Transverse Coherence of Self-Amplified Spontaneous Emission

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Abstract

Transverse coherence of Self-Amplified Spontaneous Emission (SASE) in high gain FELs is investigated through a three-dimensional eigenmode analysis. It is the first time such an analysis could be carried out to account for the effects of emittance and betatron focusing. In addition to the variational solution [1] found for the fundamental mode, E_{00} , similar solutions are derived here for two higher order eigenmodes, E_{10} and E_{01} . Based on the growth rates of these three modes, a universal criteria on transverse coherence of SASE is proposed.

Key words: Transverse Coherence; SASE; 3D, High Gain FEL theory

1 Introduction

Transversely coherent undulator radiation is highly desirable for users of synchrotron light sources. Traditionally, the approach to achieve this goal is to reduce beam emittance, ϵ , in a storage ring to satisfy the so-called diffraction limited criteria [2]: $\epsilon < \lambda/4\pi$, under optimal matching condition: $\beta_f = L/2\pi$, where λ is the minimum wavelength above which the radiation is transversely coherent and β_f is the beta function at the beam waist located in an undulator of length L . This approach, evidently, becomes increasingly difficult at shorter wavelength. Recently, the developments of fourth generation, linac-based light sources promise to generate X-rays with orders of magnitude higher brightness. This remarkable advance is made possible through a process known as self-amplified spontaneous emission, in which spontaneous radiation from an

¹ Work supported by the U.S. Department of Energy under contract No.DE- AC03-76SF00098.

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undulator or a wiggler is amplified by orders of magnitude in power and enhanced in coherence as well through FEL interaction. In light of these developments, it is important to know what is the transverse coherence property of SASE, how does the property depend on all important FEL parameters such as emittance and beta function, and in particular, could the parameter dependence be casted into an explicit criteria.

Transverse coherence of SASE can be studied with two different approaches: eigenmode analysis and simulation. The eigenmode analysis is, first of all, analytical. As such, it could offer much clear physical insights to the problem in a global scale. On the other hand, simulation could carry out the study into nonlinear region at saturation. However, earlier studies with eigenmode analysis [3] and simulation [4] were limited to the special case of parallel electron beam without angular spread. In other words, the effects of emittance and betatron focusing were neglected.

In this article, an eigenmode analysis of transverse coherence of SASE is presented, taking into account the effects of emittance and betatron focusing. This analysis is made possible by the exact solutions found for both fundamental and higher order eigenmodes [1]. Based on the growth rates of these modes, a condition for the dominance of the fundamental mode can be determined. Since the dominance of the fundamental mode inevitably implies full transverse coherence, such a condition is also a criteria on transverse coherence of SASE. In an effort to make the proposed criteria explicit and convenient to use, variational solutions are also derived for the higher order eigenmodes of concern to the problem. As shown [1,5] for the fundamental mode, the efficient and robust variational solutions can be mapped out in the entire scaled parameter space and fitted into a simple formula. Similar procedure can be carried out in parallel for the higher order modes, thus allowing the criteria to be expressed through a simple formula as well.

2 Variational Solution for E_{10} Mode

The formalism and notations used in this article follow closely that introduced in [1] wherever relevant. In particular, we seek variational solutions of higher order mode, E_{nm} , of the following scaled eigenmode equation [1]

$$\begin{aligned} & \left[2\eta_d \left[\frac{d}{RdR} \left(R \frac{d}{dR} \right) - \frac{m^2}{R^2} \right] + iq \right] E_m(R) \\ & = \int_0^\infty R' dR' G_m(R, R') E_m(R') , \end{aligned} \tag{1}$$

where

$$G_m(R, R') = \int_{-\infty}^0 \frac{\tau d\tau i^{-m} h}{\sin^2(2\sqrt{\eta_d \eta_\epsilon} \tau)} e^U J_m(V) ,$$

$$U = (q - i\eta_\omega)\tau - 2\eta_\gamma^2 \tau^2 - \frac{(1 + i\eta_\epsilon \tau)(R^2 + R'^2)}{2 \sin^2(2\sqrt{\eta_d \eta_\epsilon} \tau)} ,$$

$$V = \frac{i(1 + i\eta_\epsilon \tau) \cos(2\sqrt{\eta_d \eta_\epsilon} \tau) R R'}{\sin^2(2\sqrt{\eta_d \eta_\epsilon} \tau)} .$$

In addition to the complex eigenvalue q and a constant h , there are four scaling parameters in Eq.(1): η_d is a diffraction parameter; η_ϵ and η_γ characterize the effective spread in longitudinal velocity due to emittance and betatron focusing and due to energy spread, respectively; and η_ω is a frequency detuning parameter.

