

Bulk Structure Characterization



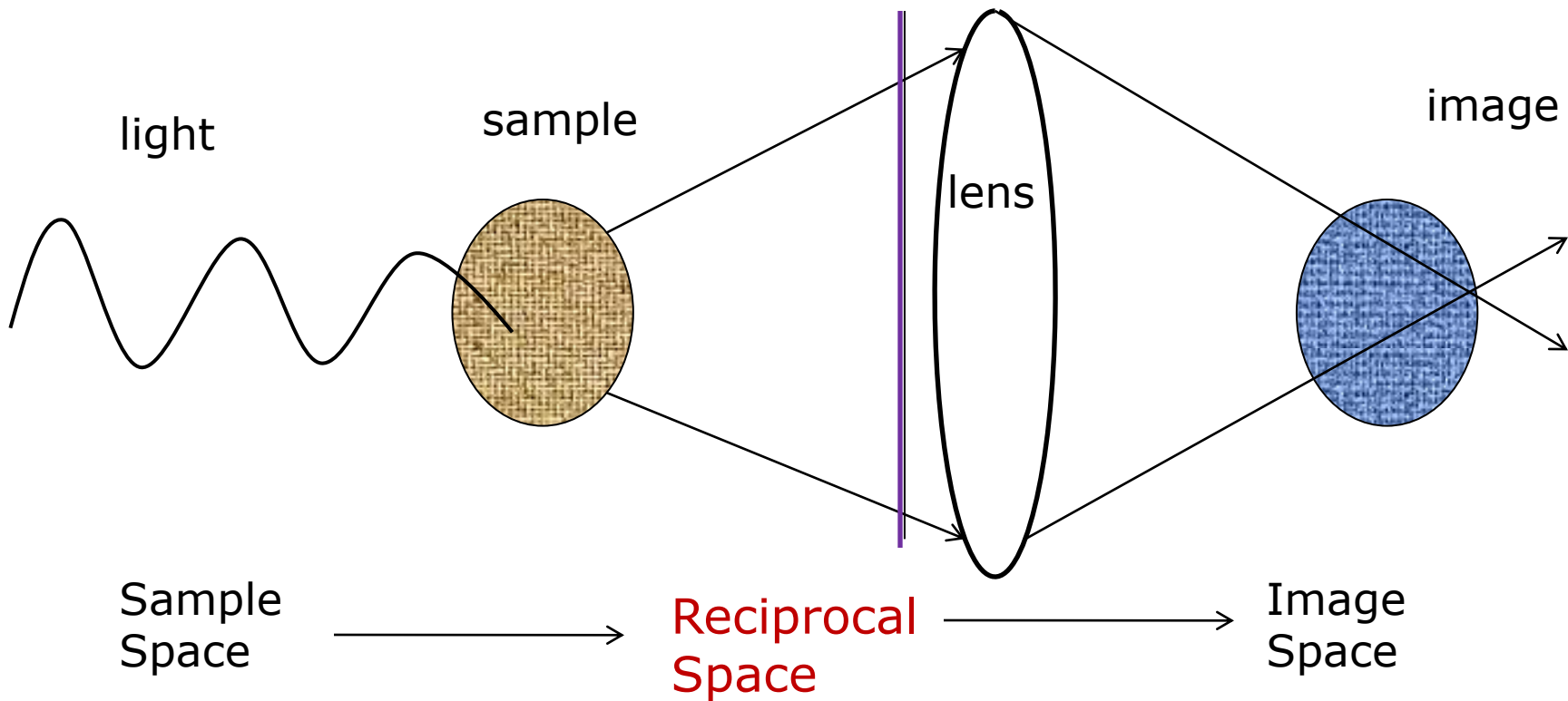
Apurva Mehta

Linda Lim

- What do diffraction peaks tell us?
 - Structure Refinement Intro
 - Refinement Details - Linda
-

Diffraction Physics

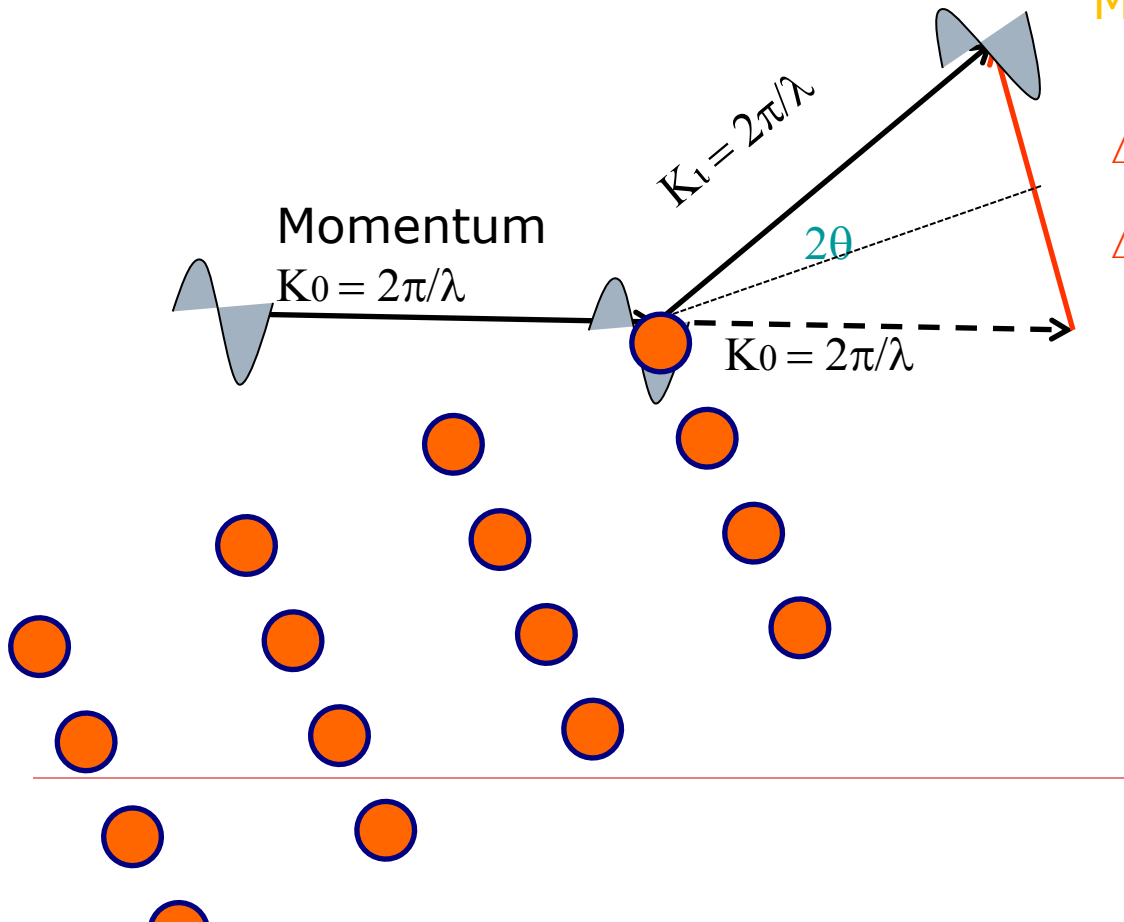
X-ray lens with resolution better than $\sim 10\text{nm}$ don't exist



