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Quadrupole Magnet Error Sensitivities for FODO-Cell and Triplet Lattices in the LCLS Undulator

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ABSTRACT

The error sensitivities of quadrupole magnets in the LCLS FEL undulator are compared for a FODO-cell lattice and for a triplet lattice. The comparisons are made at a radiation wavelength of 1.5 Å, where electron-to-photon phase errors are very sensitive to small trajectory variations in misaligned quadrupoles between the undulator sections. The results show that the triplet lattice is extremely sensitive, with triplet pitch and yaw alignment tolerances of ~100 μrad. The FODO-cell lattice, with its shorter, weaker quadrupoles is much more error tolerant with pitch and yaw tolerances of ~2.5 mrad. Several other magnet errors are examined and categorized as trajectory, phase slip, and beam size effects. In nearly all cases, the FODO-cell lattice is much less sensitive with technologically achievable tolerance levels, while the triplet lattice tolerances are, in many cases, near achievable limits and may not be sustainable over the long term. Table 2 presents a brief tolerance comparison for the two lattice types.

1 Introduction

A FODO-cell focusing lattice for the 112-meter long LCLS undulator has been designed [1] which uses twenty-six 4.32-meter long FODO cells with a mean beta function of 18 meters in both planes. An alternate design has also been proposed which uses twenty-one 5.3-m triplet cells and includes several significant advantages, including more available space between the undulator sections for diagnostics. The mean beta functions for the two schemes are the same. The Twiss parameters for the FODO-cell and triplet lattices are shown in Fig. 1.

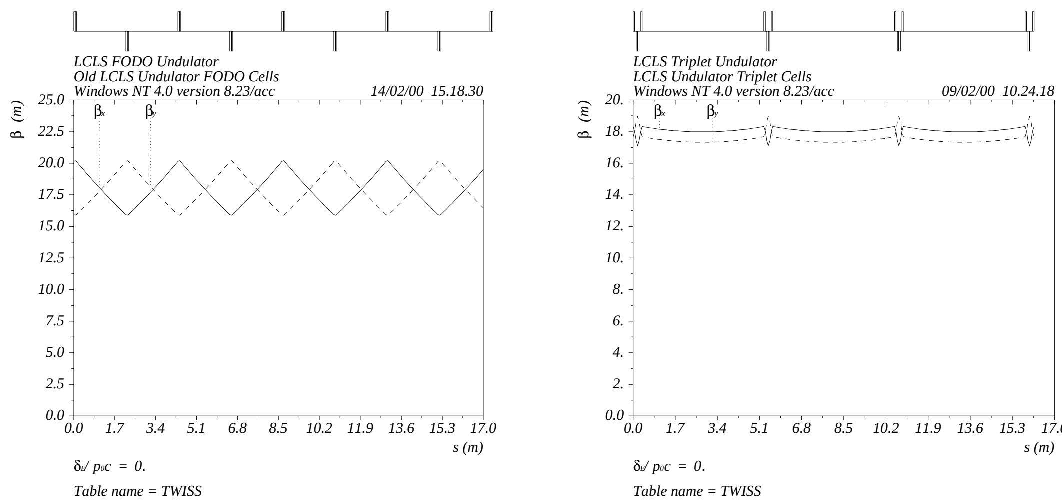

Figure 1. Beta functions for the FODO lattice (left) and for the triplet lattice (right) with equal average beta functions of ~18 m in both planes. Slight focusing effects of the undulator section are ignored here.

The full triplets are, as integrated focusing elements, weaker than the FODO-cell quadrupoles, but have internal gradients that are much stronger than the FODO-cell magnets. A fair comparison of the two systems will include the effects of the much stronger internal gradients of the triplet. Very small trajectory variations within misaligned focusing elements can easily generate path length increases for the electron beam, with respect to the photon beam, which may cause a significant degradation of the FEL gain. In addition, long term drift of the magnet alignment with temperature or ground motion can change the trajectory.

The first section of this note discusses trajectory sensitivity to quadrupole misalignments, while the second part involves the much more critical and challenging aspect of electron-to-photon phase errors generated by misalignments. Finally, magnet errors and beam size effects are discussed. The lattice parameters for the two systems are given in Table 1.

Table 1. Lattice parameters for the FODO and the triplet-based undulator, with equal mean beta functions.

parameter	symbol	**FODO**	**Triplet**	unit
Electron energy	E_0	15	15	GeV
Middle quadrupole length (only quad in FODO)	l	12	12	cm
Outer quadrupole lengths (triplet only)	$l_{1,3}$	—	6	cm
Quadrupole focusing constant	$\|k\|$	0.95	4.80	m^{-2}
Quadrupole pole-tip field (all trip. quads same)	$\|B_0\|$	0.237	1.20	T
Quadrupole gradient (all trip. quads same)	$\|G\|$	47.4	240	T/m
Integrated gradient of middle quad	$\|G\|l$	5.69	28.8	T
Integrated gradient of outer quads (triplet only)	$\|G\|l_{1,3}$	—	14.4	T
Quadrupole pole-tip radius (all quads same)	r	5	5	mm
Drift between middle and outer quads (triplet)	ΔL	—	6	cm
Betatron phase advance per cell (equal in x and y)	μ	13.8	17.1	deg
Distance between focusing stations (center-center)	L	2.160	5.272	m
Number of undulator sections	N_s	52	21	—

