Scattering Physics

\[ Q = \frac{4\pi \sin(\theta)}{\lambda} \]
Lensless Imaging

Sample Space \rightarrow Angular Space

Q = 4\pi \sin(\theta) / \lambda

Scattering Pattern
Lensless Imaging

Fourier Transform

\[ Q = \frac{4\pi \sin(\theta)}{\lambda} \]

Sample Space → Angular Space
Fourier Transform Recap
Fourier Transform Recap

- 1: $\text{FT} (\text{FT} (S)) \sim S$
Lensless Imaging

Sample Space → Angular Space → Image Space

Fourier Transform

Q = \frac{4\pi \sin(\theta)}{\lambda}
Fourier Transform Recap

• 1: $\text{FT (FT (S))} \sim S$

• 2: $\text{FT (large)} \sim 1/\text{large} \Rightarrow \text{small}$
Fourier Transform Recap

1: \( \text{FT} (\text{FT} (S)) \sim S \)

2: \( \text{FT} \text{ (large)} \sim \frac{1}{\text{large}} \rightarrow \text{small} \)

3: \( \text{FT} \text{ (periodic fn)} \sim \text{periodic} \)
Fourier Transform Recap

1. $\text{FT (FT (S))} \sim S$

2. $\text{FT (large)} \sim 1/\text{large} \Rightarrow \text{small}$

3. $\text{FT (periodic fn)} \sim \text{periodic}$

4. Convolution Theorem:
   - $\text{FT (a multiply b)} =$
     - $\text{FT (a conv FT (b))}$
   - $\text{FT (a conv b)} =$
     - $\text{FT (a) multiply FT (b)}$
FT (a multiply b) = FT (a) conv FT (b)
FT (a conv b) = FT (a) mult FT (b)
Deconstructing the Sample space

Sample = S x P * M

Sample size (S) =

Infinite Periodic Lattice (P)

Motif (M)
$\text{FT(S)}$
FT(P)
\[ \text{FT} (S \times P) = \text{FT}(S) \times \text{FT}(P) \]
$$FT(\text{sample}) = FT(S \times P) \times FT(M)$$

Along X direction
What does a diffraction pattern tell us?

- **Peak Shape & Width:**
  - Zero order peak
  - Higher order peaks
    - Crystallite size
    - Strain gradient
  
- **Peak Positions:**
  - Phase identification
  - Lattice symmetry
  - Lattice strain

- **Peak Intensity:**
  - Atom positions

Small Angle Scattering & Reflectivity

Wide Angle Scattering
1: \( \text{FT} \left( \text{FT} \left( S \right) \right) \sim S \)

2: Scattering Pattern (large) \( \sim \frac{1}{\text{large}} \rightarrow \) small features

3: Scattering Pattern (periodic lattice) \( \sim \) periodic
Diffraction Physics

\[ E(\Delta K = (s-s0)) = \sum_{i} e^{i \omega t} \sum_{f_i} e^{i (r_i \cdot Q)} \]

\[ E(\Delta K) = \sum_{f_i} e^{i (r_i \cdot Q)} \]

Phase difference

\[ 2\pi i (r_1 \sin(2\theta)/\lambda) \]

\[ 2\pi i (r_2 \sin(2\theta)/\lambda) \]

\[ \Delta K = Q = \frac{4\pi \sin(\theta)}{\lambda} \]

\[ E = \frac{q^2 E_0}{mc^2 R} e^{i[k_0 \cdot R - \vec{Q} \cdot \vec{d}] - \omega t} \]

From Kevin’s Talk

\[ E(Q) = \text{Fourier Transform (X-ray density)} \]