X-ray Scattering/diffraction is about probing the structure without a lens

Diffraction Physics

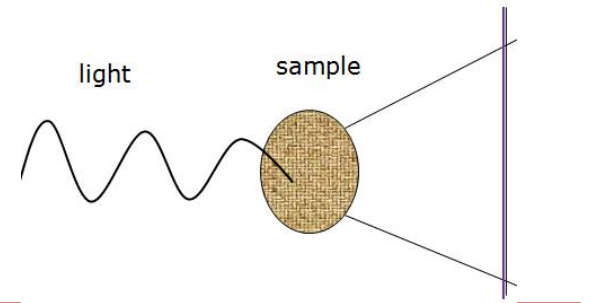
Elastic Scattering



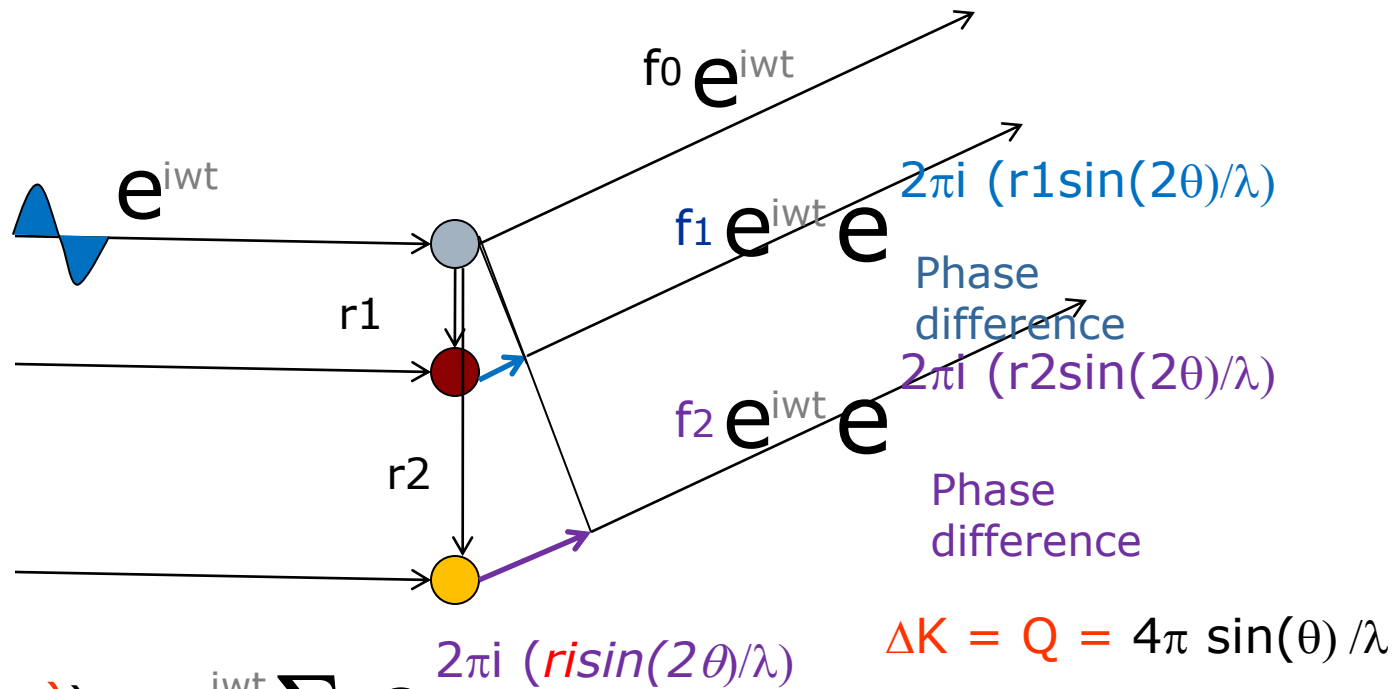
Momentum change

$$\Delta K = 2\sin(\theta) * 2\pi/\lambda$$

$$\Delta K = Q = 4\pi \sin(\theta) / \lambda$$



Diffraction Physics



$A(\Delta K = (s-s_0)) = e^{i\omega t} \sum f_i e^{i(r_i \cdot Q)}$
 amplitude

$A(\Delta K) = \sum f_i e^{i(r_i \cdot Q)}$

$A(Q) = \text{Fourier Transform} (r_i)$

Fourier Transform Recap

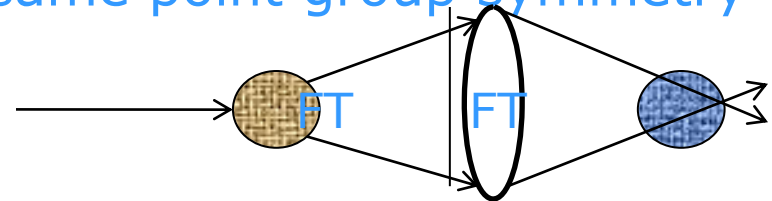
□ FT (large) $\sim 1/\text{large} \rightarrow$ small

large structures in real space \rightarrow small in reciprocal space

□ FT (periodic fn) \sim periodic

Reciprocal and space have the same point group symmetry

□ FT (FT (S)) \sim S



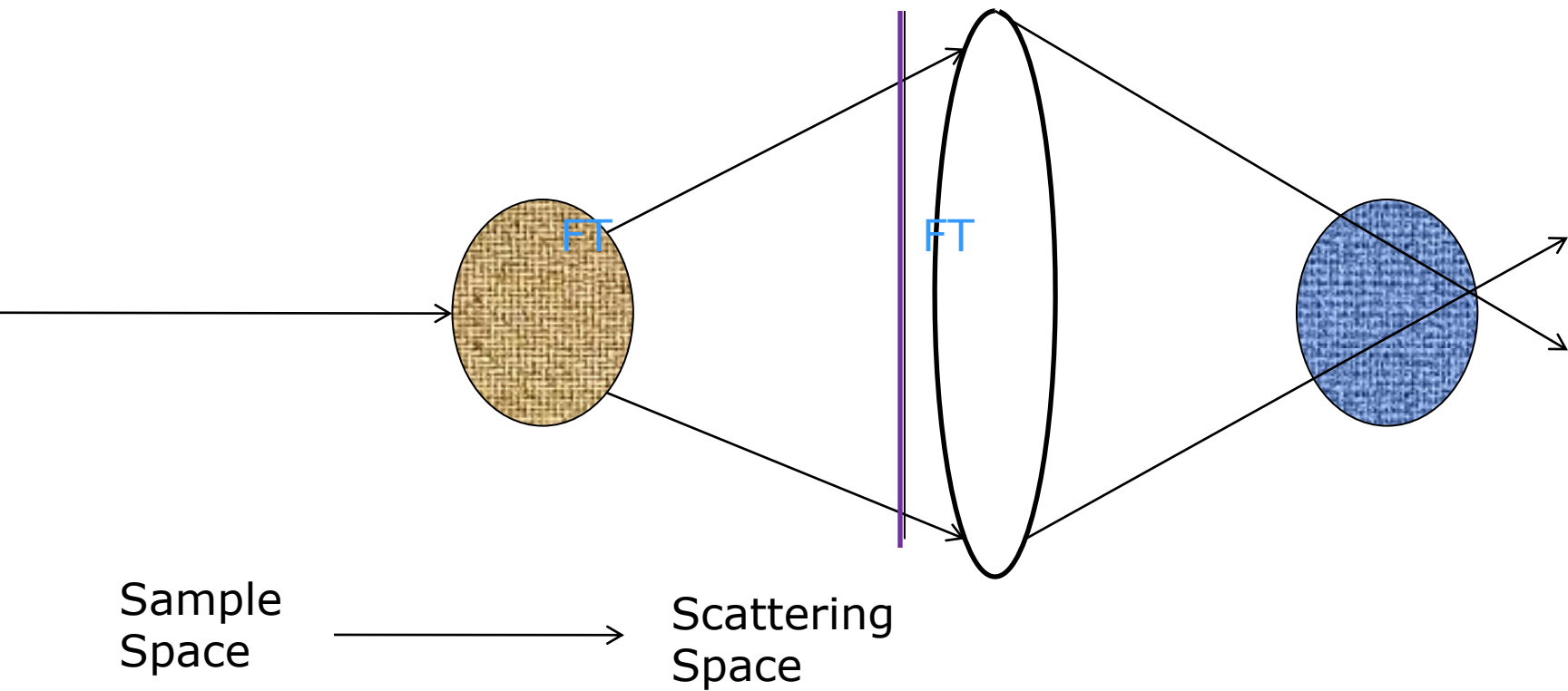
FT (real space) \rightarrow reciprocal space: FT (rec. space) \rightarrow image of real space

□ Convolution Theorem:

■ FT (a multiply b) = FT (a) conv FT (b)

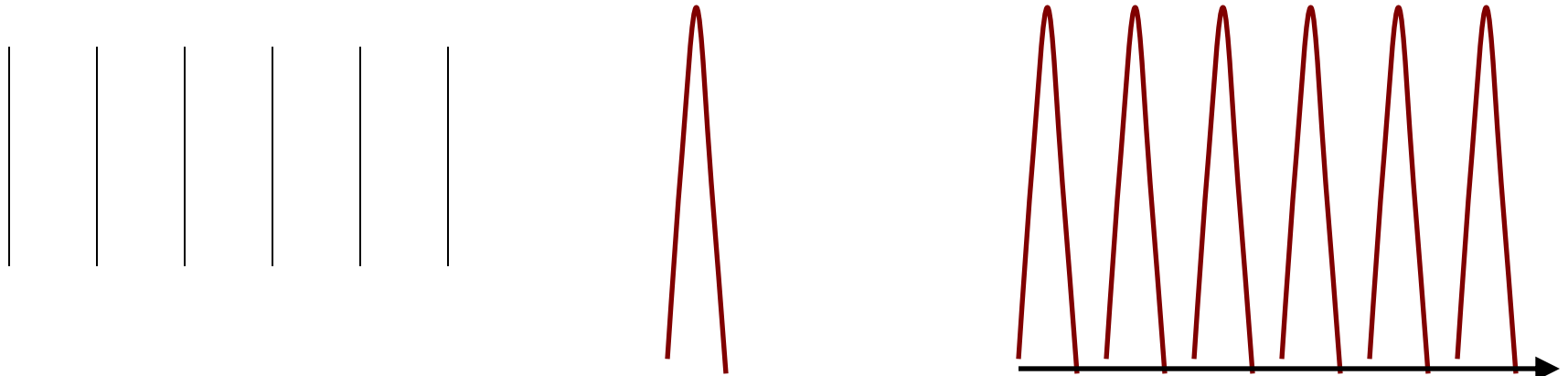
■ FT (a conv b) = FT (a) mult FT (b)