Following the same procedure as for E_{00} mode [1,6], solution for E_{10} mode can be obtained by setting $m = 0$ and substituting into Eq.(1) a trial solution of the form

$$E_{10}(R) = (1 - \beta R^2) e^{-\alpha R^2} , \quad (2)$$

where α and β are complex variational parameters to be determined by the variational conditions: $\delta q / \delta \alpha = 0$ and $\delta q / \delta \beta = 0$. Hence

$$F_1(q, \alpha, \beta) \equiv iq \left(\frac{1}{4\alpha} - \frac{\beta}{4\alpha^2} + \frac{\beta^2}{8\alpha^3} \right) - \eta_d \left(1 + \frac{\beta^2}{2\alpha^2} \right) - \int_{-\infty}^0 \tau d\tau h e^{f_1} \mathcal{F}_1 = 0 , \quad (3)$$

$$F_2(q, \alpha, \beta) \equiv \frac{\partial F_1}{\partial \alpha} = -iq \left(\frac{1}{4\alpha^2} - \frac{\beta}{2\alpha^3} + \frac{3\beta^2}{8\alpha^4} \right) + \frac{\eta_d \beta^2}{\alpha^3} + \int_{-\infty}^0 \tau d\tau h e^{f_1} \mathcal{F}_2 = 0 , \quad (4)$$

$$F_3(q, \alpha, \beta) \equiv \frac{\partial F_1}{\partial \beta} = -iq \left(\frac{1}{4\alpha^2} - \frac{\beta}{4\alpha^3} \right) - \frac{\eta_d \beta}{\alpha^2} + \int_{-\infty}^0 \tau d\tau h e^{f_1} \mathcal{F}_3 = 0 , \quad (5)$$

where

$$\begin{aligned}
\mathcal{F}_1 &= \frac{f_2^2 - \beta f_2 f_3 + \beta^2(f_4 + 2f_5)}{f_2^3} , \\
\mathcal{F}_2 &= \frac{f_2^2 f_3 - 2\beta f_2(f_3^2 - f_4) + 2\beta^2 f_3(f_4 + 3f_5)}{f_2^4} , \\
\mathcal{F}_3 &= \frac{f_2 f_3 - 2\beta(f_4 + 2f_5)}{f_2^3} ,
\end{aligned}$$

and

$$\begin{aligned}
f_1 &= (q - i\eta_\omega)\tau - 2\eta_\gamma^2 \tau^2 , \\
f_2 &= (1 + i\eta_\epsilon \tau)^2 + 4\alpha(1 + i\eta_\epsilon \tau) + 4\alpha^2 \sin^2(2\sqrt{\eta_d \eta_\epsilon} \tau) , \\
f_3 &= 4(1 + i\eta_\epsilon \tau) + 8\alpha \sin^2(2\sqrt{\eta_d \eta_\epsilon} \tau) , \\
f_4 &= 4f_2 \sin^2(2\sqrt{\eta_d \eta_\epsilon} \tau) , \\
f_5 &= 4(1 + i\eta_\epsilon \tau)^2 \cos^2(2\sqrt{\eta_d \eta_\epsilon} \tau) .
\end{aligned}$$

In the 1D limit with $\eta_d = 0$, Eqs (3,4,5) become

$$q + ih \int_{-\infty}^0 \frac{\tau d\tau e^{f_1}}{1 + i\eta_\epsilon \tau} = 0 , \quad (6)$$

$$\alpha_s = \frac{1}{4} \sqrt{\frac{h \int_{-\infty}^0 \tau d\tau e^{f_1}}{\eta_\epsilon h \int_{-\infty}^0 \tau^3 d\tau e^{f_1} / (1 + i\eta_\epsilon \tau)^2 - 1}} , \quad (7)$$

and $\beta_s = 2\alpha_s$, where in general β_s is defined by $\beta = \beta_s(\eta_d, \eta_\epsilon, \eta_\gamma, \eta_\omega) / \sqrt{\eta_d}$. For the same set of LCLS parameters used in [1], $\eta_d = 0.0367$, $\eta_\epsilon = 0.739$ and $\eta_\gamma = 0.248$, Eqs.(3,4,5) yield $q_{10} = 0.124 - i0.0326$, $\alpha_{s10} = 0.0792 - i0.0999$ and $\beta_{s10} = 0.164 - i0.232$, optimized at $\eta_\omega = -1.52$. In comparison, the exact solution is $q_{10} = 0.125 - i0.0245$, optimized at $\eta_\omega = -1.52$.