2 Misalignments and Trajectory Errors

If the triplet is considered as a rigid element, internally aligned to perfection, which focuses in both planes simultaneously, the *effective* integrated gradient for the entire triplet is $Gl_e = 0.841$ T. This 'effective triplet' generates the same Twiss function plots as at the right of Fig. 1, except for the detailed variations inside the triplet. The effective gradient is 6.8 (≈ 5.69/0.841) times weaker than the FODO-cell quadrupole and therefore the *rigid-triplet* is 6.8-times less sensitive to transverse misalignments (*e.g.* 5 μm for the FODO-quads and 30 μm for the triplet [2]).

The individual quadrupoles of the triplet, however, have very high gradients and are 5.1 (≈ 28.8/5.69) times more sensitive to internal misalignments than the FODO-cell (*e.g.* 5 μm for the FODO-quads and 1 μm for the middle triplet quad). Therefore, the middle triplet quad is 35 (≈ 28.8/0.841) times more sensitive to misalignments than the rigid-triplet assembly. If the triplet-assembly is mounted on a magnet mover and used to steer, then a 10 μm internal misalignment will require a −350 μm triplet offset to remove the net kick angle. Furthermore, a drift in the alignment of the middle triplet quad of 2.9 μm will initiate a betatron oscillation of amplitude equal to one-rms beam size (*i.e.* 30-μm peak amplitude). For the FODO-cell quad, a drift of 15 μm is required to generate the same oscillation.

These numbers reflect the magnet sensitivities. The *absolute* alignment tolerances are more clearly quantified by a consideration of the electron-to-photon phase slip, with respect to the 1.5-Å radiation wavelength, induced by trajectory errors inside the misaligned quadrupoles. This is the subject of the next section and the more decisive one in terms of lattice comparisons.

3 Phase Errors

3.1 Phase Errors due to Emittance

The path length of an electron over the undulator is, on average, longer than that of a photon, due to the finite electron beam emittance. The path length difference of one electron over a half-FODO-cell can be calculated using a small angle approximation and integrating the square of the electron beam angle, x', over the distance between thin-lens quadrupoles, L.

$$\Delta s_L = \int_0^L \sqrt{1+x'^2}\,ds - L \approx \frac{1}{2}\int_0^L x'^2 ds \qquad (1)$$

For the drift between quadrupoles, x' is constant. Then, adding the additional delay due to the vertical angle, y', and the delay of the next half-cell, and finally scaling to the full-length undulator, L_u, gives

$$\Delta s = \frac{L_u}{2}\left(x_F'^{\,2} + y_F'^{\,2} + x_D'^{\,2} + y_D'^{\,2}\right), \qquad (2)$$

where x_F' and y_F' are the beam angles at the exit of the thin-lens *focusing* quad and x_D' and y_D' are the angles at the exit of the thin-lens *defocusing* quad. The mean 'phase' error is calculated by taking the ensemble average over the electron beam where the square of the rms divergences are given by

$$\left\langle x_F'^2 \right\rangle = \varepsilon_x\left(\frac{1}{\beta_F} + \frac{k^2 l^2 \beta_F}{4}\right), \qquad \left\langle x_D'^2 \right\rangle = \varepsilon_x\left(\frac{1}{\beta_D} + \frac{k^2 l^2 \beta_D}{4}\right), \qquad (3)$$

with similar expression for y'_F and y'_D. The second term in each relation above is due to the non-zero α $(= -\beta'/2)$ at the end of a thin lens quad of strength k and length l. In addition, β_F and β_D are the maximum and minimum beta functions of the FODO-cell and ε_x is the horizontal rms emittance. The mean phase error is then`

$$\left\langle \Delta s \right\rangle = \frac{L_u}{4}\left(\varepsilon_x + \varepsilon_y\right)\left(\frac{1}{\beta_F} + \frac{1}{\beta_D} + \frac{k^2 l^2}{4}\left\{\beta_F + \beta_D\right\}\right) \approx \frac{L_u\left(\varepsilon_x + \varepsilon_y\right)}{\bar{\beta}\cos^2\left(\mu/2\right)}\ , \qquad (4)$$

where μ is the phase advance per cell (equal in x and y), and the approximation at far right assumes $\bar{\beta} \approx (\beta_F + \beta_D)/2$.

For the FODO-cells in reference [1], $\beta_F \approx 20.2$ m, $\beta_D \approx 15.9$ m, $L_u \approx 112$ m, and $\varepsilon \approx 0.51$ Å, which amounts to a net path length difference between electrons and photons of 6.4 Å over the full undulator length (or 4.3 radiation wavelengths). The mean 'phase' error can also be written more conveniently in terms of the number of FODO cells, N_c.

$$\langle \Delta s \rangle = 4N_c \varepsilon \tan(\mu/2) \tag{5}$$

Reducing the number of cells (*i.e.* increasing the cell length) and decreasing the phase advance per cell both reduce the mean phase error.