Diffraction Physics



Fourier
T

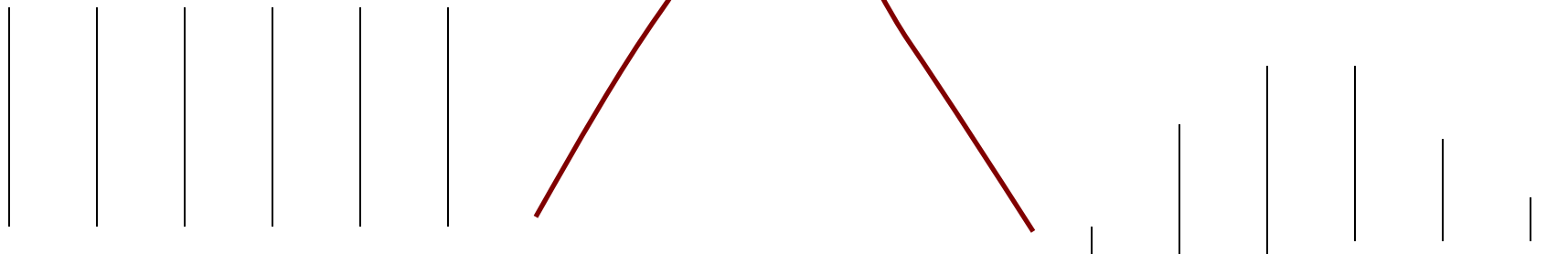
Multiplication vs Convolution



$$\text{FT (a multiply b)} = \text{FT (a) conv FT (b)}$$

$$\text{FT (a conv b)} = \text{FT (a) mult FT (b)}$$

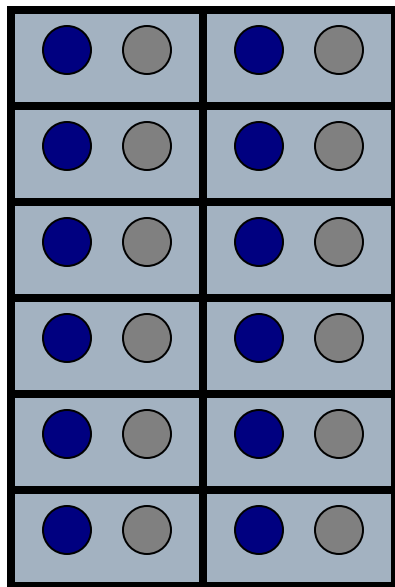
x
Convolution



Multiplication

Deconstructing the Sample space

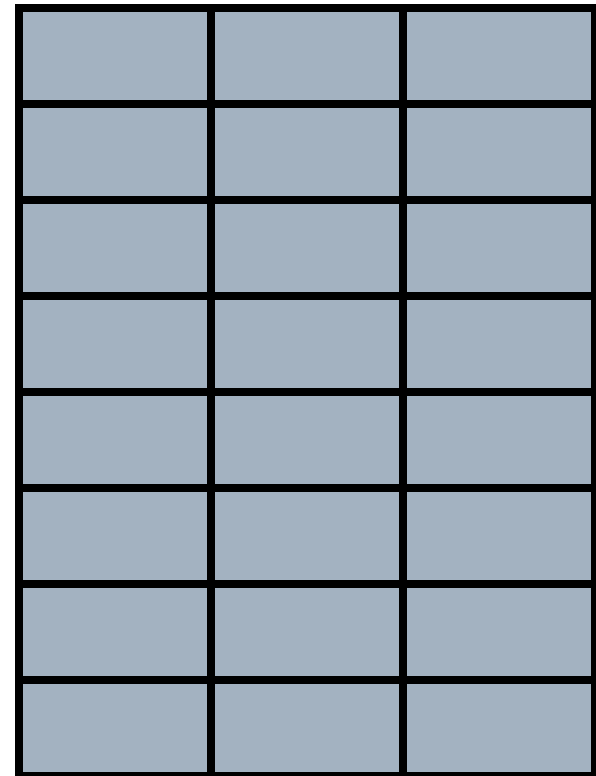
$$\text{Sample} = S \times P * M$$



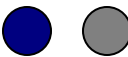
=



x



*



M
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f

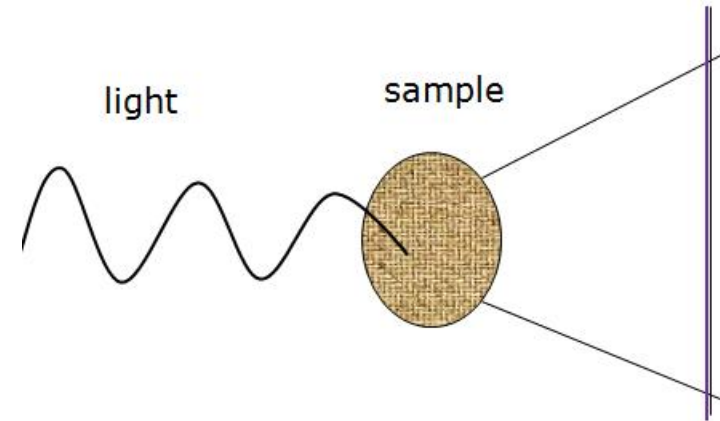
Sample size
(S)

Infinite Periodic Lattice
(P)

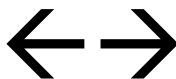
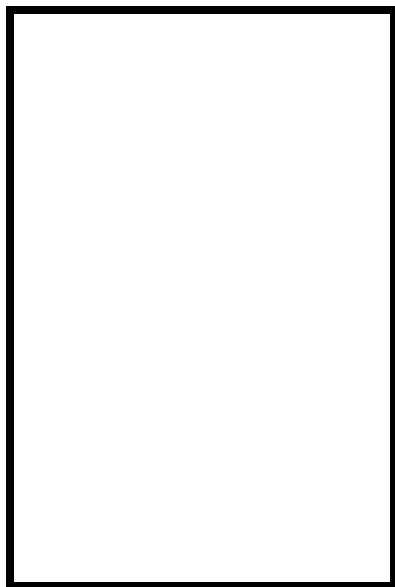
(M)

Reciprocal Space Peaks

$$\begin{aligned}
 I(Q) &= \text{FT}(\text{sample}) \times \text{FT}(\text{sample}) \\
 &= \text{FT}(S \times P * M) \times \text{FT}(S \times P * M) \\
 &= \{\text{FT}(S \times P) \times \text{FT}(M)\} \{ \dots \} \\
 &= \{\text{FT}(S) * \text{FT}(P) \times \text{FT}(M)\} \{ \dots \}
 \end{aligned}$$



FT(S)



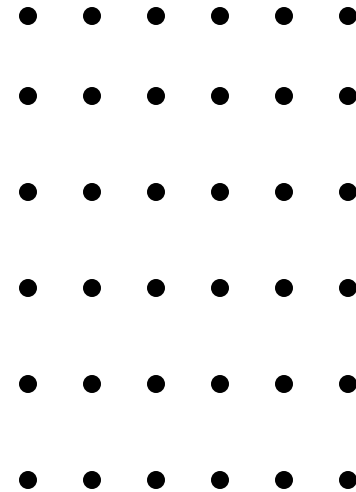
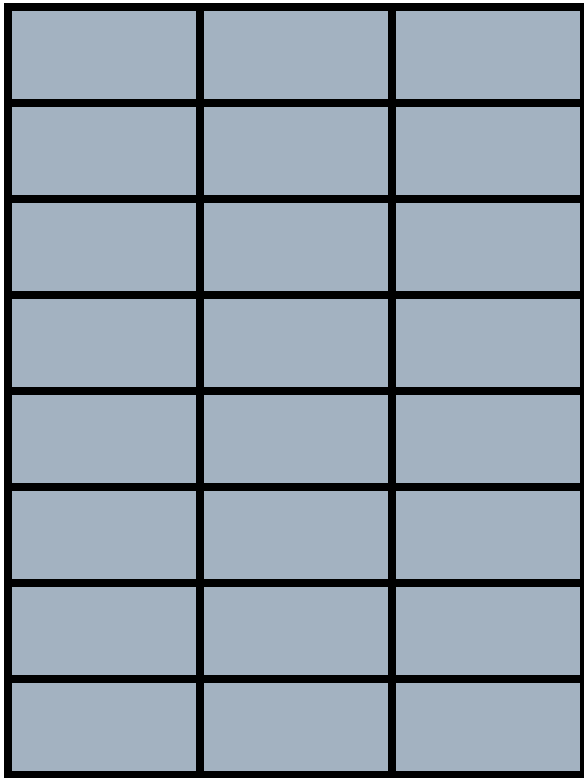
X



