \frac{\lambda_u}{2} \\
&+ \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} \left(\frac{m}{4}\right)^2\right) \frac{\lambda_u}{2} + \left(\frac{1}{2} \frac{1}{\gamma^2}\right) \frac{\lambda_u}{4}
\end{aligned} \tag{41}$$

This expression can be simplified. Combining terms, we have

$$\Delta S_{mag} = N_p \left(\frac{1}{2} \frac{1}{\gamma^2}\right) \frac{\lambda_u}{2} + (N_p - 3 + \frac{1}{2}) \left(\frac{1}{2} \left(\frac{m}{2}\right)^2\right) \frac{\lambda_u}{2} \tag{42}$$

The first term is the slippage change from drifting the length of the magnet. The second term is the added slippage change from the trajectory slope. We rewrite this one more time as follows,

$$\Delta S_{mag} = (N_p - 3 + \frac{1}{2}) \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} \left(\frac{m}{2}\right)^2\right) \frac{\lambda_u}{2} + (3 - \frac{1}{2}) \left(\frac{1}{2} \frac{1}{\gamma^2}\right) \frac{\lambda_u}{2} \tag{43}$$

The slippage change in the magnet is used to calculate the phase change. The first term in ΔS_{mag} is $(N_p - 3 + \frac{1}{2})$ times the slippage between poles in the body of the undulator. We know that there is a slippage change of λ_r for every undulator period which consists of two poles, so there is a slippage change of $\frac{\lambda_r}{2}$ between poles in the body of the undulator. Using $\Delta P_{mag} = \frac{2\pi}{\lambda_r} \Delta S_{mag}$ and the fact that $(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} (\frac{m}{2})^2) \frac{\lambda_u}{2} = \frac{\lambda_r}{2}$ allows us write down the phase change in the undulator magnet

$$\Delta P_{mag} = (N_p - 3 + \frac{1}{2})\pi + \frac{2\pi}{\lambda_r} (3 - \frac{1}{2}) (\frac{1}{2} \frac{1}{\gamma^2}) \frac{\lambda_u}{2} \quad (44)$$

The phase change in the magnet can be analyzed as follows. If there were no end effects, the phase change in the magnet would be $N_p\pi$, i.e. there would be a phase change of 2π for each of the $\frac{N_p}{2}$ undulator periods. We assume N_p is even as in the LCLS. The added terms come from the end effects. Dividing the expression this way and replacing $\lambda_r = \frac{1}{2} \frac{1}{\gamma^2} (1 + \frac{1}{2} K^2) \lambda_u$, we have

$$\Delta P_{mag} = \frac{N_p}{2} 2\pi - 2\pi - \frac{\pi}{2} + 2\pi (3 - \frac{1}{2}) (\frac{1}{2}) \frac{1}{(1 + \frac{1}{2} K^2)} \quad (45)$$

To get the phase change in the cell, we add to the magnet phase change the phase change from the drift region from the cell boundary to the magnet, ΔP_{drift} , and the phase change from the drift region at the other end from the magnet to the cell boundary, ΔP_{drift} , assumed equal. The formula for calculating ΔP_{drift} was given in equation 40.

$$\Delta P_{cell} = 2\Delta P_{drift} + \frac{N_p}{2} 2\pi - 2\pi - \frac{\pi}{2} + 2\pi (3 - \frac{1}{2}) (\frac{1}{2}) \frac{1}{(1 + \frac{1}{2} K^2)} \quad (46)$$

If we adjust the drift length to give a phase change of $\Delta P_{drift} = 2\pi$ before and after the undulator, the phase change in the cell is $(\frac{N_p}{2} + 1)2\pi$, plus small corrections. This explains, up to small corrections, why there was a phase change of $114 \times 360^\circ$ for the cell in the numerical calculation above. The correction term is $2\pi (3 - \frac{1}{2}) (\frac{1}{2}) \frac{1}{(1 + \frac{1}{2} K^2)} - \frac{\pi}{2}$. For $K = 3.630$, the correction is -0.17π , or -30.7° . Since a phase change of 360° occurs in a drift length of 0.2279 m as we saw above, the correction to the cell length is $+0.0194$ m. It is positive to make up for the negative phase correction. The total cell length is then $L_{cell} = 0.2279 + 3.390 + 0.2279 + 0.0194$ m = 3.865 m. This differs from the numerical calculation of 3.840 m by 0.025 m. In summary, the impulse model has given us a fairly simple, closed form expression for the phase change in a cell. It has given results which agree fairly well with the numerical evaluation of a sinusoidal field.

3.2 Pole Strength Error

Suppose we now make a single pole too strong by 1%. We arbitrarily choose pole number 51 because it is about 1/4 of the way through the undulator and has positive B_y (as do all the odd numbered poles). A strong pole might be caused by a manufacturing error in which the pole is placed too far into the gap. We wish to determine the consequences of this error. Since pole 51 has positive B_y , we expect the electron to be deflected toward positive x . (To see this, use superposition of a sinusoidal field plus an error field. The error field causes the deflection.) We also expect a phase error caused by the added contribution to x' after the deflection.

Figure 9 shows the stronger pole 51 in the bottom right corner.

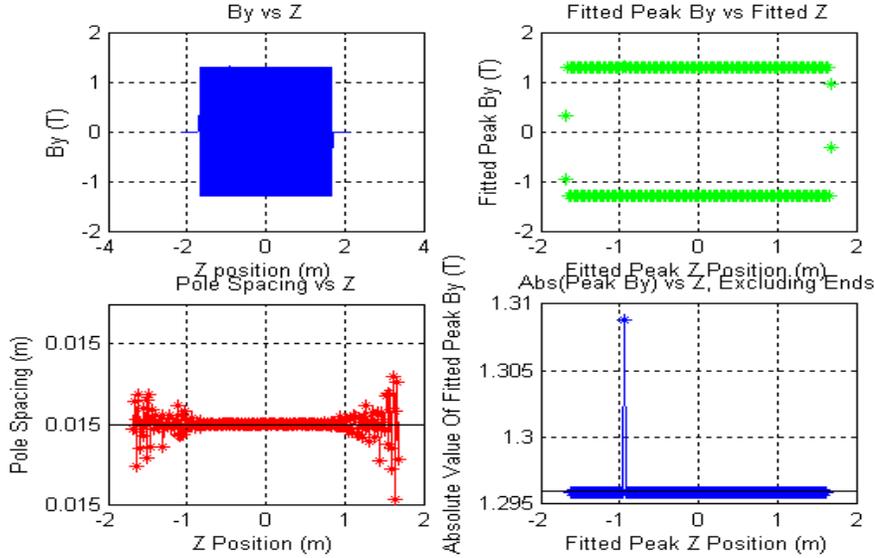


Figure 9: In this simulation, pole 51 has its strength increased by 1%. This is illustrated in the bottom right figure.