3 Variational Solution for E_{01} Mode

For E_{01} mode, set $m = 1$ in Eq.(1) and chose a trial solution of the form

$$E_{01}(R) = R e^{-\alpha R^2} . \quad (8)$$

Hence

$$F_1(q, \alpha) \equiv \frac{iq}{8\alpha^2} - \frac{\eta_d}{\alpha} - \int_{-\infty}^0 \tau d\tau h \frac{f_3 e^{f_1}}{f_2^2} = 0 , \quad (9)$$

$$F_2(q, \alpha) \equiv \frac{\partial F_1}{\partial \alpha} = -\frac{iq}{4\alpha^3} + \frac{\eta_d}{\alpha^2} + \int_{-\infty}^0 \tau d\tau h \frac{f_4 e^{f_1}}{f_2^3} = 0 , \quad (10)$$

where

$$\begin{aligned} f_1 &= (q - i\eta_\omega)\tau - 2\eta_\gamma^2 \tau^2 , \\ f_2 &= (1 + i\eta_\epsilon \tau)^2 + 4\alpha(1 + i\eta_\epsilon \tau) + 4\alpha^2 \sin^2(2\sqrt{\eta_d \eta_\epsilon} \tau) , \\ f_3 &= 2(1 + i\eta_\epsilon \tau) \cos(2\sqrt{\eta_d \eta_\epsilon} \tau) , \\ f_4 &= 8f_3[(1 + i\eta_\epsilon \tau) + 2\alpha \sin^2(2\sqrt{\eta_d \eta_\epsilon} \tau)] . \end{aligned}$$

In the 1D limit, Eqs.(9,10) reduce to Eqs.(6,7). For the LCLS case, Eqs.(9,10) give $q_{01} = 0.298 + i0.0651$ and $\alpha_{s01} = 0.0869 - i0.109$, optimized at $\eta_\omega = -1.40$. In comparison, the exact solution is $q_{01} = 0.297 + i0.0662$ at $\eta_\omega = -1.40$.

4 Criteria on Transverse Coherence of SASE

Generally speaking, the evolution of optical field along a wiggler in a high gain FEL amplifier can be completely determined before saturation by an expansion over the eigenmodes of the system as follows:

$$E(R, \phi, z) = \sum_{n,m} C_{nm} E_{nm}(R) \exp \left(\frac{q_{nm} z}{2L_{1d}} + im\phi \right) . \quad (11)$$

Here in Eq.(11), a term of integration over a continuous spectrum of modes is neglected from the right hand side, as it is significant only in the initial section of the wiggler known as the lethargy region [7–9]. The eigenmode expansion requires, first of all, a solution of eigenvalue problem which determines $\{q_{nm}, E_{nm}\}$ of the system, and secondly, a solution of initial value problem which determines C_{nm} in terms of initial condition. In the case of SASE, the expansion coefficients are generally random variables due to the stochastic nature of mode excitation by spontaneous emission. As a result, the relative phases between different modes in the expansion are random as well, giving rise to a stochastic field profile with poor transverse coherence. However, under certain circumstances, the growth rate of the fundamental mode can be so high above all other modes such that at the end of linear amplification the single mode is dominant. In this case, full transverse coherence is expected as the field profile is given entirely by the fundamental mode, regardless of initial

random mode excitation. Therefore, the condition on full transverse coherence of SASE is same as the condition on the dominance of the fundamental mode. With the newly derived solutions of higher order eigenmodes, this statement can now be further quantified into a criteria.