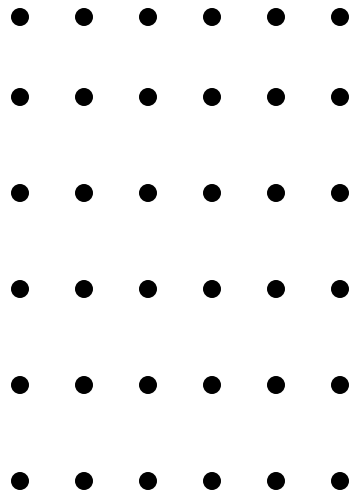
Y



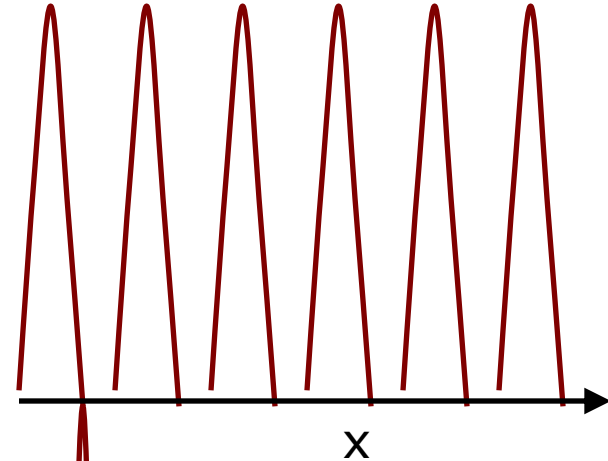
FT(P)



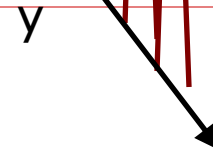
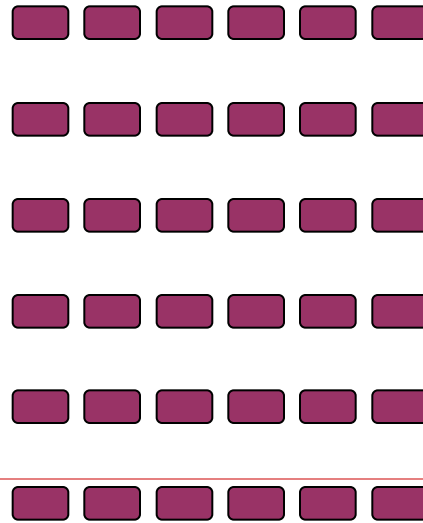
$$FT(S \times P) = FT(S) * FT(P)$$



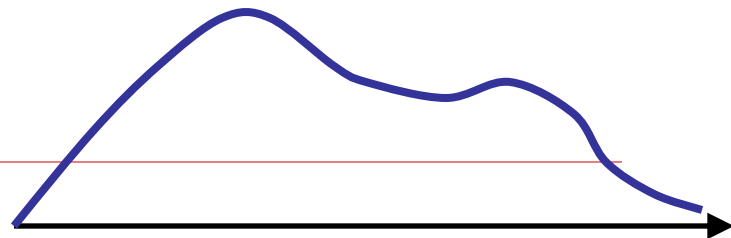
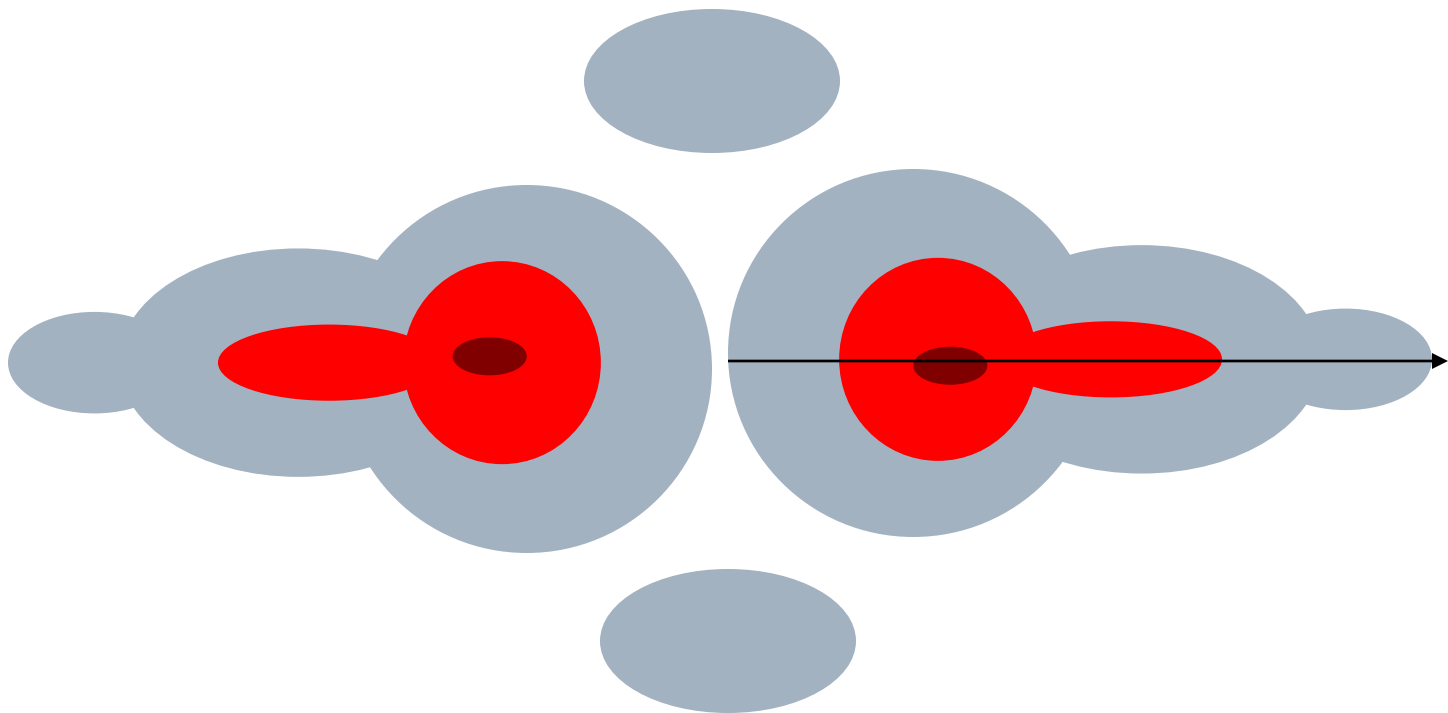
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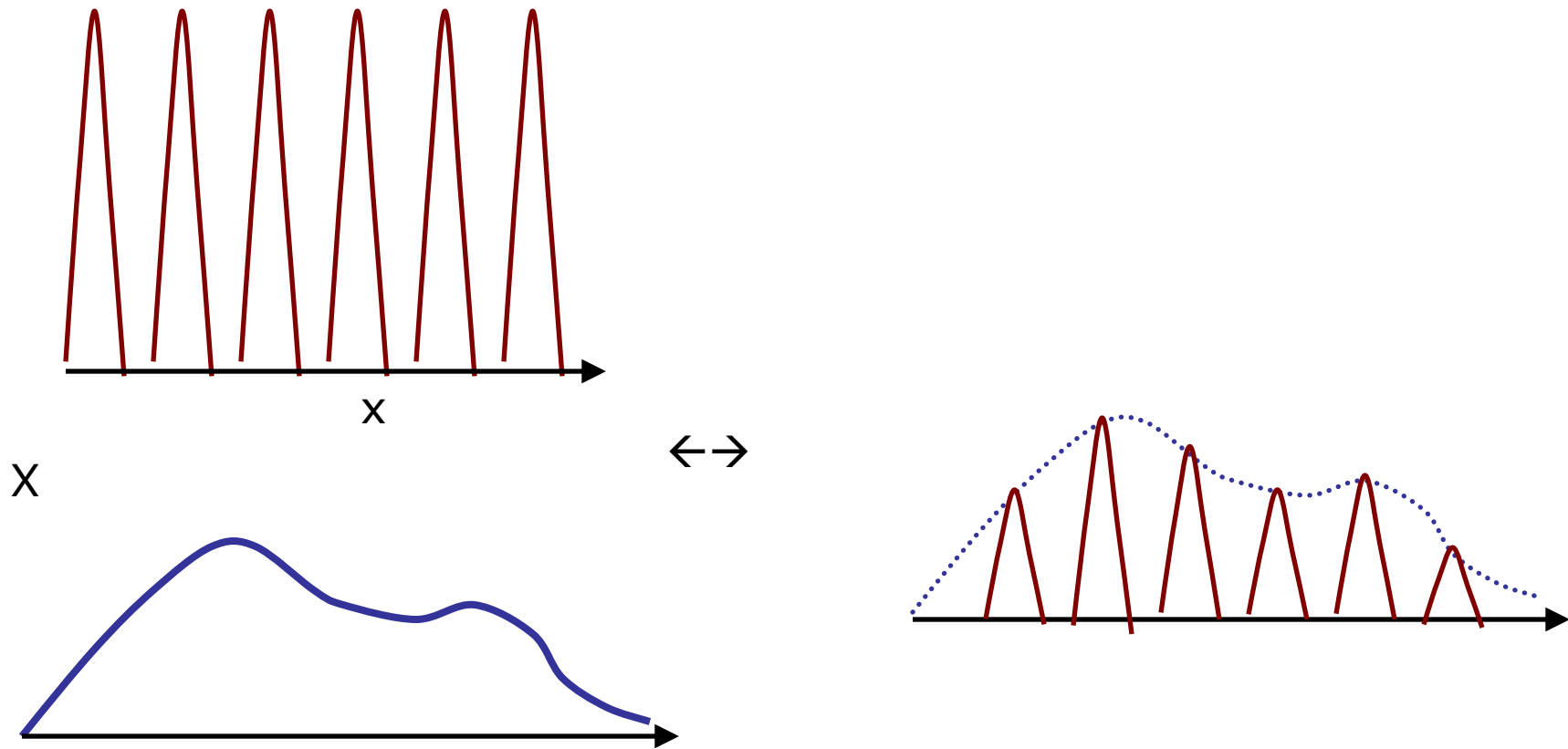