Figure 10 shows the kink in the electron trajectory caused by the kick from the strong pole. Note that the electron is about $7 \mu\text{m}$ away from the axis at the end of the undulator. This is large compared to our tuning tolerance which will be around $2 \mu\text{m}$ deviation from the axis.

We can calculate the slope of the trajectory after the kick using equation 7 and integrating over the strong pole. The 1% error over a half sinusoid has $\int B_y(z) dz = \int_0^{\lambda_u/2} (0.01) B_0 \sin(\frac{2\pi z}{\lambda_u}) dz = (0.01) B_0 (\frac{\lambda_u}{\pi}) = 1.24 \times 10^{-4} \text{ Tm}$. The other factor in equation 7 has value $-\frac{q}{\gamma m v_z} = 2.13 \times 10^{-2} 1/(\text{Tm})$. Multiplying these terms gives $x' = 2.64 \times 10^{-6}$, or $2.64 \mu\text{m}/\text{m}$ after the kick, which agrees with the plot.

Figure 11 shows the slippage outside and inside the undulator, and the slippage error at the field peaks. The slippage error is the measured slippage minus the ideal

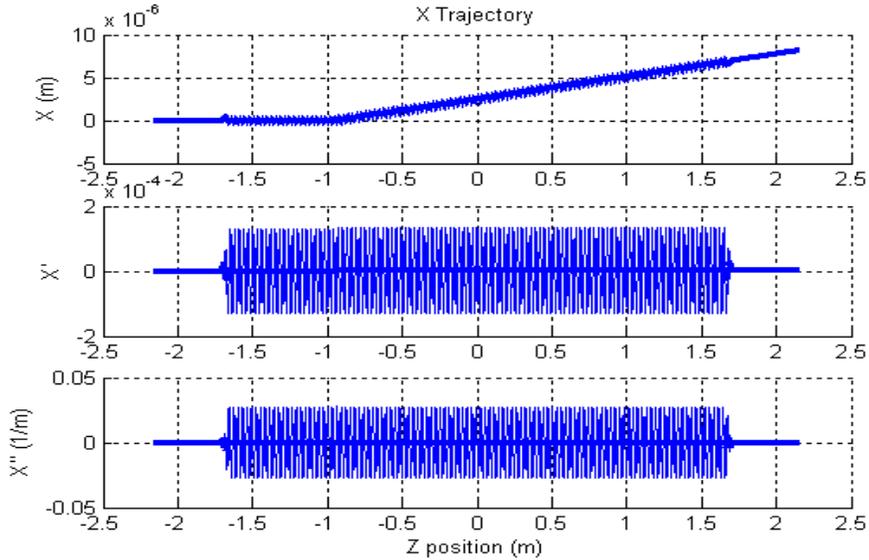


Figure 10: Pole 51 is too strong by 1%. This causes the electron trajectory to be deflected toward positive x .

value, which is calculated from the specified K value of the undulator using equation 27. There is a very visible kink in the slippage error. The slippage error increases because of the added x' due to the slope of the trajectory (see equation 16). There is also a difference between alternating poles as the electron moves toward the average trajectory angle and away from it. When the average slope adds to x' , the slippage change for that pole increases. When the average slope subtracts, the slippage change decreases.

To calculate K_{eff} , B_{eff} and λ_r , a linear fit is made to the slippage at the poles, neglecting the end poles. The slippage error in this example, however, changes the values of these parameters from the values without field errors in table 2. The new values are given in table 3. K_{eff} has changed by 0.04%, a value larger than the required accuracy we must set K_{eff} to when tuning.

Parameter	Value	Units
K_{eff}	3.63143	
B_{eff}	1.29639	T
λ_r	1.50082×10^{-10}	m

Table 3: Undulator parameters calculated with a pole strength error. Pole 51 is too strong by one percent.

Once λ_r is known, the phase change at each field peak can be found using equation 34. We use pole # 6 as the reference from which phase changes are measured. The

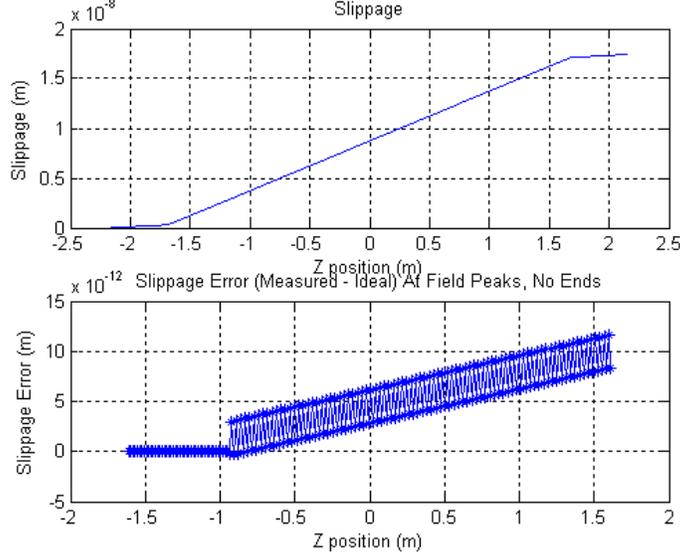


Figure 11: The slippage error shows the deviation of the slippage from the ideal value. The error comes from the added slope in the trajectory.

phase at the field peaks is shown in the upper part of figure 12. The phase errors are calculated using equation 35. They are shown in the bottom part of figure 12. Even though there is no error until pole 51, the procedure produces phase errors because the new λ_r calculated from the fit to the entire undulator does not match the λ_r from the first 50 poles. The shape of the phase error plot can also be understood from the point of view of residuals from the linear fit to the slippage vs z curve. The slippage error causes residuals from the fit, which are evident in the phase error plot.

In table 3 we saw how the pole error changed the value of K_{eff} . This was primarily due to the average slope of the trajectory, which changed x' and therefore the slippage. The fit to the slippage gave the new value of K_{eff} . A corrector magnet at the beginning of the undulator could easily take out the average slope of the trajectory. We can ask what the value of K_{eff} would be and what the phase errors would look like if we used such a corrector magnet. This is easily done in the analysis. The average trajectory slope is subtracted from x' and the phase analysis is repeated.