The rms 'phase' spread can be calculated in a similar way resulting in

$$\sigma_{\Delta s} = \frac{L_u}{\sqrt{8}}\sqrt{\varepsilon_x^2 + \varepsilon_y^2}\sqrt{\frac{1}{\beta_F^2} + \frac{1}{\beta_D^2} + k^2 l^2\left\{1 + \frac{k^2 l^2}{16}\left(\beta_F^2 + \beta_D^2\right)\right\}} \approx \frac{L_u\sqrt{\varepsilon_x^2 + \varepsilon_y^2}}{\bar{\beta}\cos^2(\mu/2)}\ , \tag{6}$$

where $\sigma_{\Delta s}$ is the standard deviation of the path length difference (*i.e.* with respect to the mean) and gaussian transverse distributions have been assumed, such that fourth moments are related to second moments by: $\langle x^4 \rangle = 3\langle x^2 \rangle^2$. For round, gaussian beams with a FODO-cell lattice, the rms phase spread is related to the mean by $\sigma_{\Delta s} = \langle \Delta s \rangle/\sqrt{2}$. In addition, the rms phase spread is a minimum for equal emittances, assuming the sum of the horizontal and vertical emittances are constant. The FODO-cell parameters result in $\sigma_{\Delta s} \approx 4.6$ Å. More detailed tracking calculations of the phase errors have been done using the computer code *Elegant* [3], written by Michael Borland at ANL.

Figure 2 shows the 'phase' errors of 10000 electrons over the full 112-m undulator for perfectly aligned FODO-cell quads with horizontal and vertical normalized emittances of 1.5 μm at 15 GeV (gaussian transverse distributions). The mean path length error is 6.45 Å and the rms error is 4.56 Å, in agreement with Eqs. (4), (5) and (6) above, where phase errors scale with emittance, undulator length and the inverse of the mean beta function.

Figure 3 shows the same plots for perfectly aligned *triplets*. The mean and rms phase errors, due to the finite emittances, are almost identical for the two lattices with equal mean beta functions and aligned quadrupoles. The important differences between lattices arise only when quadrupole magnet misalignments are examined.

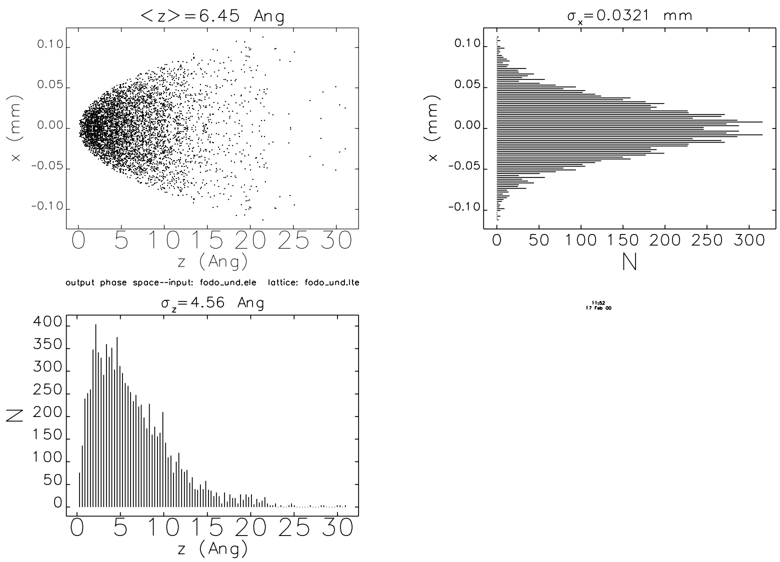

Figure 2. Path length, $\Delta z = \lambda_r\varphi/2\pi$, errors over the full 112-m undulator with perfectly aligned FODO-cell quads. Shown are horizontal position, x, vs. Δz (top-left), x distribution (top-right), and Δz distribution (lower-left). The mean 'phase' error over the full undulator is 6.45 Å and the rms 'phase' error is 4.56 Å.

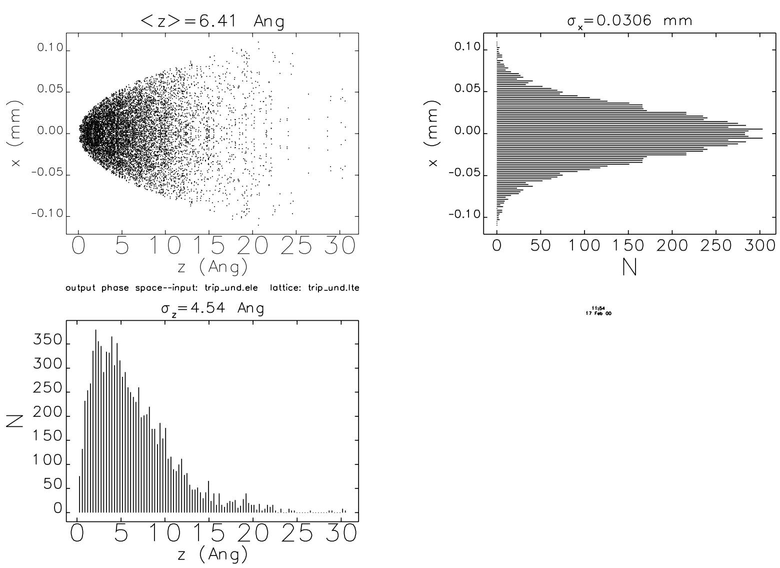

Figure 3. Same plots as Fig. 2, but for perfectly aligned *triplets*. The mean 'phase' error over the full undulator is 6.41 Å and the rms 'phase' error is 4.54 Å.