FT(M)

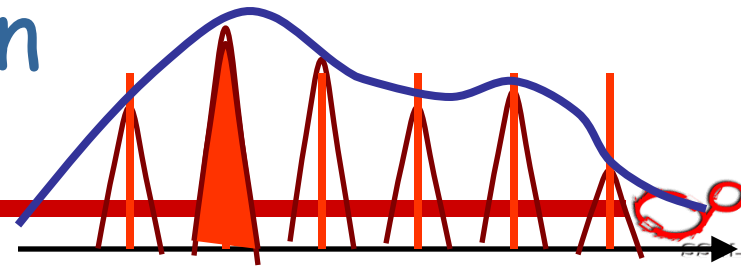


$$FT(\text{sample}) = FT(S \times P) \times FT(M)$$

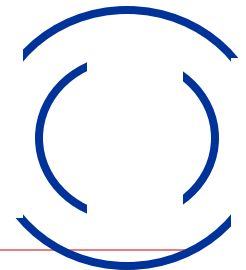
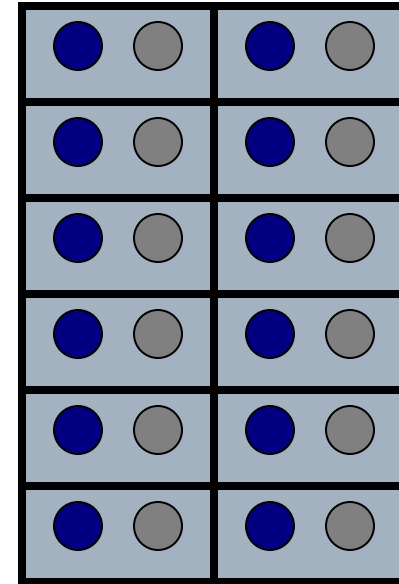
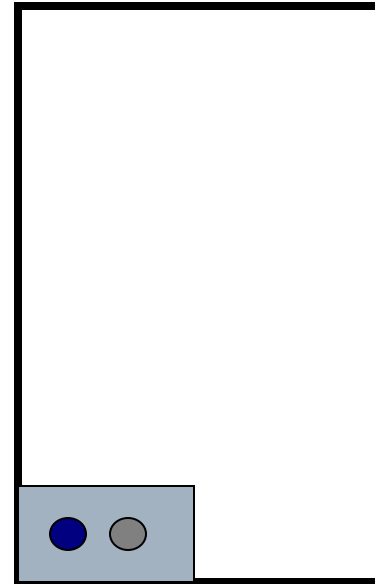
Along X direction



What does a diffraction pattern tell us?

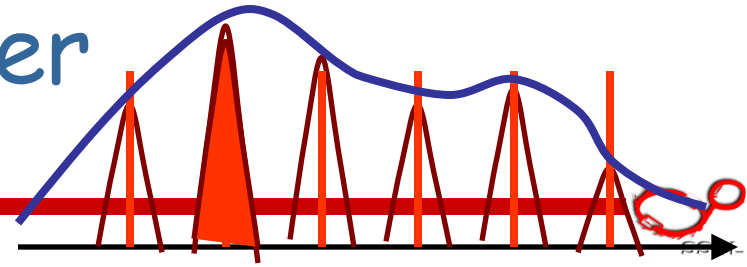


- Peak Shape & Width:
 - crystallite size
 - Strain gradient
- Peak Positions:
 - Phase identification
 - Lattice symmetry
 - Lattice strain
- Peak Intensity:
 - Structure solution
 - Crystallite orientation

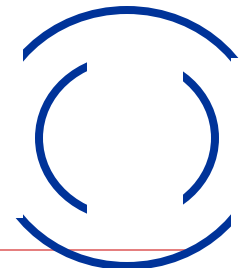
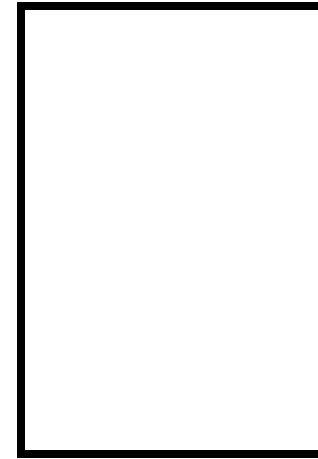


Questions?

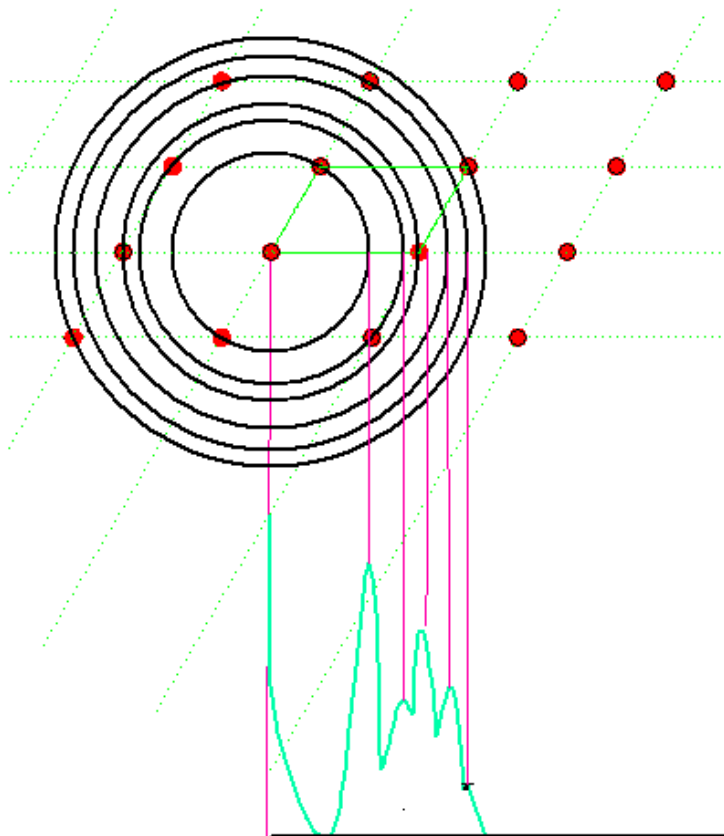
Bulk Structure via powder diffraction



- Peak Shape & Width:
 - crystallite size
 - Strain gradient
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 - Phase identification
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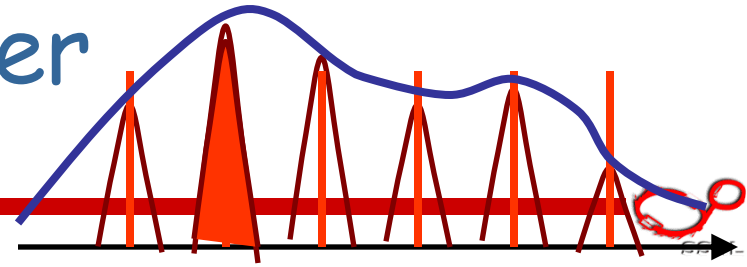
Powder Pattern