The undulator parameters with the average slope removed are given in table 4. Even though the pole 51 error is not fixed, correcting the average slope has improved the calculated undulator parameters substantially. As this example demonstrates, it is a very good idea to calculate the undulator parameters with the average slope removed so that the K_{eff} value is not heavily influenced by trajectory errors. Otherwise, tuning the K_{eff} value becomes coupled with tuning the trajectory.

Figure 13 shows the phase errors after the analysis has been repeated with the

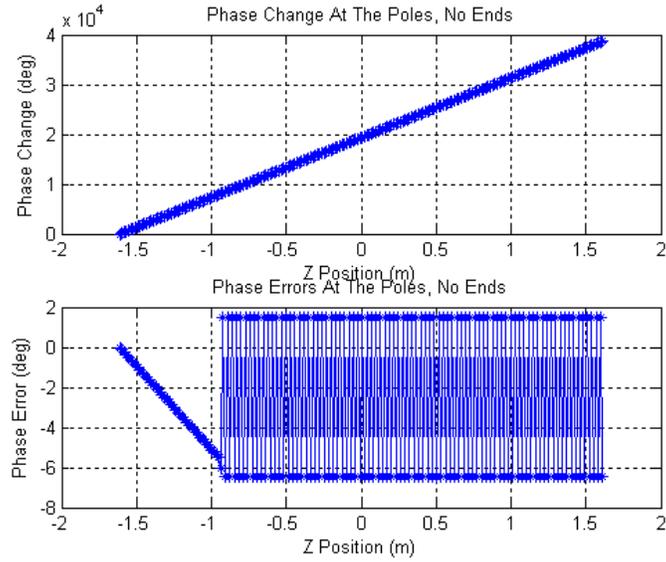


Figure 12: Phase error at the field peaks.

Parameter	Value	Units
K_{eff}	3.63031	
B_{eff}	1.29599	T
λ_r	1.50002×10^{-10}	m

Table 4: Undulator parameters calculated after the average trajectory slope had been removed.

average slope removed. The plot still shows phase errors because the section of the undulator before and after the error are operating at different λ_r than the average value. To reduce phase errors, the pole 51 error must be corrected in the tuning process. Global corrections such as making the average trajectory straight will not correct phase errors.

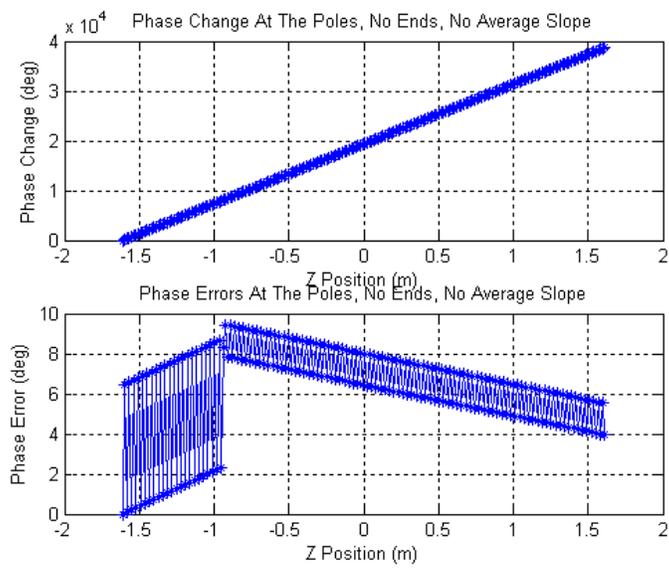


Figure 13: The average slope of the trajectory has been subtracted from x' and the phase analysis repeated.

3.3 Phase Error

Instead of making a single pole too strong, suppose we now make two neighboring poles too strong. This might be caused by too strong a magnet between the poles. We simulate this by making both poles 51 and 52 too strong by 1%. We expect the effect on the trajectory to be small because the opposite polarity poles will tend to cancel each other's deflections. The effect on the trajectory is illustrated in figure 14. The resulting trajectory is the superposition of the sinusoidal motion and a line with a sudden upward slope at pole 51 and the slope returning to zero after pole 52. A small translation results.

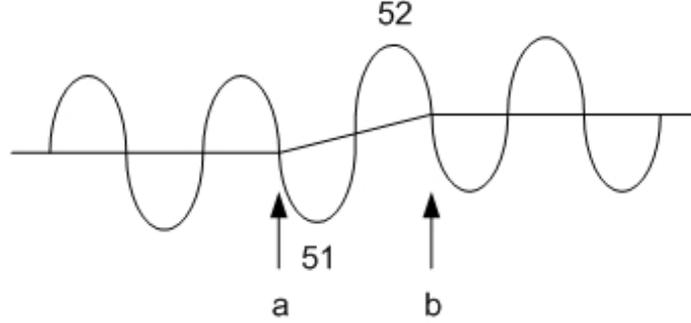


Figure 14: A phase error gives the electron two opposite kicks. No net slope to the trajectory results, but there is a small offset.

Even though the effect on the trajectory is small, the effect on the slippage is not small. We have seen how changing x' of the trajectory causes a change in slippage. We might expect a sudden change in slippage over the length of the two poles. Let us analyze this assumption within the framework of our model. The slippage between two points is given by equation 16. The slippage between points "a" and "b" in figure 14 is given by

$$S_{ab} = \int_a^b \left(\frac{1}{2} \frac{1}{\gamma^2} + \frac{1}{2} x'^2 \right) dz \quad (47)$$

The change in slippage from the case with no error to the case with the phase error is given by

$$\Delta S_{ab} = \int_a^b x' \Delta x' dz \quad (48)$$

where $\Delta x'$ is the added slope due to the phase error. The slope is given by $x' = -\frac{K}{\gamma} \cos\left(\frac{2\pi z}{\lambda_u}\right)$. The change in slope is given by $\Delta x' = -\frac{\Delta K}{\gamma} \cos\left(\frac{2\pi z}{\lambda_u}\right)$. Performing the integral, we get $\Delta S_{ab} = \frac{1}{\gamma^2} K \Delta K \frac{1}{2} \lambda_u$. We can convert to a phase change by multiplying ΔS_{ab} by $\frac{2\pi}{\lambda_r}$, where $\lambda_r = \frac{1}{2} \frac{1}{\gamma^2} (1 + \frac{1}{2} K^2) \lambda_u$. Letting ΔP_{ab} be the added

phase due to the phase error, we get

$$\Delta P_{ab} = 2\pi \frac{K\Delta K}{(1 + \frac{1}{2}K^2)} \quad (49)$$

We can use the relation $\Delta K/K = \Delta B/B$ to get

$$\Delta P_{ab} = 2\pi \frac{K^2}{(1 + \frac{1}{2}K^2)} \frac{\Delta B}{B} \quad (50)$$

Note that increasing the field between the poles, makes a positive phase change, and decreasing the field, makes a negative phase change. Using this relation with $\Delta B/B = 0.01$ and $K = 3.630$ and converting to degrees, we get $\Delta P_{ab} = 6.25^\circ$. This is a large phase shift for a small magnet error.