It is also interesting to calculate the mean phase error for a constant focusing undulator (*i.e.* with constant beta function). In this case, the beam angle, as a function of s, can be expressed in terms of the initial position, x_0, and angle, x'_0, at undulator entrance as

$$x(s) = -\frac{x_0}{\beta}\sin(s/\beta) + x_0'\cos(s/\beta)\,. \tag{7}$$

Again, as in Eq. (1), integrating the square of the beam angle over the full undulator length gives the path length change. Then adding the path length due to the similar vertical oscillations (equal emittances) and taking the ensemble average gives

$$\langle \Delta s \rangle = \frac{\varepsilon L_u}{\bar{\beta}}\ , \tag{8}$$

which, when compared to Eq. (4), shows the mean phase error with constant focusing is a factor of two smaller than with FODO-cell focusing.

The mean 'phase' error, for the FODO-cell lattice, might be compensated by a change in the undulator section-break length of

$$\Delta L = -\frac{2\varepsilon}{\bar{\beta}}\frac{\lambda_u}{\lambda_r}L\left(1+K^2/2\right), \tag{9}$$

where λ_u (= 3 cm) is the undulator period, K (= 3.71) is the undulator parameter, and L is the FODO half-cell length ($L \approx 2.16$ m) or the triplet cell length ($L \approx 5.27$ m). For the FODO-cells, with $\varepsilon \approx 0.51$ Å, then $\Delta L \approx -1.9$ cm. For the triplets, with the same mean phase error (see Fig. 3), then $\Delta L \approx -4.7$ cm. The minus sign indicates a reduction of the section break length. This removes the mean phase error, which may not be the optimal compensation depending on the transverse distributions (*e.g.* uniform or gaussian).

3.2 Magnet Misalignments

Trajectory errors between undulator sections can significantly increase the electron's path length between the sections, generating phase errors between electrons and photons. With the extremely short radiation wavelength of $\lambda_r = 1.5$ Å, even a micron-amplitude trajectory variation over the length of the quadrupole(s) can be significant. For strong gradient magnets, this level of trajectory variation can be introduced with very small misalignments. The simple picture in Fig. 4 illustrates the effect.

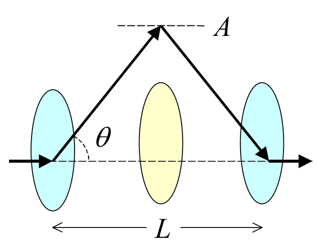

Figure 4. Simplified misalignment-induced trajectory 'bump' of amplitude A and length L in a triplet.

The mean phase error between the *bent* electrons and straight-ahead photons, for this simplified picture, is

$$\Delta\varphi = \frac{2\pi}{\lambda_r}\left(\frac{2A^2}{L}\right) = \frac{4\pi A^2}{L\lambda_r} \ , \tag{10}$$

where A is the bump amplitude, L is the bump length, and λ_r is the radiation wavelength. A bump amplitude of $A = 1$ μm over a length $L = 30$ cm produces a 16° mean phase error over the triplet for $\lambda_r = 1.5$ Å. The phase error is 48° for radiation at the 3rd harmonic. This 1-μm bump amplitude can be generated by a ~20 μm misalignment of the first quadrupole in the triplet (see discussion of Fig .5). The above is a rough description in order to describe the order of magnitude of the effect.

The mean and rms spread are each potential gain reducing effects, and each reacts differently to misalignments. The mean phase error is always a 'lag', independent of the sign of the misalignment. The phase spread, however, can be amplified by magnet misalignments. The effect is dependent on the pattern of misalignments along the undulator. A constant misalignment pattern, when integrated over the 2π of full undulator betatron phase advance, has no effect on the nominal phase spread. If, for example, the quadrupoles are misaligned in the *positive* direction during the first half of the undulator (first π of betatron phase), and misaligned in the *negative* direction during the second half of the undulator (last π of betatron phase), the nominal phase spread of 4.5 Å, shown in Figs. 2 and 3, is amplified. The amount of amplification depends on the scale of misalignment. This effect will be mostly ignored in what follows, but is another possible limitation. The effects of different types of misalignments are now examined. The resulting tolerances are listed at the end of this section in Table 2.

3.3 Magnet Offset

A phase error can be generated by a misaligned quadrupole magnet, especially for a uniformly misaligned rigid-triplet. Figure 5 shows the trajectory through a triplet with a 50 μm rigid-triplet offset simultaneously in x and y (*i.e.* all three triplet quadrupoles are misaligned transversely by the same amount).