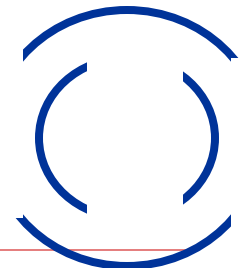
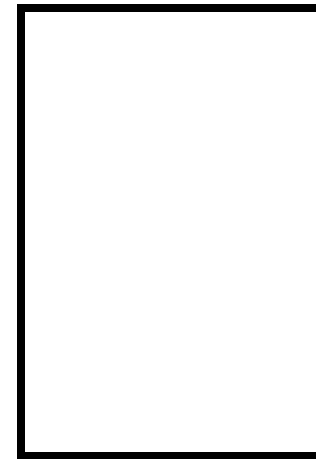
- Loss of angular information
 - Not a problem as peak position = $\text{fn}(a, b \ \& \ \alpha)$

- Peak Overlap :: A problem
 - High Resolution Synchrotron measurements help
 - But can be useful for precise lattice parameter measurements

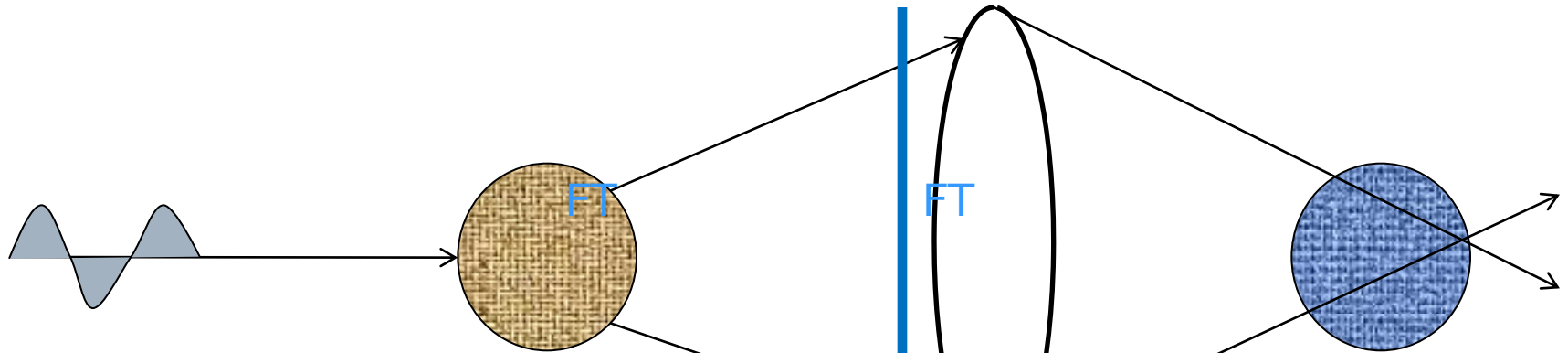
Bulk Structure via powder diffraction



- Peak Shape & Width:
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Diffraction Physics



$$A(\Delta K) = \sum f_i e^{2\pi i (r_i \sin(2\theta)/\lambda)}$$

$$A(\Delta K) = \sum f_i e^{i (r_i \cdot Q)}$$

$$A(Q) = F_{hkl} = \text{Fourier Transform} (r_i)$$

$$A(r) = \sum F_{hkl} e^{i (r \cdot Q_i)}$$

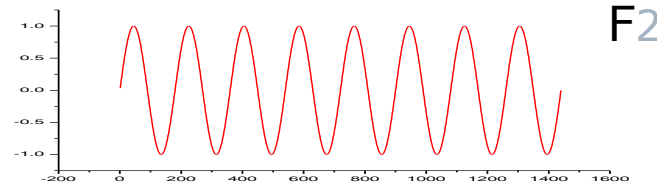
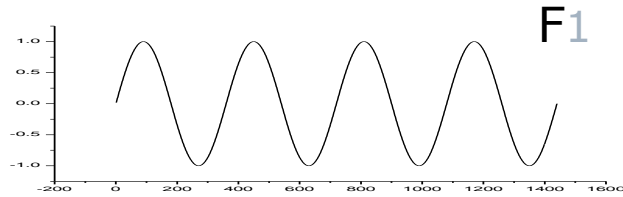
$A(Q) / F_{hkl}$ is a complex number

We measure $A(Q) * A(Q)$

→ amplitude **but not the phase** of $A(Q)$ is known

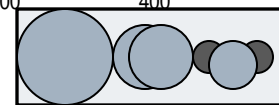
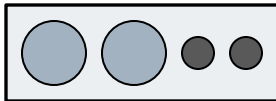
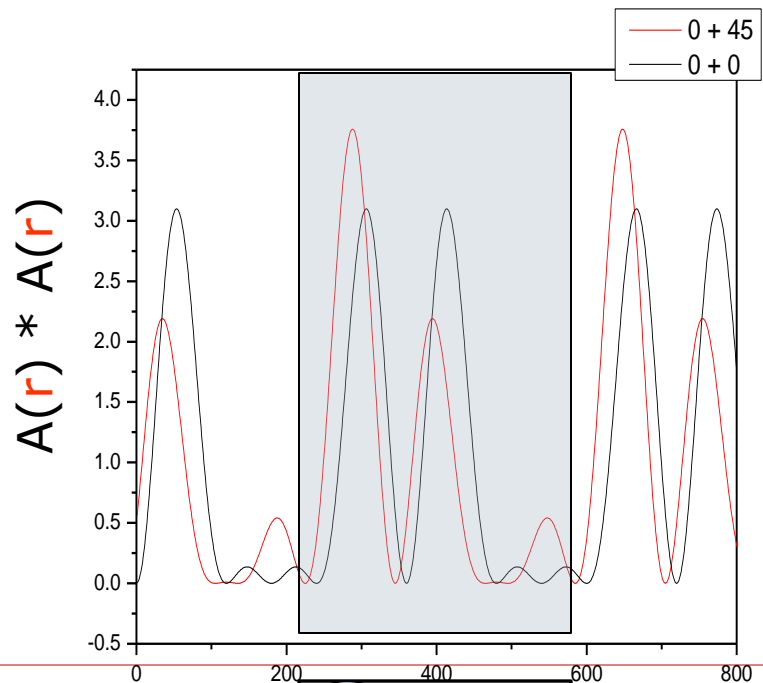
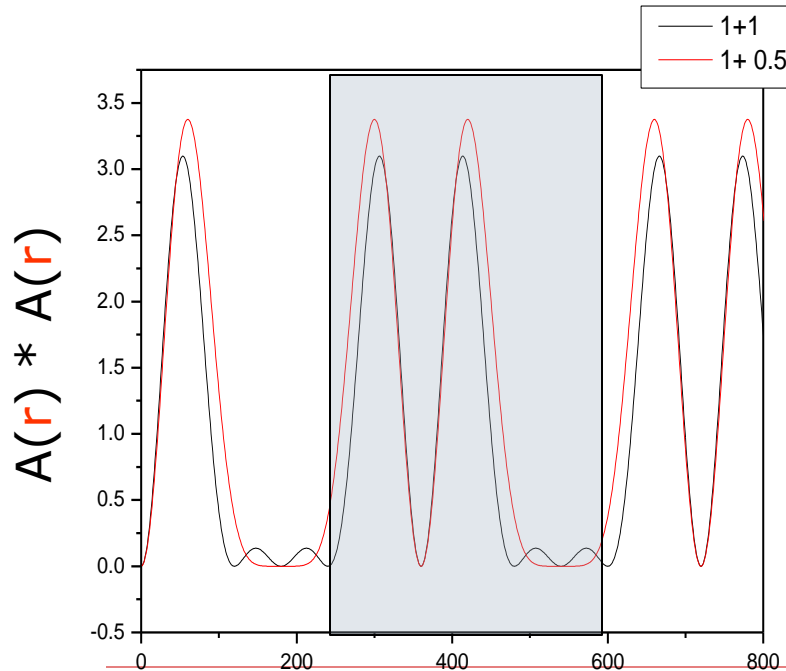
Phase Problem

$$A(r) = \text{FT} \{ F_1 + aF_2 e^{i(\text{ph})} \}$$



Amplitude unknown

Phase Unknown



Solution to Phase Problem



- Phase/Structure must be guessed
 - And then refined.