Returning to the results of the analysis program, figure 15 shows the electron trajectory. One can see a small shift in the trajectory, but no large deflection.

Figure 16 shows the slippage error. There is a positive jump, as expected.

Figure 17 shows the resulting phase error. There is a positive jump of a little over 6° , as expected.

Table 5 shows the calculated undulator parameters with the phase error. The values are very close to the values with no errors.

Parameter	Value	Units
K_{eff}	3.63033	
B_{eff}	1.29600	T
λ_r	1.50003×10^{-10}	m

Table 5: Calculated undulator parameters with phase error.

The undulator is working properly and producing the correct radiation wavelength before and after the phase error. After the error, however, the radiation has a shifted phase relative to the existing radiation wave in the undulator. In order to calculate the undulator parameters, a fit is made to the slippage throughout the undulator. Fitting the slippage with a jump at poles 51 and 52 produces a small change in the calculated average λ_r . The residuals of the fit are related to the phase error, explaining the slope in the phase error plot away from the error.

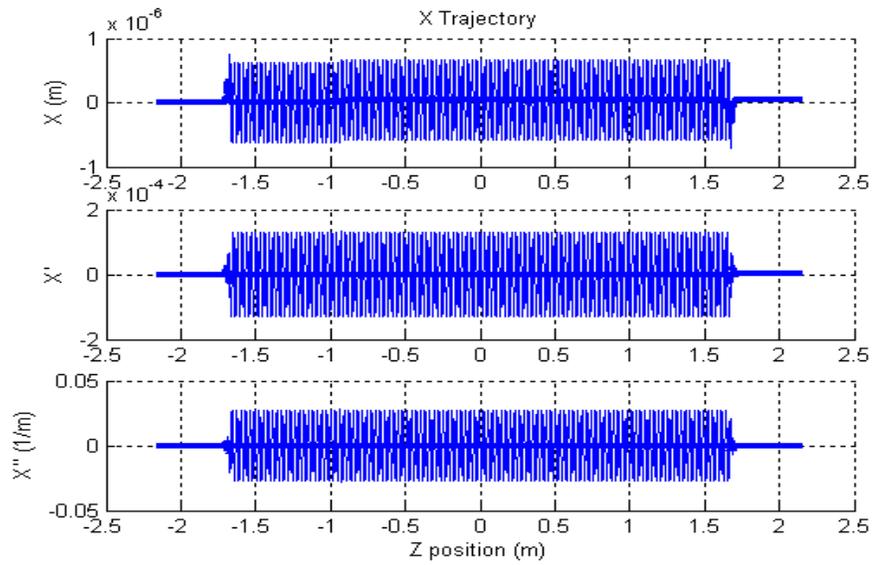


Figure 15: Electron trajectory for a phase error in which poles 51 and 52 are made too strong by 1%.

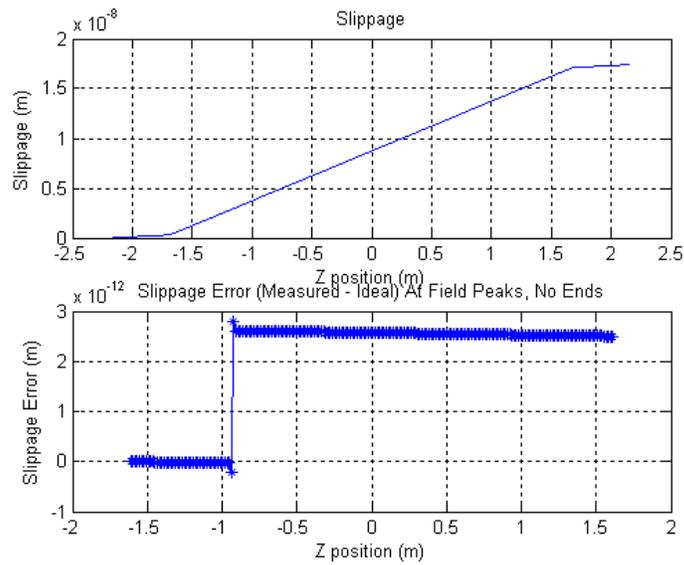


Figure 16: The slippage error has a jump at poles 51 and 52, which are too strong by 1%.

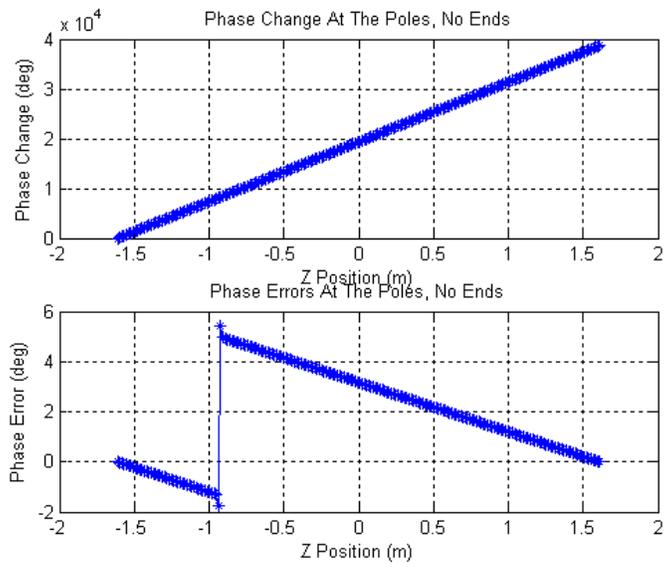


Figure 17: Phase error caused by two neighboring poles too strong by 1%.

3.4 Localized Gap Error

Suppose a region of the undulator is too strong. This could be caused by improperly shimming the gap so the gap is too small over a number of poles. Let us make poles 51 to 70 too strong by 1%. Figure 18 shows the region with strong poles.

We do not expect a large change in the trajectory because the error occurs over an even number of poles so there is no net deflection. We do expect, however, a small shift in the trajectory. The trajectory from the analysis program is shown in figure 19.

The main effect of the gap error is in the slippage. The slippage error accumulates at each pole and increases rapidly in the high field region. This is shown in figure 20. The slippage error can be thought of as the accumulation of slippage from phase errors, which we have studied previously.