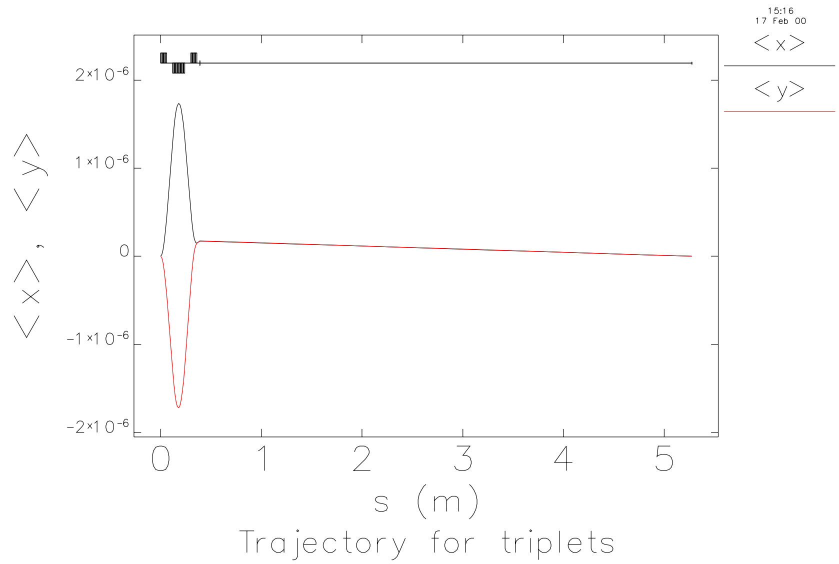

Figure 5. Electron's horizontal (solid) and vertical (dash) trajectory through a 50-μm offset triplet assembly (x and y) with steering coil 3-cm downstream of triplet to steer the trajectory to zero at entrance to the next triplet. The phase error generated between electrons and photons over this length is 99° or 19°/m.

A dipole steering coil (x and y) is placed 3 cm downstream of the triplet, and the trajectory at the entrance to the next triplet is steered to zero. The mean phase error generated over this length is 99° or 19°/m. No significant change occurs if the steering coil is placed at the center of the triplet. The same calculation for the FODO-cell quadrupole results in a 6° phase error over a 2.16-m length, or 2.8°/m, for the 50 μm misalignment.

Since there are fewer triplets than FODO-cell quadrupoles, it is best to compare the phase errors accumulated over the whole undulator for both lattices (or the phase error per meter). Figure 6 shows the 'phase' errors of 10000 electrons over the full 112-m undulator with all triplets rigidly misaligned by 50 μm horizontally and vertically (still with normalized emittances of 1.5 μm). The mean 'phase' error over the full undulator is now 15.1 Å and the rms 'phase' error is still 4.54 Å. This is an additional alignment-induced mean phase error (see Fig. 3) of 8.7 Å, which should have a significant negative impact on the FEL gain. Since the alignment-induced mean phase error is always a lag, independent of the sign of misalignment, the uniform 50-μm offset applied to all triplets here is a valid sensitivity test. The phase spread, however, is not affected in this uniform misalignment scheme, but may be amplified by another more realistic pattern.

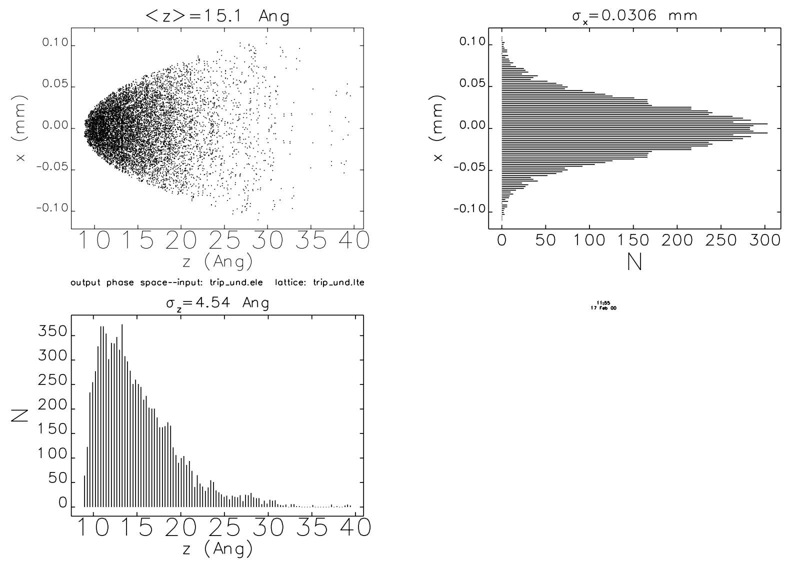

Figure 6. Path length errors over the full 112-m undulator with all triplets equally misaligned by 50μm. The mean ‘phase’ error over the full undulator is now 15.1 Å, of which 8.7 Å is due to misalignments. The rms ‘phase’ error is unchanged, in this uniform misalignment pattern, at 4.54 Å.

