BL12 Impedance Considerations

J. Sebek

August 11, 2004
1 Resistive Wall Impedance

- The macro-particle transverse equation of motion

\[
\frac{d^2 y(s)}{ds^2} + K(s) y(s) = -\frac{N r_0}{\gamma C} \sum_{k=1}^{\infty} y(s - kC) W_1(-kC)
\]

when transformed to normal coordinates becomes

\[
\ddot{w} + \nu_\beta^2 w = -\frac{N r_0}{2\pi \gamma} \nu_\beta \sum_{k=1}^{\infty} w(\phi - 2\pi k) \langle \beta W_1(-kC) \rangle
\]

where the wake function is weighted by the local $\beta$ function.

- Changing the scale of the frequency from $\nu_\beta$ to $\omega_\beta = \nu_\beta \omega_0$ one obtains the complex frequency shift

\[
\Omega - \omega_\beta = -i \frac{1}{2\gamma T_0^2} \sum_{p=-\infty}^{\infty} \langle \beta Z_1^+ (p\omega_0 + \Omega) \rangle
\]

\[
= -i \frac{I_T}{2(E/e) T_0} \sum_{p=-\infty}^{\infty} \langle \beta Z_1^+ (p\omega_0 + \Omega) \rangle
\]
and the growth rate

\[ \tau^{-1} = -\frac{I_T}{2(E/e)T_0} \sum_{p=-\infty}^{\infty} \text{Re} \left\{ \langle \beta Z_1^+ (p\omega_0 + \Omega) \rangle \right\} \]

- The resistive wall impedance for a cylindrical chamber of radius, \( b \), is

\[ \frac{Z_1^+ (\omega)}{L} \approx \frac{1 - \text{sgn} (\omega) i}{2\pi b^3} Z_0 \delta \]

where \( Z_0 = 377 \Omega \) and \( \delta = \sqrt{\frac{2}{\mu \omega \sigma}} \) is the electrical skin depth of the chamber
- Multiply \( Z_1^+ (\omega) \) by the geometric factor of \( \pi^2/12 \) to obtain the vertical parallel plate impedance

- Impedance scaling
  - \( Z_1^+ (\omega) \) varies with chamber radius \( b \) as \( b^{-3} \)
  - \( Z_1^+ (\omega) \) varies with conductivity \( \sigma \) as \( \sigma^{-\frac{1}{2}} \)
  - \( Z_1^+ (\omega) \) varies with frequency \( \omega \) as \( \omega^{-\frac{1}{2}} \)
Some contributions of the resistive wall impedance for a four bunch fill pattern with a vertical betatron tune of 0.25.
• BL12 ID impedance
  – Stainless steel chamber with 5 mm gap has the equivalent impedance of standard vacuum chambers for 3 SPEAR rings
  – Must reduce the conductivity to that of Cu to reduce the impedance by a factor of 6.5
2 Plating

2.1 Plating Thickness

- If a thickness $d$ of metal of conductivity $\sigma_1$ is plated on a metal of conductivity $\sigma_2$, the beam-generated longitudinal electric field, $E_s$, on the metal is

$$
\tilde{E}_s = -\frac{(1 - i)}{2} \sqrt{\frac{\omega}{2\pi \sigma_1}} \frac{2q}{b} \left( 1 + \sqrt{\frac{\sigma_2}{\sigma_1}} \right) + \left( 1 - \sqrt{\frac{\sigma_2}{\sigma_1}} \right) e^{2(i-1)d/\delta_1}
$$

- The impedance of the compound wall has the same dependence on $\sigma_1$, $\sigma_2$, and $d$.

- Only about 66 $\mu$m of copper plating, one $\delta$ at 1 MHz, is needed to obtain the desired ID resistive wall impedance
Decrease of resistive wall impedance, at a fixed frequency, of a stainless steel chamber as the plating thickness of copper increases.
2.2 Plating Width

- The Green function for the electric field generated by a charge between two parallel plates is

\[
G(x, y; x', y') = \sum_{k=1}^{\infty} \frac{4}{k} e^{-\frac{k\pi}{d}|x-x'|} \sin \frac{k\pi y}{d} \sin \frac{k\pi y'}{d}.
\]

- The surface charge distribution on the plates is

\[
\sigma(x, 0; x', y') = -\frac{1}{4\pi} \frac{\partial}{\partial y} G(x, y; x', y') \bigg|_{y=0} = -\frac{1}{d} \sum_{k=1}^{\infty} e^{-\frac{k\pi}{d}|x-x'|} \sin \frac{k\pi y'}{d}
\]

\[
= -\frac{1}{d} \left[ e^{-\frac{\pi}{d}|x-x'|} \sin \frac{\pi y'}{d} + e^{-\frac{2\pi}{d}|x-x'|} \cos \frac{\pi y'}{d} + e^{-\frac{2\pi}{d}|x-x'|} \right].
\]

which drops off exponentially fast away from the source.