- How to guess?
 - Heavy atom substitution, SAD or MAD
 - Structure with one strong scatterer – e.g. U in Phosphates matrix -
 - Similarity to homologous compounds, e.g. NaCl → KCl

- ~~■ Patterson function or pair distribution analysis.~~

M. Michel

Structure Solution



Single Crystal

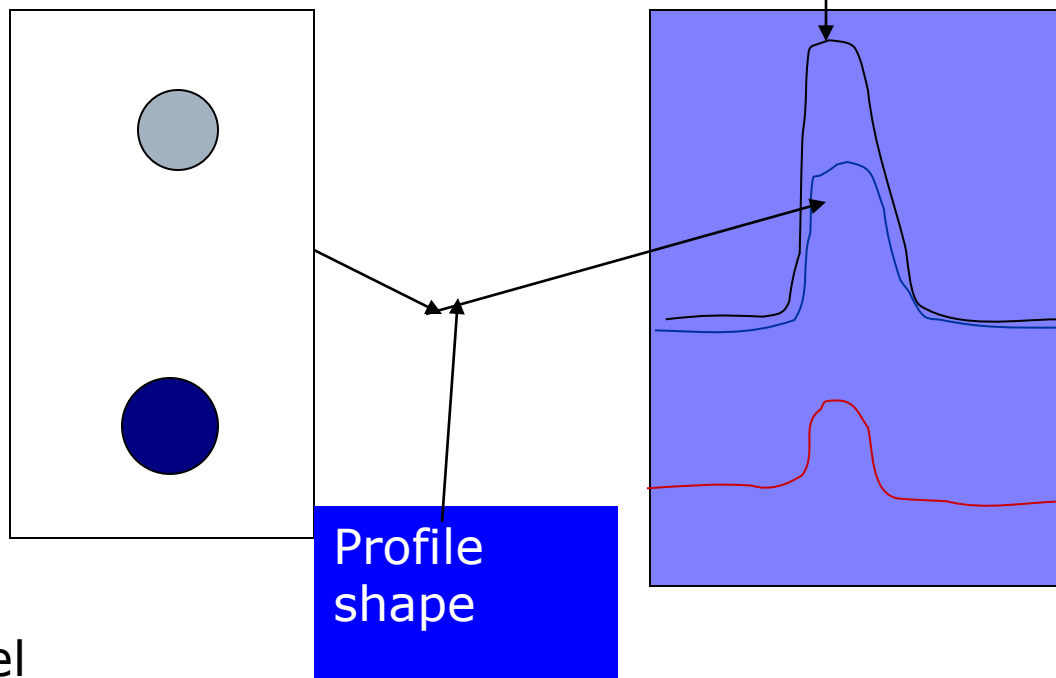
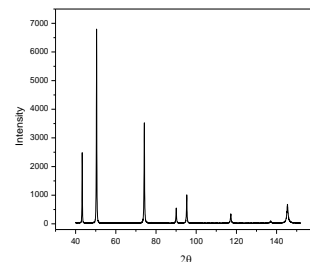
- Protein Structure
- Sample with heavy Z problems Due to
 - Absorption/extinction effects
- Mostly used in Resonance mode
 - Site specific valence
 - Orbital ordering.

Powder

- Due to small crystallite size kinematic equations valid
- Many materials can not be readily prepared in single-crystal form: **their structures obtained via synchrotron powder diffraction**
- Peak overlap a problem – high resolution setup helps
- Much lower intensity – **loss on super lattice peaks** from small symmetry breaks. (Fourier difference helps)

Inverse Modeling Method 1

□ Reitveld Method Data



Model

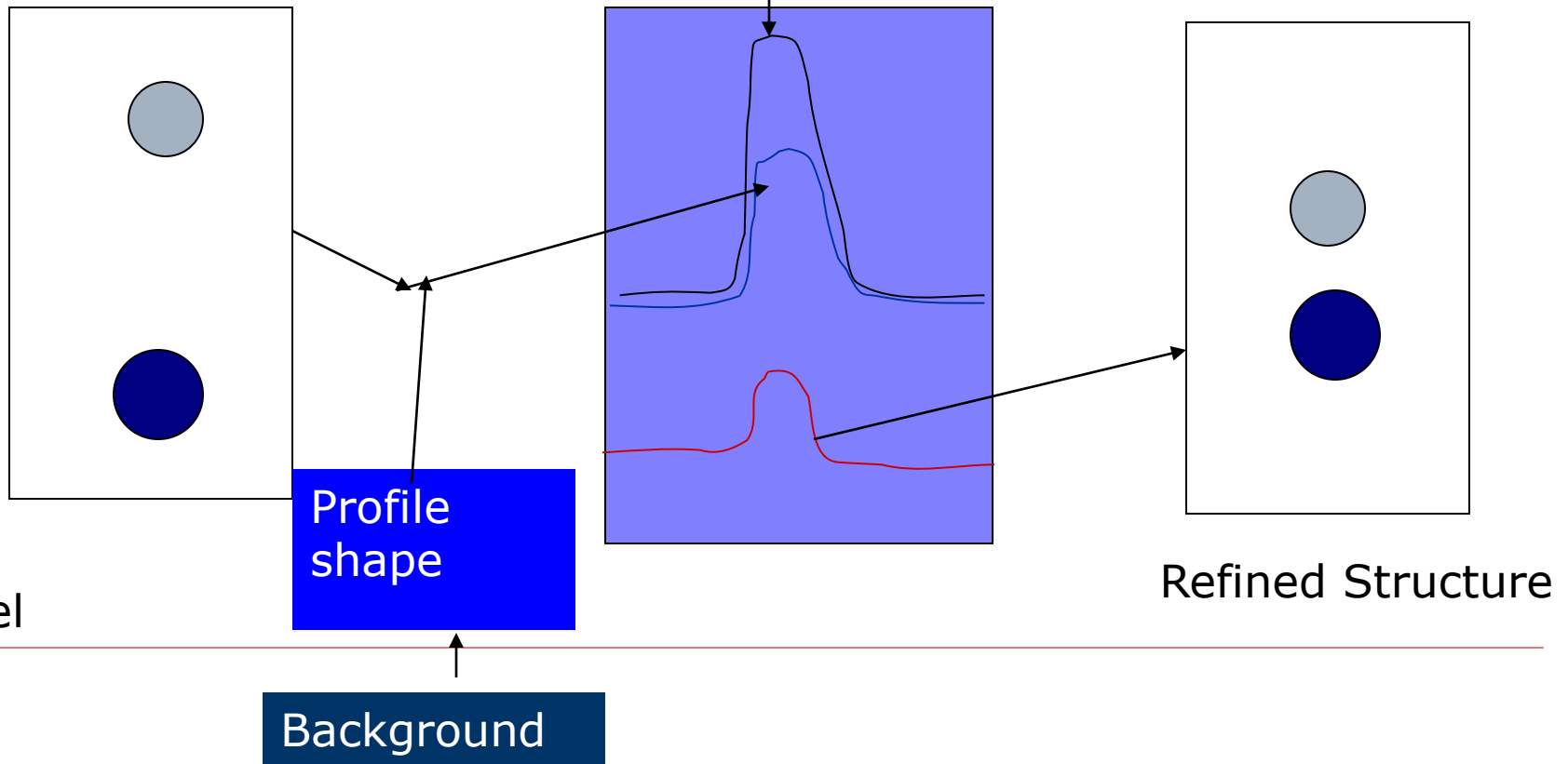
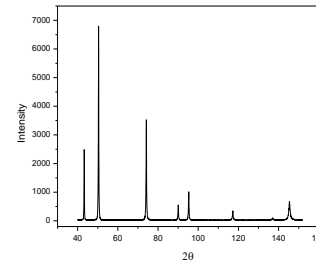
Background

- Empirical
 - Voigt function modified for axial divergence (Finger, Jephcoat, Cox)
 - Refinable parameters – for crystallite size, strain gradient, etc...