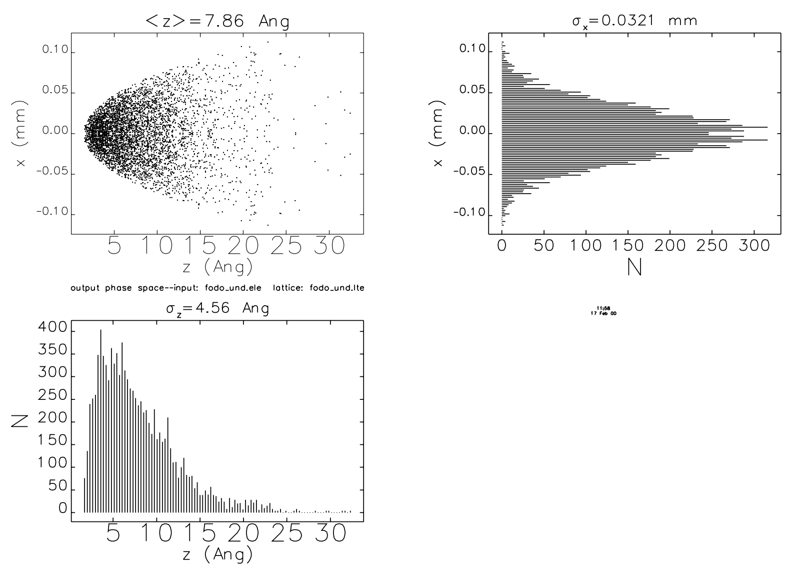

Figure 7. Path length errors over the undulator for all FODO-cell quads equally misaligned by 50 μm. The mean ‘phase’ error over the undulator is now 7.86 Å, of which 1.4 Å is due to misalignments. The rms ‘phase’ error is unchanged at 4.56 Å.

In Fig. 7, the same test is applied to the FODO-cell lattice. The mean ‘phase’ error over the full undulator is now 7.86 Å. This is an additional mean phase error (see Fig. 2) of

1.4 Å (or 3°/m), which is six times less sensitive than the triplets. The rms 'phase' error is unchanged at 4.56 Å. In all cases (triplet and FODO), the same steering is used.

The quadratic scaling of phase error with misalignment means that a 68-μm triplet offset will slip the phase by π per triplet cell, which will critically reduce the FEL gain. Therefore, if the BPM readings include offsets at the level of ~60 μm, steering the undulator trajectory may destroy the FEL gain. Furthermore, this effect is not fully correctable with beam-based alignment [4], which until now, has only been evaluated based on the straightness of the resulting trajectory. The alignment simulation results show that, due to resolution limitations, the final rigid-triplet misalignments are limited to a level of ~20 μm in the vertical plane [2]. This 20-μm residual will be larger in the horizontal plane (due to dipole errors), and may be further increased in the vertical plane by dipole roll errors. A 20-μm misalignment will generate an average phase shift of 16° per triplet cell, or 3°/m, which may increase the saturation length. Beam-based alignment simulations for the FODO-cells indicate the magnets can be aligned to ~15 μm, a mean phase shift of only ~0.5°/m.

It is also useful to study the phase errors over a single misaligned triplet cell. In the above tests, the 2π betatron phase across the full undulator, in conjunction with the uniformly misaligned magnets, masks the phase spread amplification effect. In Fig. 8, only one 50-μm misaligned triplet cell is studied. In this case the rms 'phase' error is *amplified* here (when scaled to 21 triplet cells) to nearly twice the phase spread of Figs. 3 and 6. The asymmetry in the *x*-*z* scatter plot helps to explain the phase spread amplification effect. Particles with initial offsets of the level of +50-μm are not lagged by the offset triplet (with Δx = 50 μm), but the more densely populated beam core, and the minus side of the *x*-distribution, is lagged significantly according to $(x - \Delta x)^2$.

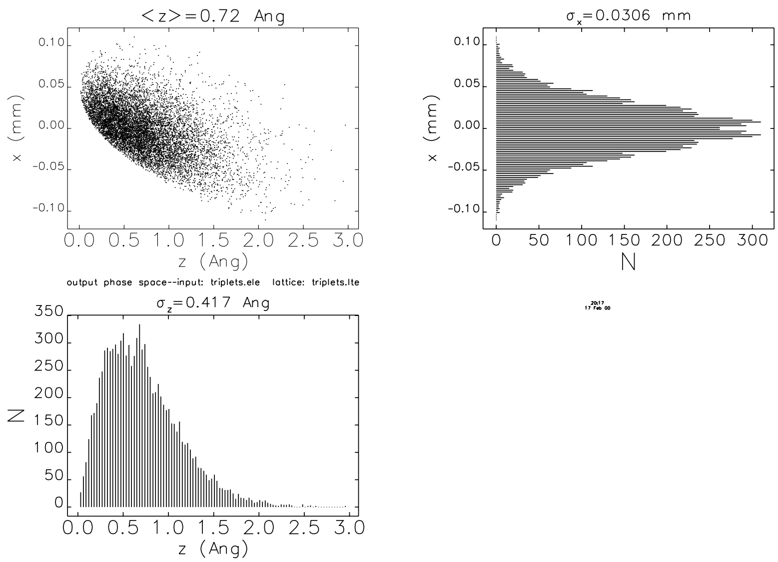

Figure. 8. Path length errors over one 50-μm misaligned triplet cell. The mean 'phase' error over the cell is 0.72 Å (≈ 15.1 Å/21 cells, see Fig. 6), of which 0.41 Å is due to the misalignment. The rms 'phase' error is *amplified* here to 0.417 Å, which amounts to 8.8 Å over 21 triplet cells, or nearly twice the phase spread in Figs. 3 and 6.