The phase error becomes very large as shown in figure 21. The phase error changes by about 60° in this region. This agrees with the previous example which gave 6° phase change per period, times 10 periods. As discussed previously, the gradual slope in the rest of the phase error plot is due to the proportion between phase errors and the residuals of the fit to the slippage.

Table 6 gives the calculated undulator parameters. Notice that in addition to K_{eff} and λ_r having errors, the phase change through the undulator and the phase change through the cell have errors.

Parameter	Value	Units
K_{eff}	3.63385	
B_{eff}	1.29725	T
λ_r	1.50256×10^{-10}	m
$\Delta P_{enter, \text{ mod } 360^\circ}$	87.82	deg
$\Delta P_{und_ctr, \text{ mod } 360^\circ}$	171.81	deg
$\Delta P_{exit, \text{ mod } 360^\circ}$	87.82	deg
$\Delta P_{cell, \text{ mod } 360^\circ}$	347.46	deg

Table 6: Calculated undulator parameters with a localized gap error.

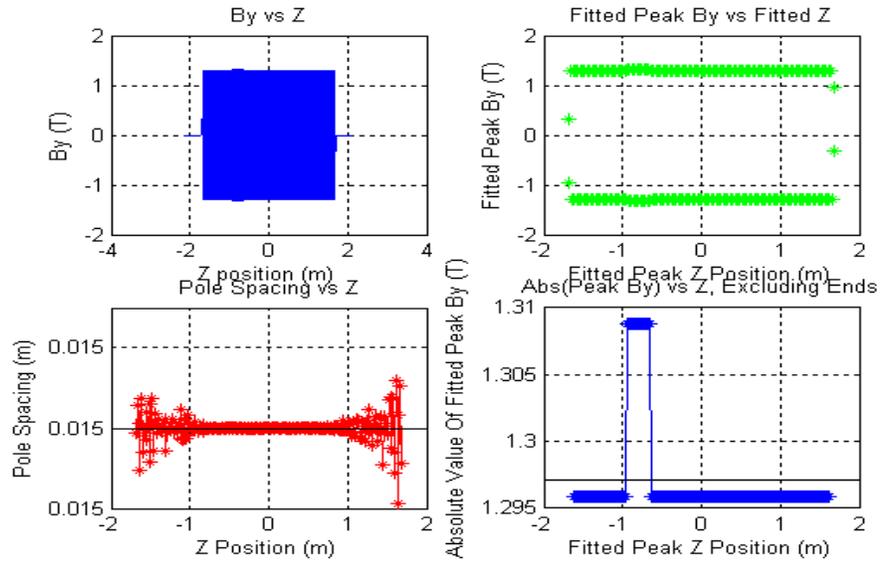


Figure 18: Poles 51 to 70 are too strong by 1%. This is shown in the bottom right figure.

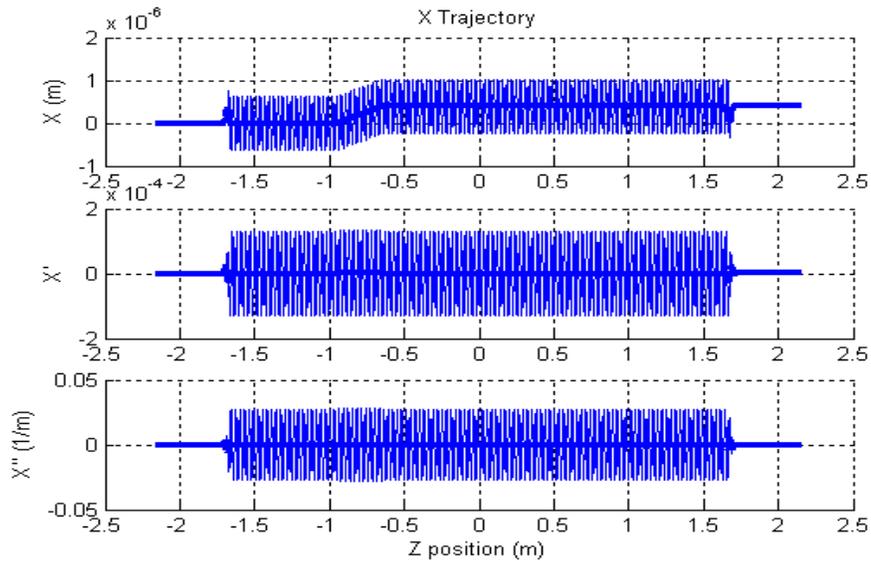


Figure 19: Gap error from poles 51 to 70 causes a small shift in the trajectory.

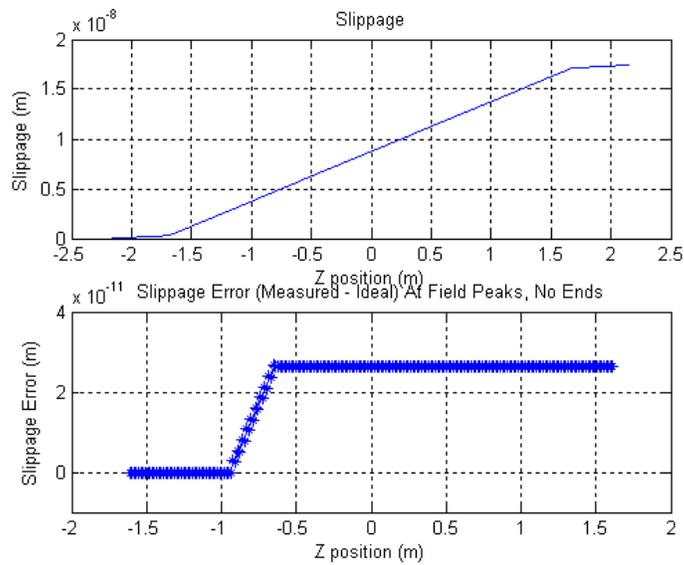


Figure 20: The slippage error increases rapidly in the high field region.

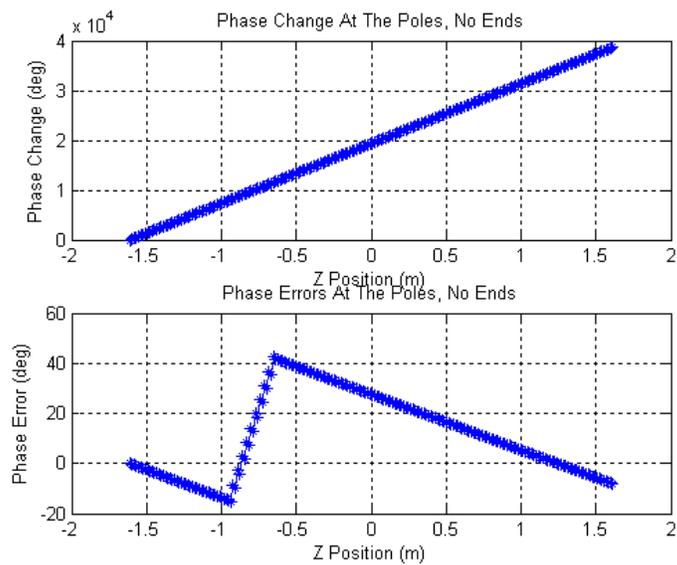


Figure 21: The high field region causes very large phase errors.