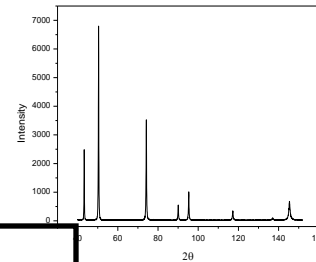
 - From Fundamental Principles
-

Inverse Modeling Method 1

□ Reitveld Method Data



Inverse Modeling Method 2



Data

subtract

Background

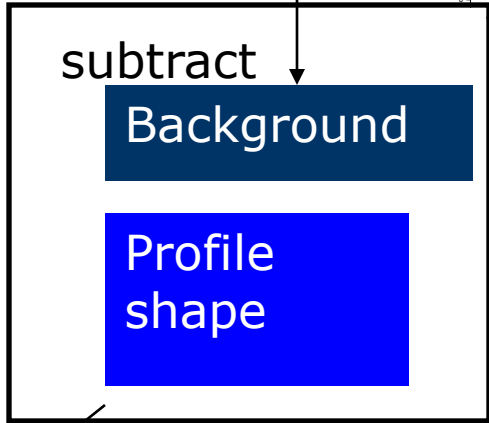
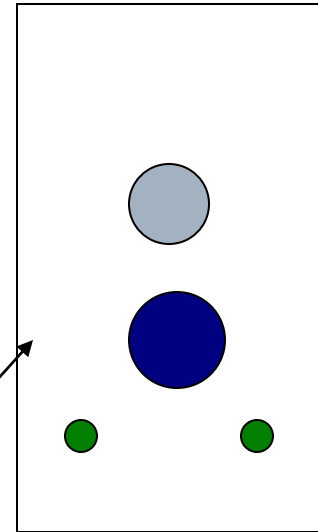
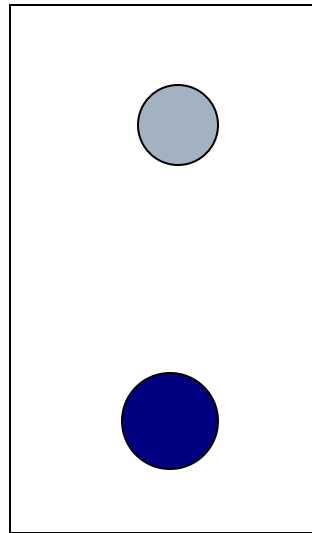
Profile shape

Integrated Intensities

Refined Structure

Model

phases

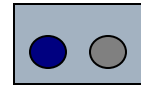


Inverse Modeling Methods

- Rietveld Method
 - More precise
 - Yields Statistically reliable uncertainties
 - Fourier Method
 - Picture of the real space
 - Shows “missing” atoms, broken symmetry, positional disorder
 - Should iterate between Rietveld and Fourier.
 - Be skeptical about the Fourier picture if Rietveld refinement does not significantly improve the fit with the “new” model.
-

Procedure for Refinement/Inverse Modeling

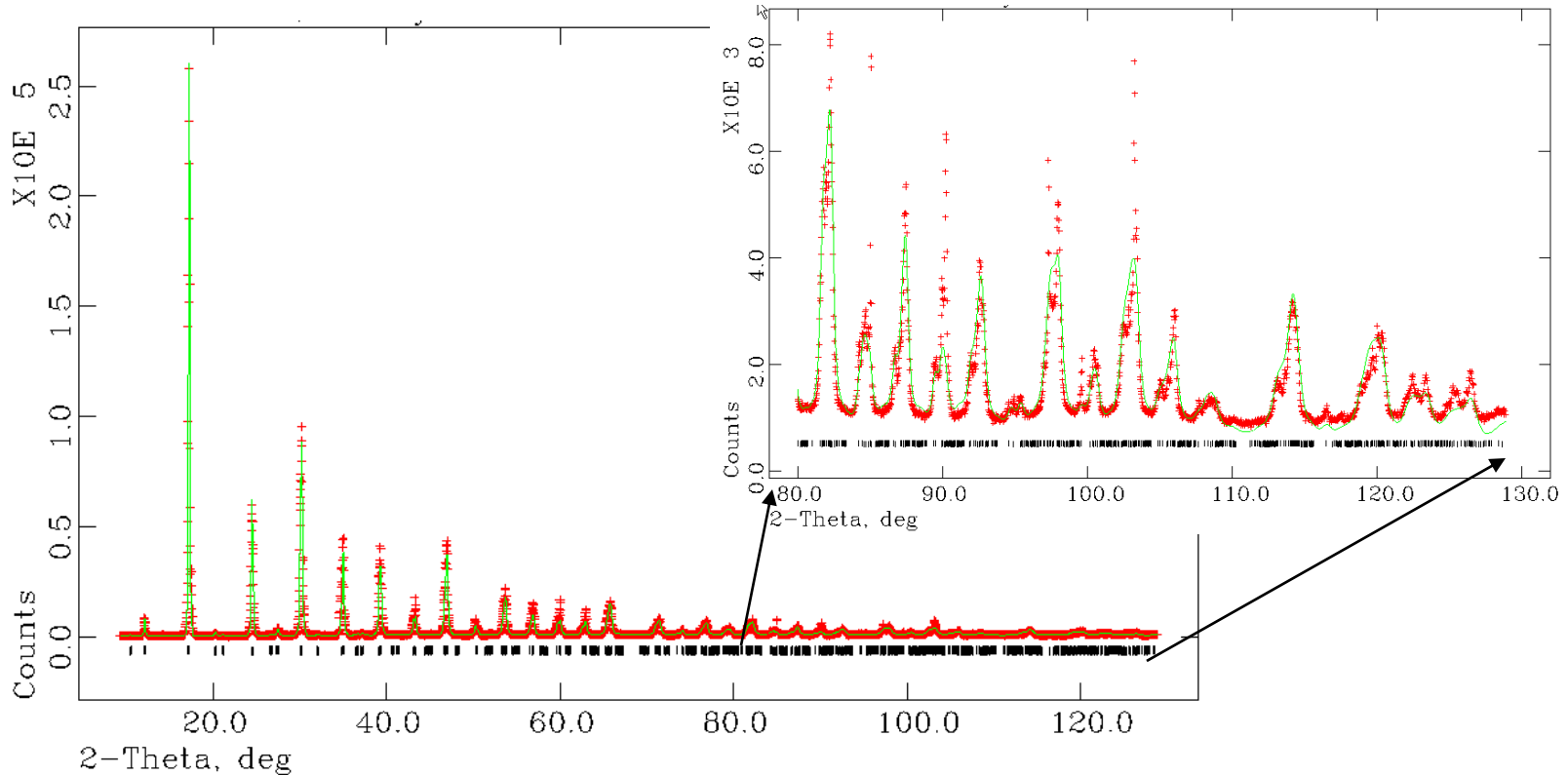
- Measure peak positions:
 - Obtain lattice symmetry and point group
 - Guess the space group.
 - Use all and compare via F-factor analysis
- Guess the motif and its placement
 - Phases for each hkl
- Measure the peak widths
 - Use an appropriate profile shape function
- Construct a full diff. pattern and compare with measurements



Collect data on Calibrant under the same conditions

- Obtain accurate wavelength and diffractometer misalignment parameters
- Obtain the initial values for the profile function (instrumental only parameters)
- Refine polarization factor
- Tells of other misalignment and problems

Need for High Q



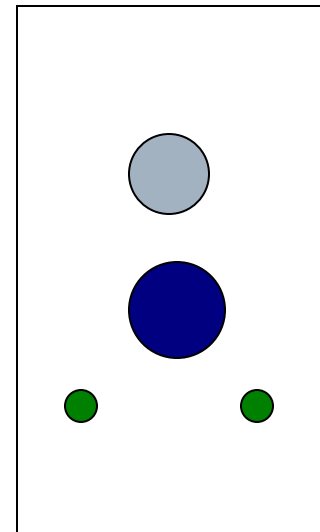
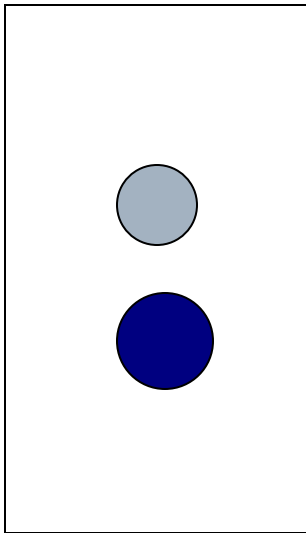
Many more reflections at higher Q.

Therefore, most of the structural information is at higher Q

Structure Refinement Method

Reitveld Method

Fourier Difference Method



Thanks

Questions?