CSR Effects – Theory

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Coherent Synchrotron Radiation (CSR) Wake

A relativistic particle moving in vacuum in a circular orbit of radius $R$, in steady state, generates a CSR wake (per unit length of path) (Murphy et al., 1995; Derbenev et al. 1995)

$$w_l(s) \approx -\frac{E_\parallel}{q} = -\frac{2}{3^{4/3} R^{1/3} s^{4/3}}$$

- For $R \approx 30$ m and $\gamma \approx 10^4$, $R/\gamma^3 = 3 \cdot 10^{-11}$ m
- We will be talking about $s \sim$ microns — the wake is large for short wavelengths
- Shielding effects are negligible if $h \gtrsim s^{2/3} R^{1/3}$
One might expect a short-wavelength instability of the bunch and modulation of the beam density.

In computer simulations, M. Borland observed beam microbunching in a bunch compressor (Borland, 2001).

Recently M. Borland and P. Emma found microbunching in elegant simulations of the LCLS bunch compressor BC2.

There are observations of coherent radiation of the bunch at wavelengths $\ll \sigma_z$ in several rings: NSLS VUV at BNL, SURF at NIST, and ALS at LBL.

What are predictions of the theory?
Linear theory of the CSR instability in rings

If the wavelength of the instability $\lambda \ll \sigma_z$, we used local approximation for the beam current and applied Keil-Schnell approach for the coasting beam (Heifets & Stupakov, 2001).

- Threshold due to finite energy spread

$$\frac{\lambda}{R} \gtrsim \pi \delta_0^3 \left[ \frac{\eta \gamma}{(I/17 \text{ kA})} \right]^{3/2}$$

- Far from the threshold the growth rate $\Gamma$ is

$$\Gamma \sim \frac{1}{\lambda^{2/3}}$$

If the instability develops, it most likely proceeds into a nonlinear stage.
CSR instability in LCLS BC2

Analytical theory of the instability in a bunch compressors (Heifets & Stupakov, 2001) predicts an amplification factor of a sinusoidal initial perturbation, in linear approximation. The theory has been applied for the LCLS bunch compressor BC2 assuming $Q = 1$ nC and $\delta_0 = 1.6 \cdot 10^{-5}$.

![Graph showing Log (Amplification) vs s (m)]

Conclusion: large amplification will drive the instability into nonlinear regime.
Transient effects in CSR

The problem has been studied by Saldin et al. in 1997. The resulting equation for finite $\gamma$ cannot be used for calculations with $\gamma \approx 10^4$. We obtained analytical formulas in the limit $\gamma \rightarrow \infty$ (Stupakov & Emma), which are now included in **elegant**.

Example: $R = 1.5$ m, $E = 150$ MeV, $q = 1$ nC, $\sigma_z = 50$ $\mu$m.
Assumptions and limitations in theory and simulations

- CSR shielding is neglected
- 1D CSR wake is used, 3D effects are neglected
- Self-consistent calculation of the wake. Might be important for very short wavelength.
- Transient effects in CSR (E. Saldin et al. 1997). Transients were neglected in analytical theory.

Some of those issues will be discussed at ICFA workshop “Coherent Synchrotron Radiation (CSR) and its impact on the beam dynamics of high brightness electron beams” in Berlin, Germany, January 14-18, 2002.