3.5 Temperature Change / Overall Gap Error

The strength of the permanent magnets in the undulator changes with temperature. The on-axis undulator field is expected to change by 0.054% per degree Celsius if the undulator is not compensated.⁷ Suppose the temperature increases by one degree Celsius. In our model, we can lower the strength of every pole by 0.054%. The trajectory will have no errors since it is still a perfect undulator, just with a different field value. The main effect of this change is that the undulator will have the wrong K value and it will radiate at the wrong wavelength.

Figure 22 shows the resulting slippage error, the actual slippage minus the desired value. The slippage error is negative because the field is lower than the ideal field, causing less slippage. The negative slippage error accumulates through the undulator.

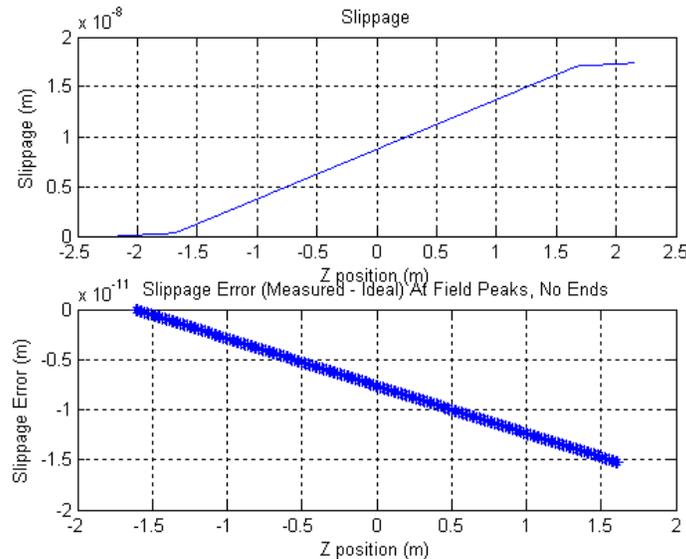


Figure 22: Slippage error corresponding to a 1 degree Celsius temperature increase. The higher temperature makes the magnetic field weaker, which causes less slippage. This makes the slippage error negative.

The calculated undulator parameters are given in table 7. The new $K_{eff} = 3.62802$. The value at the proper field strength is $K_{eff} = 3.63000$. The difference between K_{eff} and the ideal value is a 0.054% reduction, as expected.

The phase changes going into the undulator, through the undulator, out of the undulator, and through the cell all have their ideal values. These phase changes are calculated using the emitted radiation wavelength, not the desired wavelength. The undulator behaves as if it had no errors, it just emits at the wrong wavelength.

⁷LCLS Conceptual Design Report, SLAC-R-593.

Parameter	Value	Units
K_{eff}	3.62802	
B_{eff}	1.29517	T
λ_r	1.49838×10^{-10}	m

Table 7: Calculated parameters for a 1 degree C temperature rise.

Errors in the strength of the field in the entire undulator can also be caused by an overall gap error. If the gap is too small, the field will be too large, and visa versa. The symptom of this will be the measurement of K_{eff} with a value different than the design value. Conversely, if we measure the wrong K_{eff} , we can correct it by shimming the gap.

3.6 Earth's Field

So far, we have been primarily concerned with the electron's x trajectory. The y trajectory is also important in terms of keeping the electrons within the radiation field. The y trajectory also contributes to slippage errors, as briefly noted above. Small errors in the undulator and also the Earth's magnetic field can cause a small B_x which deflects the electron in the y direction. We will consider here the effect of the horizontal component of the Earth's magnetic field.

Suppose we have a uniform field of $B_x = 0.2$ G in the undulator. This is approximately the horizontal component of the Earth's field at SLAC. The undulator poles will provide some shielding of the horizontal component of the Earth's field in the gap, however, we will use 0.2 G as a rough estimate for illustration.

Figure 23 shows the y trajectory of an electron in the undulator when $B_x = 0.2$ G. The electron deflects about 3 microns in the undulator. This is a large fraction of the allowed deviation from the axis.

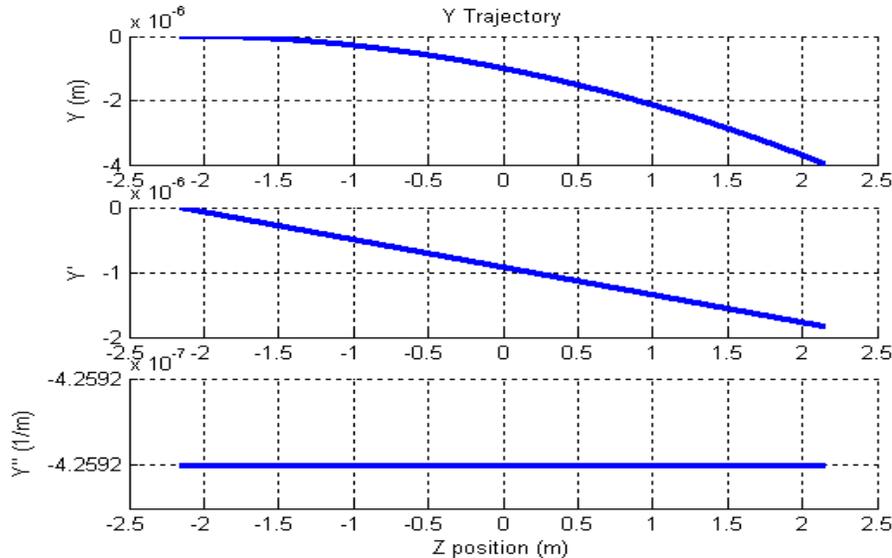


Figure 23: A horizontal field of 0.2 G over the length of the undulator deflects the electron in y by about 3 microns in the undulator.

It is important that we tune the undulators with the same orientation they will have in use. If the undulator is turned around about a vertical axis, the compensating field we put into the undulator changes sign. If the undulators were tuned with B_x from the Earth, we would put in $-B_x$ to compensate it. If we now turn the undulator around for use, the effective field would be $2B_x$.