- Plating width will be greater than four times the maximum gap opening.
Surface charge density distribution for positive surface charge. The source particle is located at \((x, y) = (0, d/2)\) and the horizontal position is normalized in units of \(d\). The leading term of this expansion is also plotted.
3 Spear Resistive Wall Contributions

<table>
<thead>
<tr>
<th>BL</th>
<th>gap cm</th>
<th>len m</th>
<th>mat</th>
<th>$\beta_y$ m</th>
<th>$Z_{1}^{\perp} (\omega_0) / L$ kΩ/ m²</th>
<th>$Z_{1}^{\perp} (\omega_0)$ kΩ/ m</th>
<th>$\beta_y Z_{1}^{\perp} (\omega_0)$ kΩ</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.48</td>
<td>2.34</td>
<td>Cu</td>
<td>4.8</td>
<td>13.4</td>
<td>33.0</td>
<td>159</td>
</tr>
<tr>
<td>5</td>
<td>1.96</td>
<td>2.04</td>
<td>Al</td>
<td>4.8</td>
<td>3.9</td>
<td>7.9</td>
<td>38</td>
</tr>
<tr>
<td>6</td>
<td>1.19</td>
<td>2.46</td>
<td>Cu</td>
<td>4.8</td>
<td>13.4</td>
<td>33.0</td>
<td>159</td>
</tr>
<tr>
<td>7</td>
<td>1.19</td>
<td>2.46</td>
<td>Cu</td>
<td>4.8</td>
<td>13.4</td>
<td>33.0</td>
<td>159</td>
</tr>
<tr>
<td>9</td>
<td>1.78</td>
<td>2.27</td>
<td>ss</td>
<td>4.8</td>
<td>26.3</td>
<td>59.6</td>
<td>286</td>
</tr>
<tr>
<td>10</td>
<td>1.96</td>
<td>2.25</td>
<td>ss</td>
<td>4.8</td>
<td>19.7</td>
<td>44.3</td>
<td>213</td>
</tr>
<tr>
<td>11</td>
<td>1.48</td>
<td>2.40</td>
<td>ss</td>
<td>4.8</td>
<td>45.7</td>
<td>109.7</td>
<td>527</td>
</tr>
<tr>
<td>12</td>
<td>0.50</td>
<td>1.83</td>
<td>Cu</td>
<td>1.77</td>
<td>181.1</td>
<td>330.9</td>
<td>587</td>
</tr>
<tr>
<td>straights</td>
<td>3.40</td>
<td>28.0</td>
<td>Cu</td>
<td>10.0</td>
<td>0.6</td>
<td>11.7</td>
<td>117</td>
</tr>
<tr>
<td>vac cham</td>
<td>3.40</td>
<td>188.0</td>
<td>Cu</td>
<td>10.0</td>
<td>0.6</td>
<td>108.3</td>
<td>1083</td>
</tr>
</tbody>
</table>

- Total $\beta_y Z_{1}^{\perp} (\omega_0)$ around the ring is 3346 kΩ
- BL12 accounts for 18% of the total resistive wall contribution
4 Stability

- Relevant machine parameters
  - $\omega_0 = 2\pi \times 1.28 \text{ MHz}$
  - $\nu_y = 6.22$

- Instability thresholds (at zero chromaticity)
  - 269 mA for existing configuration
  - 223 mA including BL12

- Calculated chromaticity (normalized) needed to stabilize beam (based on $1\Omega$, $Q = 1$
  BBR model centered around 15 GHz)
  - 0.09 for existing configuration
  - 0.13 for ring with BL12

- No instabilities seen at 200 mA with zero chromaticity during initial run gives some
  confidence to these calculations
5 Broad Band Impedance

- Taper from $34 \text{ cm}$ to $5 \text{ cm}$ extends over more than $35 \text{ cm}$
- Slope is less than $1 : 20$
- ABCI calculations
  - Longitudinal impedance
    * $270 \text{ pH}$
    * $k_{Loss} = 3.2 \text{ mV/ pC}$
  - Transverse $(m = 1)$ impedance
    * $Z_{1}^{\perp} (\omega) \sim k\Omega/ \text{ m}$
6 BL12 Power Loss Calculations

- Power loss formula (MKS)
  \[
  \frac{P_{tot}}{L} \approx \frac{1}{(2\pi)^2} \Gamma \left( \frac{3}{4} \right) T_0 \frac{I_{tot}^2}{N} \frac{1}{b\sigma_t^{3/2}} \sqrt{\frac{\mu_0}{2\sigma}}
  \]

- Operational modes
  - Normal running is 500 mA in 279 bunches
  - Possible “single bunch” mode of 150 mA in 6 bunches

- Normal total resistive wall power loss along BL12 chamber wall is 19 W
- “Single bunch” power loss is 81 W
7 Other Considerations (Bane/Krinsky 1993)

7.1 Tune Shift Across ID

- Maximum tune shift from resistive wall wake

\[
\Delta \nu = \frac{Q}{4\pi (E/e)} \beta W_{1}^{\perp} l
\]

where

\[
W_{1}^{\perp} \approx \frac{2}{\pi b^3} \sqrt{\frac{c}{4\pi \varepsilon_0 \sigma}} \sqrt{\frac{2}{\pi \sigma_z}}
\]

- For head tail stability, \( \Delta \nu < \nu_s = 8.2 \times 10^{-3} \)

- Aggressive current
  - 25 mA = \( 1.2 \times 10^{11} e^- \times f_0 \)

- Calculated tune shift

\[
\Delta \nu \approx 1.7 \times 10^{-4}
\]

\( \ll \nu_s \)
7.2 Emittance Blow-up

- Calculate vertical kick due to ID wake
  \[ \Delta y' = \frac{Q}{(E/e)} W_{1}^{\perp} l y \]

- Compare with angular divergence of
  - beam of \( \sim 90 \mu \text{rad} \)
  - radiation of \( \sim 12 \mu \text{rad} \)

- Use aggressive parameters
  - \( Q = 1.2 \times 10^{11} e^- \)
  - \( y = 1 \text{mm} \)

- Calculated angular kick
  \[ \Delta y' \approx 1.2 \mu \text{rad} \]