3.4 Magnet Pitch and Yaw

Figure 9 shows the electron trajectory through a triplet where an internally aligned triplet has been pitched (vertically) and simultaneously yawed (horizontally) about its center by just 160 μrad. (The quadrupole magnets were sliced into 1.2-cm long sections to simulate more accurately the pitch and yaw.) Again, the same steering coil is used and no significant change occurs if the steering coil is placed at the center of the triplet. The resulting mean phase error over the 5.3-m triplet cell is 34°, of which 32° occurs along the 30-cm long triplet trajectory and just 2° is generated by the beam angle in the following 5-m undulator section. This is a mean phase slip rate of 6.4°/m. In this case, the phase error spread is not magnified by the pitch and yaw. As reflected in Eq. (10), the phase error is sensitive to the square of the trajectory amplitude. Therefore, a triplet pitch and yaw of 370 μrad will generate a mean phase error of π, which may cancel the FEL gain. The same calculations for the FODO-cell quadrupole show that the 6.4°/m error is generated with a 3-mrad pitch and yaw, or a sensitivity that is nearly 20 times more forgiving than the triplets.

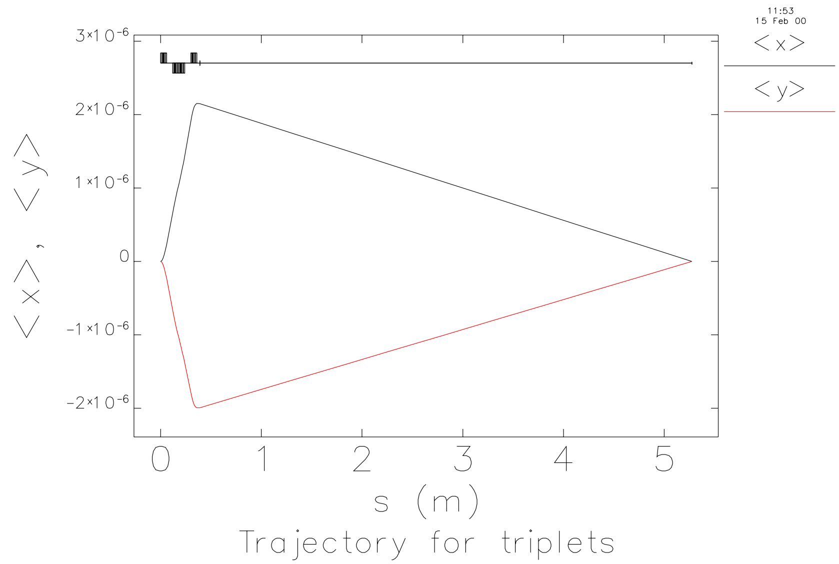

Figure 9. Electron's horizontal (solid) and vertical (dash) trajectory through a 160-μrad pitched and yawed triplet assembly with steering coil 3-cm downstream of triplet to steer trajectory to zero at entrance to next triplet. The phase error generated here between electrons and photons is 34° over this 5.3-m distance.

The pitch and yaw tolerance is very critical since it is not recoverable by any BPM-driven correction technique. The tiny trajectory variations of the 1-μm scale which occur inside the triplet are very significant yet not measurable. This point appears to be the most decisive in the lattice comparison.

3.5 Internal Triplet Misalignments

For the triplet, internal misalignments of the individual quadrupoles can also generate significant trajectory variations, even if the electron beam is aligned with the average position of the triplet. A 20-μm misalignment of the middle (12-cm long) quadrupole magnet, with respect to the two outer magnets, generates a mean phase error rate of 10.5°/m. Again, beam-based-alignment cannot correct these internal misalignments. Here the same steering coil is used, but the 20-μm sensitivity loosens to 50-μm if the steering coil is somehow placed at the center of the triplet. Of course, the FODO-cell offset sensitivities are also reduced if the steering is precisely at the center of the quadrupoles.

As discussed in section 2, an internal misalignment of the center quad of the triplet steers the beam 35 times more than the same misalignment of the whole triplet. If the triplet assembly is mounted on a magnet mover for steering purposes, then a 1-μm misalignment of the center quad requires a 35 μm move of the whole triplet to correct the trajectory. This results in a 1.2-μm bump inside the triplet and slips the mean phase by an additional

9.2˚/m. Therefore, if the triplets are internally misaligned by 1-μm and beam-based alignment is used, the triplets will remain misaligned by at least 35 μm (not including the 20-μm resolution limit) and a phase slip of more than 9˚/m is to be expected, which is beyond a tolerable level. If beam-based alignment is not used, then the triplets need to be aligned to ~20 μm over the entire undulator using the initial alignment survey, which does not seem reasonable.