Although this example was intended to illustrate an error causing the y trajectory to deviate from a straight line, we will briefly mention the effect on the x trajectory.

The Earth's field also has a vertical component. This component is the same no matter how the undulator is oriented, so it can be tuned out without regard to the undulator orientation.

If we did not tune out the Earth's field, the x trajectory would be as shown in figure 24. An Earth's field of $B_y = -0.3$ G was used, which is a typical value at SLAC. The x trajectory has a superimposed curvature, similar to the effect on the y trajectory. The trajectory moves about $5 \mu\text{m}$ away from the axis.

The curvature in the x trajectory adds to x' , giving slippage errors and phase errors. (The effect of y' is not included here.) The phase errors are shown in figure 25. As discussed previously, the error changes from pole to pole depending on whether the average x' is adding or subtracting from the local x' .

The trajectory could be initialized with a corrector which deflects the beam so the deviation from the axis is minimized. One still has the phase errors, however. It is best to compensate for the Earth's field in the tuning process by putting in corrections to keep the trajectory straight.

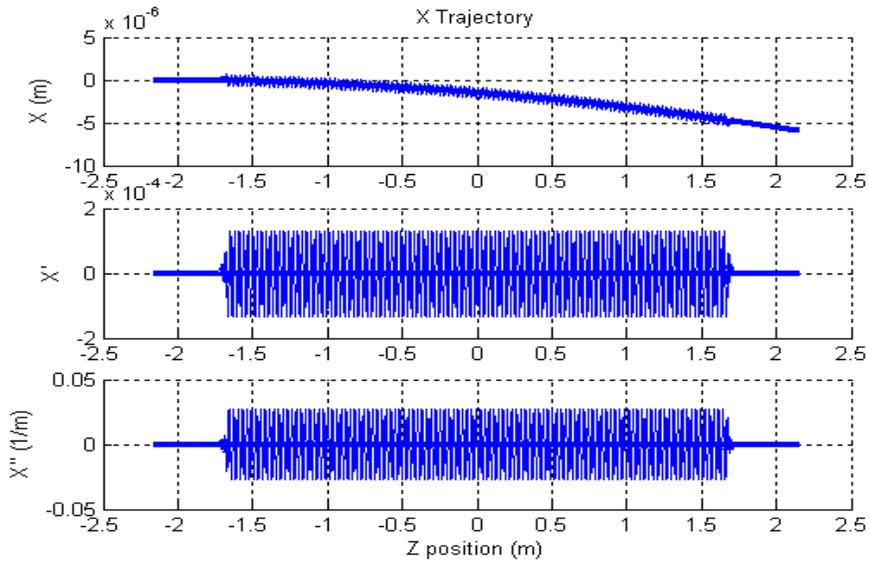


Figure 24: The vertical Earth's field affects the electron x trajectory.

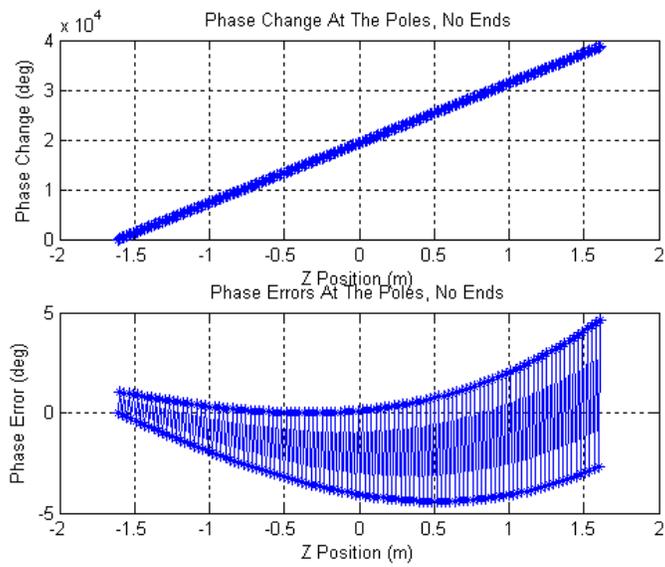


Figure 25: The curvature in the x trajectory gives added x' , which causes phase errors.

4 Shims

We have modeled various errors in the undulators which will certainly occur. We need a method to correct the errors. The devices used to correct the errors are called shims. Several types of shims have been developed for the LCLS. They are shown in figures 26 and 27.⁸ We now discuss each type of shim in turn.



Figure 26: Pole shims and phase shims.

4.1 Pole Shims

Pole shims are cylindrical pieces of steel which are screwed up next to a pole. A pole shim is shown in the upper left in figure 26. The steel is magnetic. Flux leaves the pole and goes into the shim instead of into the gap. This effectively weakens the pole. In figure 9 we considered the case in which pole number 51 was too strong. By placing a pole shim on pole 51, we could weaken it to the proper value so that the trajectory would be straight.

Suppose pole 51 was instead too weak. Adding a pole shim would not work. Instead, we add a pole shim to pole 52 and weaken it by an appropriate amount. This effectively creates a small phase error, which we studied above. The trajectory will be straight, but a small phase error will persist.

Note that in figure 27 the pole shims are placed both on the top and the bottom of the undulator. There is a reason for this. As the flux leaves the pole, it creates a small horizontal field. Suppose there is a south pole on top and a north pole on the bottom giving a $+B_y$ field. The north pole sends flux out to the shim in the

⁸R. Dejus (Editor), "LCLS Prototype Undulator Report", ANL/APS/TB-48, January, 2004.



Figure 27: Shims mounted on an undulator.

$-x$ direction creating $-B_x$ as shown in figure 28. The south pole, however, receives flux from the shim. This makes a field in the $+x$ direction creating $+B_x$. The two horizontal fields cancel. This allows a correction to B_y without affecting B_x .

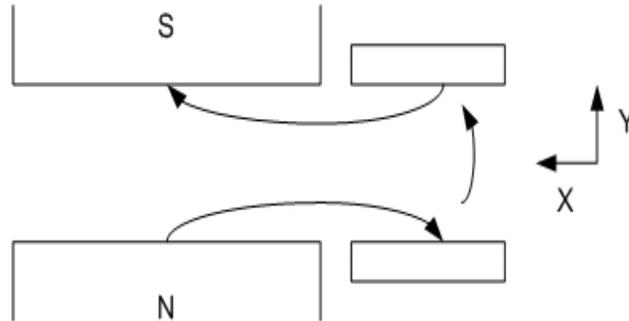


Figure 28: This figure is a schematic illustrating how the horizontal fields cancel by using a shim on the upper and lower pole.