Let's consider three low order modes and denote for simplicity, $E_1 = E_{00}$, $E_2 = E_{10}$, $E_3 = E_{01}$, in addition, $L_1 = L_g^{00}$, $L_2 = L_g^{10}$, $L_3 = L_g^{01}$. The total power at saturation length ($z = L_{sat}$) can be expressed as

$$P = P_1 \exp\left(\frac{L_{sat}}{L_1}\right) + P_2 \exp\left(\frac{L_{sat}}{L_2}\right) + P_3 \exp\left(\frac{L_{sat}}{L_3}\right) + P_{12} \exp\left(\frac{L_{sat}}{2L_1} + \frac{L_{sat}}{2L_2}\right) + \dots \quad (12)$$

where only four out of six terms from the three modes are kept, since $P_{13} = P_{23} = 0$ due to orthogonality between modes of different azimuthal mode index. The modes with the same azimuthal mode index are not power orthogonal to each others, thus, $P_{12} \neq 0$. A criteria may be constructed by taking the ratio of the sum of the last three terms to the first one on the right hand side of Eq.(12), and requiring the ratio to be much less than unity for full transverse coherence. However, the quantities P_1, P_2, P_3 and P_{12} can not be calculated due to the lack of a solution to the initial value problem other than in the simpler case of parallel beam with $\eta_e = 0$ [3,7–11]. Nevertheless, one may impose a reasonable assumption, $P_1 \geq P_2, P_3, P_{12}$, implying that the spontaneous emission excites preferably the fundamental mode which has a better overlap with the electron beam. As a result, a stronger condition which tends to overestimate power in higher order modes may be formulated by defining

$$\Delta = \exp\left(\frac{L_{sat}}{L_3} - \frac{L_{sat}}{L_1}\right) + \exp\left(\frac{L_{sat}}{2L_2} - \frac{L_{sat}}{2L_1}\right), \quad (13)$$

and requiring $\Delta \ll 1$ as a criteria for full transverse coherence of SASE. A term due to P_2 is neglected from Eq.(13) for it has a smaller exponential factor than the term due to P_{12} .

For the LCLS case, $L_g^{00} = 6.04\text{m}$, $L_g^{10} = 23.7\text{m}$, $L_g^{01} = 9.97\text{m}$ and $L_{sat} = 110\text{m}$, where approximately, $L_{sat} = L_g^{00} \ln(P_{sat}/\alpha_c P_n)$, $P_{sat} \approx 1.6\rho(L_{1d}/L_g^{00})^2 P_{beam}$, $\alpha_c = 1/9$, $P_n \approx \gamma_0 m c^3 \rho^2 / \lambda_r$ and P_{beam} is the beam power [5,12]. In this case, $\Delta = 0.0019$, the fundamental mode is still dominant, though not overwhelmingly. This result is remarkable in the sense that, with $\epsilon = 4.5\lambda_r/4\pi$, the usual criteria for diffraction limited undulator radiation is violated by a large factor, and with $\sqrt{2}a_w = 3.7$, the initial spontaneous radiation is generated in the wiggler instead of undulator regime, as such the transverse coherence of the initial seed radiation is expected to be further degraded [2].

5 Conclusions

A universal criteria, Eq.(13), on transverse coherence of SASE is proposed. It contains all the essential physics of FEL interaction in the high gain regime, including the effects due to energy spread, emittance and betatron focusing of the electron beam, as well as diffraction and optical guiding of the radiation field. As such, it can be used in system design and optimization over a broad parameter space. The new criteria can also be used to elucidate the intrinsic difference between undulator radiation from uncorrelated radiators and coherent amplification in SASE, despite the fact that the former initiated the latter. In particular, the role of emittance can be different for the two as it is shown that the emittance criteria valid for undulator radiation can be violated for SASE by a large factor. Due to the lack of a solution to the initial value problem, the criteria is approximate in the sense that its prediction on the degree of transverse coherence is on the pessimistic side for it has assumed an upper bound estimate of power in the higher order modes. Nevertheless, it has captured the crucial physics in the process of coherence enhancement and power amplification, as revealed by the growth rates of different eigenmodes. In addition, the special variational technique [1,6] is applied to higher order modes and the solutions found to be highly accurate. Based on these efficient and robust solutions, the growth rates for higher order eigenmodes can be interpolated into simple formulas with full dependence on all scaled parameters. A full account of this work to be published elsewhere will include an explicit form of the criteria expressed through fitting formulas for the growth rates of the eigenmodes and exploration of the criteria and its implications for FEL systems in conjunction with other major design considerations.

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