Table 2 lists the triplet and FODO-cell magnet sensitivities to various misalignments. In each case listed, the misalignment produces a mean phase error rate of 4°/m as an average over the 112-m undulator (in addition to the emittance-induced phase error). This 4°/m phase slip rate is defined here as the tolerable level. The triplet 'tolerances' are extremely tight with questionable stability, while those of the FODO-cell quad are achievable.

Table 2. Triplet and FODO-cell magnet alignment 'tolerances' for a 4°/m mean phase slip rate. In each case, a steering coil 3 cm downstream of the final magnet is used to correct the trajectory at the next break.

misalignment mode	**FODO**	**Triplet**	units
Horizontal and vertical offset of magnet(s)	58	23	μm
Pitch and yaw of entire focusing package	2400	126	μrad
Offset of middle quadrupole only (triplet only)	—	12	μm
Offset of the 1st quadrupole (triplet only)	—	16	μm
Offset of the 3rd quadrupole (triplet only)	—	45	μm
Offset of middle quadrupole (triplet-mover used to steer)	—	0.7	μm

4 Beam Size Issues

The beam's beta functions and emittances are also sensitive to quadrupole magnet errors such as gradient and roll. A gradient error generates a beta function mismatch with respect to the periodic solution. This mismatch is best quantified in terms of the Courant-Snyder invariant

$$\zeta = \frac{1}{2}\left(\beta\gamma_0 - 2\alpha\alpha_0 + \gamma\beta_0\right) \geq 1 \ , \tag{11}$$

where β, α and γ are the Twiss parameters of the perturbed electron beam and β_0, α_0 and γ_0 are the periodic Twiss parameters of the matched lattice. For demonstration purposes, when $\alpha_0 = \alpha = 0$, a value of $\zeta = 1.02$ is generated by $\beta \approx 22$ meters at a point in the lattice where $\beta_0 = 18$ meters. For a gradient error in a single thin lens quadrupole, the mismatch parameter is

$$\zeta \approx 1+\frac{1}{2}\Delta k^2 l^2 \beta_0^2 \ , \tag{12}$$

where Δk is the gradient error and l is the magnet length. The relative gradient tolerance is then proportional to the inverse of the nominal gradient, k_0.

$$\left|\frac{\Delta k}{k_0}\right| < \frac{\sqrt{2(\zeta-1)}}{|k_0|\, l \beta_0} \tag{13}$$

Since the mean beta functions are the same for the two lattices (18 m), the tolerance depends only on the value of k_0. With $|k_0| \approx 4.8\ \mathrm{m}^{-2}$ for the middle quadrupole of the triplet, and $|k_0| \approx 0.95\ \mathrm{m}^{-2}$ for the FODO-cell quadrupoles. The relative gradient error tolerances are 5.1 times tighter for the middle triplet quadrupole than for the FODO quad. Including a factor of $\sqrt{N}$ for the different number of quads in each lattice, (for simplicity treating the outer triplet quads as one 12-cm quad), and allowing $\zeta < 1.02$, the triplet quad 'tolerances' are approximately $|\Delta k/k_0| < 0.3$ % ($N = 42$), while the FODO quad tolerances are approximately $|\Delta k/k_0| < 1.4$ % ($N = 52$). These are not necessarily the absolute tolerances. Rather they are sensitivity numbers for comparison purposes.

Quadrupole magnet roll errors (rotation about the longitudinal quad axis) can couple and increase the projected emittances. The roll tolerance for a thin lens quad can be written as

$$|\phi| < \frac{\sqrt{\Delta\varepsilon/\varepsilon_0}}{|k_0|\, l\sqrt{2\beta_x\beta_y}} \ , \tag{14}$$

where $\Delta\varepsilon/\varepsilon_0$ is the relative emittance growth (equal in x and y for a 'round' beam). Including the factor of $\sqrt{N}$, and allowing <5 % emittance increase in each plane, the 'tolerance' for the roll of the individual quadrupoles within the triplets is $|\phi| < 2.3$ mrad. For the FODO quads the 'tolerance' is $|\phi| < 10.7$ mrad.

Table 3. Triplet and FODO-cell magnet alignment tolerances for a 2% beta mismatch and a 5% emittance increase in each plane.

quadrupole error	**FODO**	**Triplet**	units
Relative gradient errors	1.4	0.3	%
Quadrupole roll errors	10.7	2.3	mrad

Field quality tolerances, such as sextupole and dodecapole content, are very loose for both lattices due to the extremely small beam size in a relatively large bore quadrupole.

5 Summary

The triplet lattice for the LCLS undulator is a clear prescription for failure.

6 References

[1] *LCLS Design Study Report*, SLAC-R-521, (1998).

[2] See LCLS Technical Advisory Committee meeting notes from February 11, 2000.

[3] Michael Borland, *Elegant*, ?.

[4] P. Emma, R. Carr, H.-D. Nuhn, *Beam Based Alignment For The LCLS FEL Undulator*, Proceedings of the 1998 Free Electron Laser Conference, Newport News, Virginia, August 1998.