A variation of this effect of creating horizontal fields is used to shim the vertical trajectory. To do this, we want a net horizontal field, but no net vertical field. Suppose we put a shim on a south upper pole as shown in figure 29 lower left. This reduces B_y and makes $+B_x$. On the next pole, we put a shim on the lower south pole as shown in the upper left of the figure. This reduces $-B_y$ and makes $+B_x$. The net effect in y is that B_y has been reduced on two successive poles creating a small

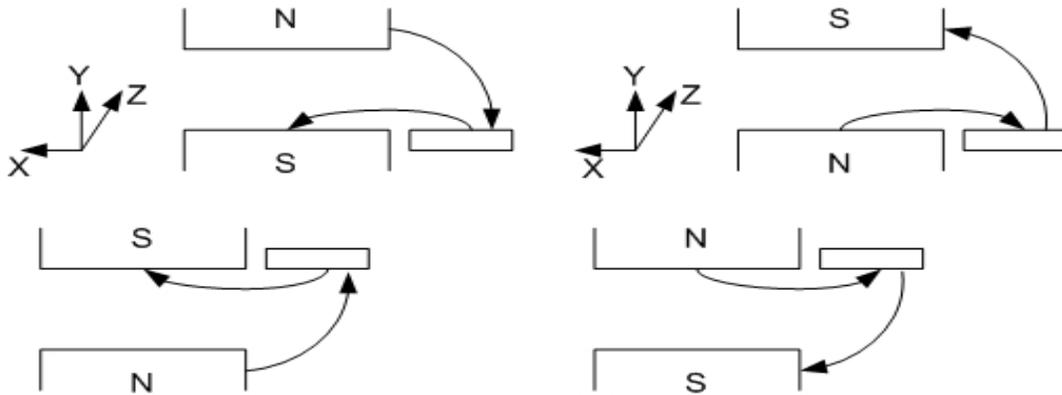


Figure 29: This figure illustrates how shims placed on successive poles can create a horizontal field. The vertical fields have opposite directions and are weakened by the same amount so the horizontal trajectory is minimally affected.

phase shift, but no effect on the x trajectory. In both cases, however, we have $+B_x$. This allows us to give the y trajectory a downward kick.

By placing shims on successive north poles, we get a $-B_x$ field giving the y trajectory an upward kick. In practice, shims of slightly different shape are used to enhance this effect.

4.2 Phase Shims

We have considered the case in which two successive poles are too strong. This created extra slippage, but no trajectory error. We can correct this by weakening both poles by the same amount. We do this with a phase shim. Phase shims are the metal strips shown in figure 26. They are placed on the magnet between two poles. This is illustrated in figure 30. Phase shims shunt some of the flux from the magnet out of the gap, weakening both poles. This allows us to decrease the slippage without affecting the trajectory.

Unfortunately, we have no way to increase slippage. The undulators are not built in such a way that the poles can be moved to increase the field. In theory we could add steel shims to the poles to increase the field. This is very difficult in the small gap. Also the clearance for the beam pipe is very small. Instead we employ a less precise method using base shims.

4.3 Base Shims

Figure 27 shows the poles and magnets held on an aluminum base. The base is then attached to the titanium housing. The base and housing are separated by shims, called base shims. These shims can be used in two ways.

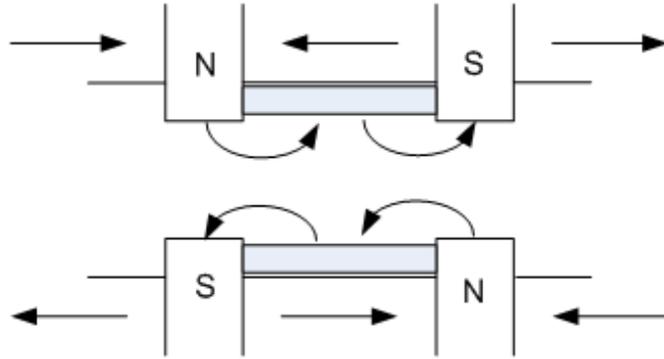


Figure 30: A phase shim is a metal strip placed on the magnet between two poles. It shunts flux out of the gap weakening both poles.

To increase slippage, we can locally reduce the gap in the undulator by adding a base shim between the base plate and the housing. This effectively bends the base, creating a smaller gap over several poles. This is illustrated in an exaggerated way in figure 31. In practice, we only need gap changes of microns. Although we can use

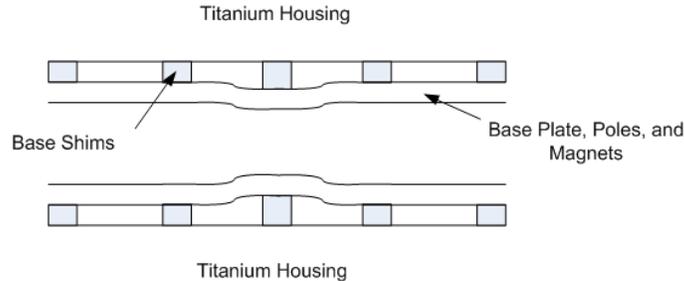


Figure 31: A base shim can be used to change the gap over several poles.

this approach to increase the slippage if needed, it is a fairly blunt instrument.

A more important use of the base shims is to set the K value. We have seen that if the field in the gap is wrong along the length of the undulator, the trajectory is straight, but we get a large slippage error. The slippage error causes the electron oscillation phase to be out of sync with the radiation wave. We can adjust the gap of the undulator using the base shims to get the proper field in the gap and the proper K value. Again, however, with standard shims, this is a fairly coarse adjustment.

Recently, it was proposed to put a small angle into the poles so the field changes with x . This would be implemented with tapered base shims. The tapered base shims allow fine adjustment of the gap. For example, a shim taper angle of $3 \mu\text{rad}$ will change the gap by 3 nm for every 1 mm movement of the shim in x . Fine adjustment of the K value will result. There is another benefit to adding a taper

angle to the poles. Because the field changes with x , the K value changes with x . By measuring the field at different x positions, a plot of K vs x can be made. We can very finely set the correct K value by determining the proper x position for the electron beam.

5 Conclusion

The LCLS will be made up of 33 undulators. Each undulator must keep the electron beam within the radiation field and moving in phase with it. This will require careful tuning of the undulators.

In this note we have presented the equations governing undulator tuning. The equations have been used in a computer program to analyze simulated field measurements. Errors in the field have been simulated and discussed. We discussed the shims used to correct field errors.

The next step is to come up with algorithms which allow the tuning process to be automated for a production environment. Computer programs are being written to calculate the shim sizes and placement. This will greatly increase the speed and accuracy of the LCLS undulator tuning.

Acknowledgements

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